- The deadline for submitting your solutions is 24:00 hrs on September 23, 2022. Submit handwritten solutions to either Prajakta or Nithin. If you cannot find us, slide the solutions under our office doors.
- Write down your name and CMI ID at the top right corner of the first page of your solutions sheet.
- Collaboration on homeworks is encouraged. However, you must write down the solutions on your own and list the names of all your collaborators on the first page of your solution sheet. If you have no collaborators, you must write "Collaborators: None". You will lose 0.5 points per problem for not listing your collaborators.
- CMI's academic honesty policy regarding cheating and plagiarism applies to this homework.

1. In this problem, you will derive a lower bound on the randomized private coin 2-sided error one-way communication complexity of the greater than function GT: $[2^n] \times [2^n] \to \{0,1\}$ defined as $\mathrm{GT}(x,y) = 1$ if and only if $x > y$. Throughout, the one-way communication is assumed to be from Alice to Bob.
   - Given a randomized private coin 2-sided error one-way protocol for GT with error at most $1/3$ and communication complexity R, describe another protocol with error improved to $n^{-\Omega(\log n)}$.
   - Let $x \in [2^n]$ denote Alice's input. Argue that for every set $\{y_1, y_2, \ldots y_n\} \subseteq [2^n]$, Alice and Bob can compute (in the same communication model), with probability at least $1 - n^{-\Omega(\log n)}$, $\mathrm{GT}(x, y_i)$ for all $i \in [n]$.
   - Prove that Bob can determine the exact value of Alice's input x using $O(R \log n)$ bits of communication from Alice to Bob.
   - Argue that $R = \Omega(n/\log n)$. *Hint: You might want to use the lower bound on the one-way communication complexity of the INDEX problem.*
2. Consider a stream of elements $x_1, \ldots x_m \in [n]$. Let f_j denote the number of times $j \in [n]$ occurs in the stream. We define F_∞ to be equal to $\max_{j \in [n]} f_j$. Show that any streaming algorithm that, with probability at least $2/3$. estimates F_∞ within a factor of 1 ± 0.2 requires $\Omega(\min\{m, n\})$ space. *Hint: Show that you can solve DISJOINTNESS using $s+1$ bits of communication if you have a streaming algorithm for the above problem that uses at most s bits of space.*