Extremal Surfaces, de Sitter Entropy and Entanglement in Ghost Theories

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- de Sitter space and dS/CFT
- Extremal surfaces and de Sitter entropy
- Ghost CFTs, “ghost-spins” and entanglement

1904.01223 (KN), *Honorable Mention, Gravity Research Foundation Essay Awards 2019*

[Refs: arXiv:1711.01107 (KN), work in progress;
also ’18 (Dileep Jatkar, Kedar Kolekar, KN), ’17, ’16 (DJ, KN), ’16, ’15 (KN)]

Partially related refs: Arias, Diaz, Sundell; Miyaji, Takayanagi; Sato; Dong, Silverstein, Torroba

Holography, de Sitter space, dS/CFT

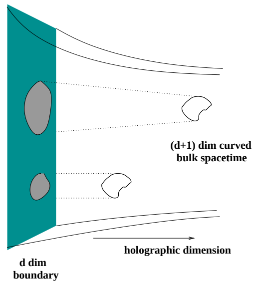
20^+ yrs since AdS/CFT '97 Maldacena; '98 Gubser,Klebanov,Polyakov; Witten.

Holography: quantum gravity in $\mathcal{M} \leftrightarrow$ dual without gravity on $\partial\mathcal{M}$ ('t Hooft, Susskind).

(Witten@Strings'98, '01) Gauge/gravity duality and asymptotics —

$\Lambda < 0$: AdS $\rightarrow$ asymptotics at spatial infinity.

Dual: unitary Lorentzian CFT, includes time.



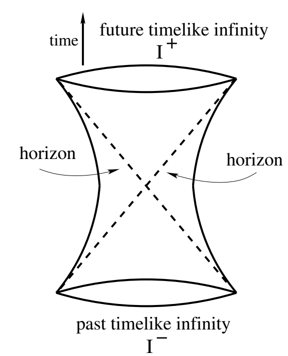
$\Lambda = 0$: flat space $\rightarrow$ null infinity $\rightarrow$ S-matrix, symmetries...

$\Lambda > 0$: de Sitter space

Fascinating for various reasons. Less clear.

Boundary at future/past timelike infinity $\mathcal{I}^{\pm}$.

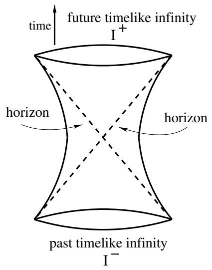
Dual $\rightarrow$ Euclidean CFT ...



[note: gravity dual of ordinary Euclidean CFT $\longrightarrow$ Euclidean AdS]

de Sitter space and dS/CFT

dS/CFT: dual Euclidean non-unitary CFT on dS boundary at future/past timelike infinity $\mathcal{I}^{\pm}$ ('01 Strominger; Witten).



$ds^2 = \frac{R_{dS}^2}{\tau^2}(-d\tau^2 + d\vec{x}^2)$

(Maldacena '02) analytic continuation $r \to -i\tau,\ R_{AdS} \to -iR_{dS}$ from Eucl AdS

$\rightarrow$ Hartle-Hawking wavefunction of the universe $\boxed{\Psi_{dS} = Z_{CFT}}$.

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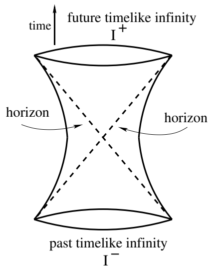

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EAdS interior regularity $\to$ Bunch-Davies dS initial conditions, $\varphi_k \sim e^{ik\tau}$.

$Z = \Psi[\varphi] \sim e^{iS_{cl}[\varphi]} \sim e^{-\int_k R_{dS}^2 k^3 \varphi_{-k}^0 \varphi_k^0 + \ldots}$ $\longrightarrow$ dual CFT: $\langle O_k O_{k'} \rangle \sim \frac{\delta^2 Z}{\delta\varphi_k^0 \delta\varphi_{k'}^0}$

$\underline{dS_4}$: Energy-momentum $\langle TT \rangle$ 2-pt fn $\to$ $\mathcal{C}_3 \sim -\frac{R_{dS}^2}{G_4}$, ghost-CFT?

Dual CFT central charge $\mathcal{C}_d \sim i^{1-d} \frac{R_{dS}^{d-1}}{G_{d+1}}$ negative/imaginary more generally.

Anninos,Hartman,Strominger: higher-spin dS_4 dual to $Sp(N)$ ghost $CFT_3, \ldots$

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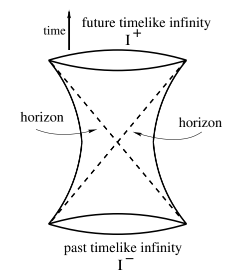

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Bulk expectation values $\langle \varphi_k \varphi_{k'} \rangle \sim \int D\varphi\ \varphi_k \varphi_{k'} |\Psi|^2$

Ψ^* and Ψ in bulk vevs $\rightarrow$ dual involves two CFT copies.

