

CHENNAI MATHEMATICAL INSTITUTE

Discrete Mathematics

Quiz 3:

Date: June 25, 2021.

Each question is 10 marks.

- (1) Let (P, \leq) be a poset and let ζ denote the zeta function of P , defined as $\zeta(x, y) = 1$ if and only if $x \leq y$. Let δ denote the Kronecker delta function, $\delta(x, y) = 1$ if and only if $x = y$. For $x \leq y$, a chain of length k from x to y is a sequence of the form $x = x_0 < x_1 < x_2 \cdots < x_{k-1} < x_k = y$ with $x_i \in P$ (note, the inequalities here are strict).

- i Show that the number of distinct chains of length k from x to y is $(\zeta - \delta)^k(x, y)$. 4 marks
- ii Denote by c_i the number of distinct chains of length i from x to y . Prove that

$$\mu(x, y) = c_0 - c_1 + c_2 \cdots + (-1)^k c_k + \cdots$$

4 marks

- (2) Let G be a graph on n vertices having m edges. Let S be a random subset of vertices of $[n]$ picked as follows - for $i \in [n]$ toss a coin - put vertex i in S if and only if you see heads. The coin tosses are independent and the probability that the coin shows heads is p with $0 < p < 1$.

- i What is $\mathbb{E}(|S|)$? 4 marks
- ii Let Y be the random variable denoting the number of edges of G both of whose end points are in S . What is $\mathbb{E}(Y)$? 4 marks
- iii Draw the sample space and the value of Y on each sample point when G which is a triangle on three vertices. 4 marks