COMBINATORICS 1
ASSIGNMENT 3
(DUE DATE: 08/09/2016)

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(1) (5 points) Draw the Hasse diagrams of all (non-isomorphic) posets with 4 elements that are self-dual.

(2) (5 points) Characterize natural numbers N such that the lattice D_N is a Boolean algebra.

(3) (10 points) Let L be a lattice. Then show that for all $a, b, c \in L$:
  (a) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$ and $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$.
  (b) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.

(4) (5 points) An element c in a lattice L is a *relative pseudo complement* of a w.r.t. b, if c is the largest element such that $a \wedge c \leq b$ and if exists, it is denoted by $a \Rightarrow b$. A *Heyting algebra* is a lattice with $\hat{0}, \hat{1}$ such that $a \Rightarrow b$ exists for all $a, b \in L$. Then show that every finite distributive lattice is a Heyting algebra. Also, for a Boolean algebra B_n find an explicit description for the relative pseudo complement in terms of meets and/or joins.

(5) (5 points)Let L be the set of all convex subsets. Show that this is a lattice by defining appropriate binary operations and an order. Is this lattice distributive? Justify.

(6) (5 points)If L is a lattice satisfying

$$(a \vee b) \wedge c \leq (a \wedge c) \vee (b \wedge c),$$

for all a, b, c then show that L is distributive.

(7) (5 points)Show that a Heyting algebra is a distributive lattice.