

You have  $n$  objects with weights  $w_1, \dots, w_n$  and values  $v_1, \dots, v_n$ .  
 You have a sack which can carry at most wt.  $W$ .

$$\hookrightarrow P(W, w[1..n], v[1..n])$$

The  $n^{\text{th}}$  object may or may not be present:

$$n^{\text{th}} \text{ object not present: } P(W, w[1..n-1], v[1..n-1])$$

$$n^{\text{th}} \text{ object present: } P(W - w[n], w[1..n-1], v[1..n-1])$$

Recursion:  $dp(W, w[1..n], v[1..n]) \rightarrow$  optimal value

$$dp(W, w[1..n], v[1..n]) = \max \left\{ dp(W, w[1..n-1], v[1..n-1]), v[n] + dp(W - w[n], w[1..n-1], v[1..n-1]) \right\}$$

Now memoize:

$dp[\text{max weight } W][k] \rightarrow$  if  $W$ -sack allowed then what is the max value I can get out of the first  $k$  objects.

weights  $\rightarrow$

	0	1	2	3	...
0	0	0	0	0	...
1	0	□			
2	0				
3	0				
⋮	⋮				
⋮	⋮				

Either ignore the first obj ( $\rightarrow 0$ )  
 or put the first object if you can  
 $w[i] \leq 1$

You are given a string. Find <sup>the length</sup> of a subsequence of max length which is palindromic.  
 (not necessarily contiguous)

→ "abca" ans: "aba"

$dp[i][j]$  = answer for string  $[i..j]$

Transition:

$dp[i][j] =$

if  $(str[i] = str[j])$  then  $2 + dp[i+1][j-1]$

else  $\max \{ dp[i+1][j], dp[i][j-1] \}$

abca:  
1 2 3 4

	j →			
i ↓	1	2	3	4
1	1	1	1	1
2	0	1	1	1
3	0	0	1	1
4	0	0	0	1

Given a list  $L$  of  $n$  non-neg int, and an integer  $x$ , determine if there is a subset with sum  $x$ .  
 (multiset)

$dp[i][j]$  → Is there a subset of  $L[1..i]$  which sums to  $j$ ? (T/F)

$dp[i][j] = dp[i-1][j]$  OR  $dp[i-1][j - L[i]]$

check if  $\geq 0$   
 (if  $< 0$  then just  $dp[i-1][j]$ )