1. Prove that the reflection groups of type $A_3$ and $D_3$ are isomorphic by exhibiting an explicit isometry between the corresponding root systems.

2. Draw the hyperplane arrangement corresponding to the root system $BC_2$ and label its chambers by the elements in the dihedral group of order 8.

Use the following information for the remaining problems. $\Phi$ is a root system in $\mathbb{R}^n$, $\Sigma$ is the corresponding arrangement of reflecting hyperplanes, $W$ is the group generated by reflections which acts transitively on $\mathcal{C}$, the chambers of the reflection arrangement. For two chambers $C$ and $D$ let $S(C, D)$ denote the set of all hyperplanes in $\Sigma$ that separate them (i.e., the set of all those hyperplanes such that $C$ and $D$ lie on their opposite sides). Recall that for a chamber $C$ the chamber opposite to it is denoted by $-C$.

3 Prove that for chambers $C_1, C_2, C_3$ the following is true:

$$S(C_1, C_3) = [S(C_1, C_2) \setminus S(C_2, C_3)] \cup [S(C_2, C_3) \setminus S(C_2, C_1)].$$

4 Given a face $F$ and a chamber $C$ of $\Sigma$ prove that there is a unique chamber $C_F$ satisfying the following:

- $F \subseteq \overline{C_F}$,
- $\text{gd}(C, C_F) = \min\{\text{gd}(C, D) \mid D \in \mathcal{C}, F \subseteq D\}$.

Moreover, conclude that if $F \subseteq \overline{C}$ then $(C_F)_G = C_G$ and that if $F \subseteq \overline{C}$ then $C_F = C$.

5 Prove that $\text{gd}(C, D) = |S(C, D)|$.

6 For a chamber $C$ prove the following assertions:

(a) $\text{gd}(C, -C) = \text{gd}(C, D) + \text{gd}(D, -C)$ for every $D \in \mathcal{C}$.

(b) $\text{gd}(C, -C) = |\Sigma|$.

(c) $\text{gd}(C, D) = \text{gd}(-C, -D)$ for every $D \in \mathcal{C}$. 

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