Reflection Groups

Homework 5

Due date: 15/04/2013

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1. Let W be a finite reducible Coxeter group with $\mathcal{G}$ as its Coxeter graph. Let $\mathcal{G}_1, \dots, \mathcal{G}_k$, for $k \geq 2$, denote the connected components of $\mathcal{G}$. Finally, let W_j denote the parabolic subgroup of W generated by the simple reflections corresponding to the nodes of $\mathcal{G}_j$ for $1 \leq j \leq k$. Then prove that
$$W \cong W_1 \times \cdots \times W_k.$$

2. Prove that the following Coxeter graphs are positive definite by explicitly calculating the determinant of the associated real, symmetric matrix $2G$. Graphs of type $A_n (n \geq 1)$, of type $BC_n (n \geq 2)$, of type $D_n (n \geq 4)$ and of the type E_6.

3. Prove that the graphs of type $\tilde{A}_n (n \geq 2), \tilde{C}_n (n \geq 3), \tilde{D}_n (n \geq 4)$ and of the type $\tilde{E}_8$ are positive semidefinite by showing that the determinant of the associated real, symmetric matrix is zero.

4. Prove that a labelled subgraph of a positive definite is itself positive definite.

5. Consider the reflection group of type BC_3 whose Coxeter presentation is the following:
$$W = \langle r_1, r_2, r_3 \mid r_1^2 = r_2^2 = r_3^2 = (r_1 r_2)^4 = (r_2 r_3)^3 = 1 \rangle.$$
Find a reduced expression for the following word in W -
$$r_1 r_2 r_3 r_2 r_3 r_2 r_1 r_2 r_3 r_2 r_3 r_2 r_3 r_1 r_3 r_2 r_3 r_2 r_3 r_2.$$
Can you identify the above element, concretely, as a signed permutation?