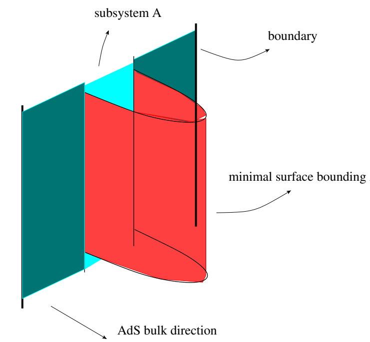
[In general $\Psi = \Psi[g^3]$, final 3-metric is g^3; sum over final boundary condns for bulk vevs.]

Entanglement as probe of dS/CFT?

Entanglement entropy: entropy of reduced density matrix of subsystem.

EE for spatial subsystem A, $S_A = -tr\rho_A \log \rho_A$, with partial trace $\rho_A = tr_B \rho$.

Ryu-Takayanagi: $EE = \frac{A_{min.surf.}}{4G}$



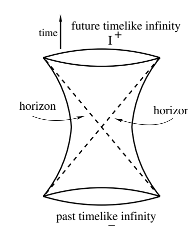
[$\sim$ black hole entropy] Area of codim-2 minimal surface in gravity dual.

Non-static situations: extremal surfaces (Hubeny, Rangamani, Takayanagi).
(Lewkowycz, Maldacena, . . .)

Operationally: const time slice, boundary subsystem $\rightarrow$ bulk slice, codim-2 extremal surface

A speculative generalization of Ryu-Takayanagi to de Sitter space
$\equiv$ bulk analog of setting up entanglement entropy in dual CFT $\rightarrow$
restrict to some boundary Eucl time slice $\rightarrow$ codim-2 dS surfaces.

de Sitter entropy as some sort of entanglement entropy?



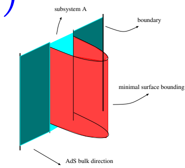
dS isometries $\Rightarrow$ all boundary Eucl time slices equivalent.

Entanglement entropy in ghost-like theories? Positive norm subsectors?

Extremal surfaces, de Sitter entropy

dS extremal surfaces (Poincare)

Ryu-Takayanagi: CFT ground state = empty AdS_{d+1}, $ds^2 = \frac{R^2}{r^2}(dr^2 - dt^2 + dx_i^2)$



$$S_A = \frac{V_{d-2} R^{d-1}}{4G_{d+1}} \int \frac{dr}{r^{d-1}} \sqrt{1 + (\partial_r x)^2} \quad \rightarrow \quad (\partial_r x)^2 = \frac{(r/r_*)^{2d-2}}{1-(r/r_*)^{2d-2}} \, .$$

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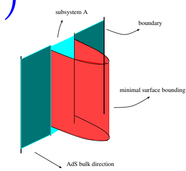

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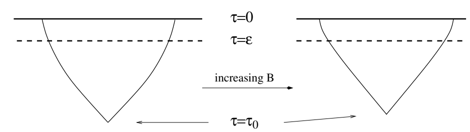
de Sitter, Poincare : $ds^2_{d+1} = \frac{R^2_{dS}}{\tau^2}(-d\tau^2 + dw^2 + dx_i^2)$ $\rightarrow$ EE in dual Eucl CFT

$\rightarrow$ bulk Eucl time slice $w = const$, subregion at I^+ $\rightarrow$ codim-2 extremal surface.

[strip] $S_{dS} = \frac{R^{d-1}_{dS} V_{d-2}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}} \sqrt{1 - (\partial_\tau x)^2}.$

$Extremize \rightarrow (\partial_\tau x)^2 = \frac{B^2 \tau^{2d-2}}{1+B^2\tau^{2d-2}}\ , \quad B^2 = const.$



Real surfaces: sign difference from AdS $\Rightarrow$ no real "turning point".

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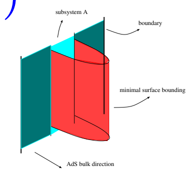

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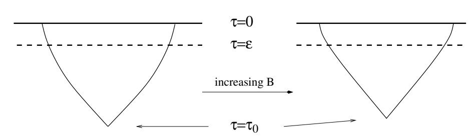

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Real surfaces: sign difference from AdS $\Rightarrow$ no real "turning point".

- $B^2 < 0$. Near $\tau \rightarrow 0$: $\dot{x}^2 \sim B^2\tau^4$ $\quad i.e.$ $x(\tau) \sim \pm i|B|\tau^3 + x(0)$ $\quad [dS_4]$.

