CHENNAI MATHEMATICAL INSTITUTE

Discrete Mathematics

Quiz 1:
Date: May 21, 2021.
Each question carries 5 marks.

(1) State Turan's theorem. Let $G = (V, E)$ be a simple undirected graph on n vertices. We define the complement graph of G to be the graph $\bar{G} = (V, \bar{E})$ with vertex set V and whose edge set is the complement of E. So $(x, y) \in \bar{E}$ if and only if $(x, y) \notin E$. If G has a clique of size at least t, what can you say about independent sets in $\bar{G}$. Using Turan's theorem show that if G has exactly $\frac{nk}{2}$ edges then G has an independent set of size at least $n/(k+1)$.

(2) State Dilworth's theorem. Let $0 < a_1 < a_2 < \ldots < a_{rs+1}$ be $rs+1$ distinct integers. Using Dilworth's theorem show that we can select $r+1$ of them each dividing the following one, or $s+1$ of them none of which divides the other.

(3) Show that if you select any $n+1$ distinct numbers among $\{1, 2, \ldots, 2n\}$ there will be a pair of numbers which add up to $2n+1$.

(4) Guess the pattern for the sum of the following expression, and prove it by induction.

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \cdots + \frac{1}{(n) \cdot (n+1) \cdot (n+2)}$$