Introduction to Manifolds

Assignment 7
Due Date: 16/11/2017

Problem 1: If V is a finite dimensional vector space and $a \in \bigwedge^k(V)$ and k is odd, then show that $a \wedge a = 0$.

Problem 2: Calculate $a \wedge b$ in the following cases.

1. $a = b = v_1 \wedge v_2 + v_2 \wedge v_3 + v_3 \wedge v_1$.
2. $a = v_1 \wedge v_2 + v_3 \wedge v_1 \quad b = v_2 \wedge v_3 \wedge v_4$.
3. $a = v_1 + v_2 + v_3 \quad b = v_1 \wedge v_2 + v_2 \wedge v_3 + v_3 \wedge v_1$.

Problem 3: Which of the following 2-tensors is decomposable? (recall that a tensor is decomposable if it can be expressed as wedge of two tensors of lower rank.)

1. $v_1 \wedge v_2 + v_2 \wedge v_3$.
2. $v_1 \wedge v_2 + v_2 \wedge v_3 + v_3 \wedge v_1$.
3. $v_1 \wedge v_2 + v_2 \wedge v_3 + v_3 \wedge v_4$.

Problem 4: If $x = r\cos\theta$ and $y = r\sin\theta$, calculate $\mathrm{d}x$, $\mathrm{d}y$ and $\mathrm{d}x \wedge \mathrm{d}y$.

Problem 5: Denote the standard coordinates on $\mathbb{R}^2$ by x, y, and let

$$X = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y} \quad \text{and} \quad x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$$

be two vector fields on $\mathbb{R}^2$. Find a 1-form ω on $\mathbb{R}^2 \setminus \{(0,0)\}$ such that $\omega(X) = 1$ and $\omega(Y) = 0$.