$x(\tau)$ real $\Rightarrow$ imaginary path $\tau = iT \rightarrow$ complex extremal surface
$\equiv$ analytic continuation $r \rightarrow -i\tau, R \rightarrow -iR_{dS}$ from AdS Ryu-Takayanagi.

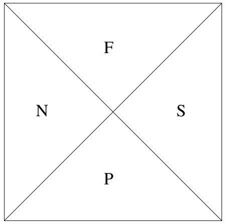
Complex surfaces: no canonical action $\rightarrow$ $S_{dS_4} \sim -\frac{R^2_{dS}}{G_4} V_1 (\frac{1}{\epsilon} - c\frac{1}{l})$ $\rightarrow$ negative area $[dS_4]$

(also spherical extr surfaces)

de Sitter, static coordinatization

dS_{d+1}: $ds^2 = -(1 - \frac{r^2}{l^2})dt^2 + \frac{dr^2}{1-\frac{r^2}{l^2}} + r^2 d\Omega_{d-1}^2$.

N, S $(0 \leq r < l)$: static patches. t is time $\rightarrow$ translations are symmetries. Event horizons for observers in N, S.



de Sitter entropy = area of cosmological horizon. (Gibbons,Hawking)

Euclidean continuation $t \rightarrow -it_E$ is sphere (no boundary): Eucl action $I_E = -\log Z = \beta F$.

de Sitter entropy $S_{dS_{d+1}} = -I_E = \frac{l^{d-1} V_{S^{d-1}}}{4G_{d+1}} \quad \rightarrow \quad \frac{\pi l^2}{G_4}$ $[dS_4]$.

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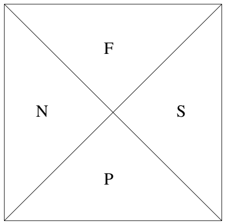

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$dS/CFT \rightarrow$ future/past universes $F, P \quad \rightarrow \quad \tau = \frac{l}{r},\ w = \frac{t}{l} \quad \rightarrow$

$$ds^2 = \frac{l^2}{\tau^2}\Big(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2\Big)$$

τ is bulk time now.

Real extremal surfaces stretching from I^+ to I^-?

· no real turning point earlier, surfaces do not return to I^+: maybe end at I^-?

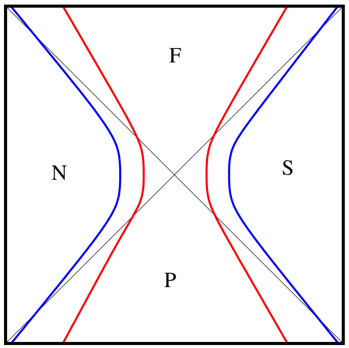
· bulk physics $\rightarrow \Psi^*\Psi \rightarrow$ two boundaries?

Extremal surfaces, de Sitter entropy

$ds^2 = \frac{l^2}{\tau^2}\big(-\frac{d\tau^2}{f} + f dw^2 + d\Omega_{d-1}^2\big), \quad [f = 1-\tau^2] \quad [\tau = \frac{l}{r}]$

Boundary Euclidean time slice $\longrightarrow$ codim-2 surfaces, area $\sim \frac{l^{d-1}}{G_{d+1}}$

S^{d-1}, all equatorial planes equivalent.



Area $S = l^{d-1} V_{S^{d-2}} \int \frac{d\tau}{\tau^{d-1}} \sqrt{1/f - f(w')^2}$

Extremize $\longrightarrow$ $\dot{w}^2 \equiv (1-\tau^2)^2 (w')^2 = \frac{B^2 \tau^{2d-2}}{1-\tau^2+B^2\tau^{2d-2}}$

B=const, $S = \frac{2 l^{d-1} V_{S^{d-2}}}{4 G_{d+1}} \int_\epsilon^{\tau_*} \frac{d\tau}{\tau^{d-1}} \frac{1}{\sqrt{1-\tau^2+B^2\tau^{2d-2}}}$

Hartman-Maldacena surfaces (AdS bh) rotated.

Turning point τ_* at $|\dot{w}| \to \infty$: $1 - \tau_*^2 + B^2 \tau_*^{2d-2} = 0.$

$\underline{dS_4}$: real τ_* for $0 < B < \frac{1}{2}$

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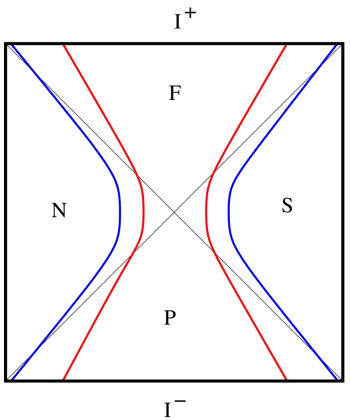

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Limiting surface as $B \to \frac{1}{2}$: $\tau_* \to \sqrt{2}$. Whole space, $\Delta w \to \infty$.

