# CHENNAI MATHEMATICAL INSTITUTE

## Discrete Mathematics

Quiz 1:
Date: June 07, 2021.
Each question is 5 marks.

(1) In how many different ways can you arrange the elements of the multiset $\{1, 2, 2, 3, 3, 3, 4, \ldots, n, n, n, \ldots, n\}$, where each $i$ appears exactly $i$ times, in a line?

(2) Give a bijective proof for the identity given below. First guess what the right hand side is counting. You may want to use problem 1 above.

$$\binom{n}{1} + (6 + 8)\binom{n}{2} + 36\binom{n}{3} + 24\binom{n}{4} = n^4$$

(3) We represent a permutation of the set $[n]$ in one line notation as $\pi_1 \pi_2 \ldots \pi_n$. For example $4312$ represents the permutation sending $1, 2, 3, 4$ to $4, 3, 1$ and $2$ respectively. We know what a decreasing subsequence in $\pi_1 \pi_2 \ldots \ldots \pi_n$ is. Show that the number of permutations of $[n]$ with no decreasing subsequence of length three or more is the Catalan number. Show this using a bijection. Do prove that the functions you construct actually give you a bijection and indicate clearly where you use the condition of no decreasing subsequence of length three or more.