DAA Problem Set 1

April 30 2021

1. Consider the following three functions. *The input integers are always assumed to be non-negative for this problem.* In these functions, `pair` and `int` are used as data types (as you might know from python). `pair p` is to be thought of as a 2-tuuple `(x,y)` where both `x,y` are of `int` type. `fst` and `snd` denote the first and second components, respectively.

```
function f1(pair p):
    x = p.fst
    y = p.snd
    return (x+1,x)
```

```
function f2(int n):
    p = (0,0)
    for i = 1 to n:
        p = f1(p)
    ans = p.snd
    return ans
```

```
function f3(int x, int y):
    if y == 0:
        return x
    else:
        x1 = f2(x)
        y1 = f2(y)
        return f3(x1,y1)
```

   (a) Describe in simple (high-school) math terms what the function `f2` does. Your answer should look something like "`f2(n)` gives the square of `n`" or "`f2(n)` gives `-n` if `n` is even and `n` if `n` is odd".

   (b) Describe in a similar way (as above) what the function `f3` does.

   (c) Let $T(n)$ be the *number* of 'basic steps' executed by the computer when you call `f2(n)`. Is $T(n)$ in $\mathcal{O}(n)$? In $\Omega(n)$? In $\Theta(n)$?

2. Consider the following functions. Each of them take two inputs $\texttt{a} \geq 1$ and $\texttt{n} \geq 0$.

```
function f4(int a, int n):
    x = 1
    for i = 1 to n:
        x = x * a
    return x
```

```
function f5(int a, int n):
    if n == 0:
        return 1
    if n == 1:
        return a
    b = f4(a, n/2)
    if (n % 2) == 0:
        return (b * b)
    return (b * b * a)
```

   In the above, `n % 2` is the least non-negative remainder obtained upon division of `n` by `2`, and `n/2` is the floor value of half of `n`.

   (a) Show that $\texttt{a}^{\texttt{n}} = \texttt{f4(a,n)} = \texttt{f5(a,n)}$.

   (b) Let $\texttt{a} \geq 1$ be a fixed integer. Let $T_1(n), T_2(n)$ denote the number of basic steps involved in the execution of `f4(a,n)` and `f5(a,n)` respectively.