

DMATH

Practice problems

April 23, 2021

Question 1.

$$\begin{aligned} a_n &= 4(a_{n-1} + 1) \\ &= 4^2 a_{n-2} + 4 + 1 \\ &\vdots \\ &= 4^k a_{n-k} + 4^{k-1} + \dots + 1 \\ &= 4^n a_0 + \frac{4^n - 1}{3} \end{aligned} \quad (\text{for } k = n - 1)$$

Now verify by induction

Question 2.

to find the accurate recursion observe for $af(n) + bf(n-1) + cf(n-2) = 0$ if α, β are the real roots of the polynomial $p(x) = ax^2 + bx + c$ then $\forall p, q f(n) = p\alpha^n + q\beta^n$ will satisfy the recursion

Now solve for p, q using the value of $f(0), f(1)$

Question 3.

observe whenever you divide a square into four parts you are removing one as well. Hence number of squares is increased by 3

Question 4.

Let p be the AM of a_1, \dots, a_n

If each a_i is equal with p then the equality holds

otherwise WLOG $a_n > p, a_{n-1} < p$

Take $b = a_n + a_{n-1} - p$

observe p is still AM of $a_1, \dots, a_{n-2}, b \implies p^{n-1} \geq a_1 \cdot a_2 \dots a_{n-2} b$

Now prove $bp \geq a_n a_{n-1}$

Now the following questions I have tried to discuss in the tutorial.

1. Any multivariate polynomial in \mathbb{F}_2 has a equivalent multilinear polynomial in \mathbb{F}_2 .

[**Hint:** observe that in \mathbb{F}_2 $p(x, y) = (1 - y)p(x, 0) + yp(x, 1)$]

2. any Boolean expression in the variables x_1, \dots, x_n can be written in conjunctive normal form, using the variables x_1, \dots, x_n and their negations $\neg x_i$

[**Hint:** Try to use the idea of hint in the previous problem]