Real connected surfaces from I^+ to I^-: limiting surface as $\Delta w \to \infty$.
Area law divergence $\to$ $S^{div} = \frac{\pi l^2}{G_4} \frac{l}{\epsilon_c}$; Finite part (limiting surface) $S^{fin} \sim \frac{\pi l^2}{G_4} \Delta w$

Area coefficients scale as de Sitter entropy

$\to$ akin to the number of degrees of freedom in the dual CFT.

Entanglement in ghost systems; “Ghost-spins”

$dS_4/CFT_3 \rightarrow -ve$ central charge, ghost-like CFT.

$-ve$ norm states $\rightarrow\ EE < 0$?

$+ve$ norm, $+ve$ EE?

bc-ghosts, $c=-2$: replica and EE

$T(w) = (\partial_w z)^2 T(z) + \frac{c}{12}\{z,w\}$, Schwarzian $\{z,w\} = \frac{z'''}{z'} - \frac{3}{2}(\frac{z''}{z'})^2$ (Calabrese,Cardy)

Subsystem A: interval betw $x=u,v$; replica w-space $\rightarrow$ $z=(\frac{w-u}{w-v})^{1/n}$ $\rightarrow$ z-plane.

z-plane: $\langle T(z)\rangle_{\mathbb{C}} = 0$ $\rightarrow$ z-plane maps to $SL(2,Z)$ inv vacuum.

$\langle T(w)\rangle_{\mathcal{R}_n} = \frac{c}{12}\{z,w\} = \frac{\langle T(w)\Phi_n(u)\Phi_{-n}(v)\rangle}{\langle \Phi_n(u)\Phi_{-n}(v)\rangle}$ $\rightarrow$ $tr\rho_A^n \equiv \frac{Z_n}{Z_1^n} \sim$ twist op 2-pt fn.

- Replica argument is applicable for the ghost ground state if it is the $SL(2)$ vacuum: $c=-2$ bc-ghost CFT $\rightarrow$ $|\downarrow\rangle = |0\rangle$ with $L_0 = 0$.
- Regularity condition $\langle T(z)\rangle_{\mathbb{C}} = 0$ $\rightarrow$ $\langle -Q|\, T(z)\, |0\rangle = 0$

Incorporate background charge, or $\langle T(z)\rangle = 0$ trivially from zero modes. $[c=-2 \rightarrow Q=-1]$

Replica formulation formally applies now: $\boxed{c<0 \;\Rightarrow\; S_A<0}$

$\mathbb{Z}_N$ bc-orbifold CFTs (Saleur, Kausch, Flohr, … '90s) confirm negative conf dims of twist ops $[l \equiv v-u]$

$$tr\rho_A^n = \prod_{k=1}^{n-1} \langle 0|\sigma^-_{k/N}(v)\sigma^+_{k/N}(u)|0\rangle = l^{\frac{1}{3}(n-1/n)} \rightarrow S_A = -\lim_{n\rightarrow 1}\partial_n\, tr\rho_A^n = -\frac{2}{3}\log\frac{l}{\epsilon}$$

"Ghost-spins"

Abstract away from technicalities of ghost CFTs, replica subtleties: simple QM toy models of ghost-like theories with negative norm states $\rightarrow$ reduced density matrix (RDM) after partial trace $\rightarrow$ EE.

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Recall ordinary spin: $\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1, \qquad \langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0$

$$|\psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle \;\; \rightarrow \;\; \langle\psi|\psi\rangle = |c_1|^2 + |c_2|^2 > 0$$

cook up

"Ghost-spin" $\rightarrow$ 2-state spin variable with indefinite norm.
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$|\psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle \;\; \rightarrow \;\; \langle\psi|\psi\rangle = c_1 c_2^* + c_2 c_1^* \not> 0.$ $\; e.g.\ |\uparrow\rangle - |\downarrow\rangle$ has norm -2.

$|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$; $\;\langle+|+\rangle = \gamma_{++} = 1,\;\; \langle-|-\rangle = \gamma_{--} = -1,\;\; \langle+|-\rangle = \langle-|+\rangle = 0$

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Two ghost-spins: $\rho = |\psi\rangle\langle\psi| \;\rightarrow$ trace over one ghost-spin $\rightarrow$ RDM for remaining ghost-spin $\rightarrow$ von Neumann entropy.

Even number of ghost-spins $\rightarrow$ calculations, interpretation sensible.

Two ghost-spins

$|\psi\rangle = \sum \psi^{ij}|ij\rangle$: $\langle\psi|\psi\rangle = \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl*} = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 = \pm 1$

Trace over one ghost-spin $\rightarrow$ ρ_A for remaining ghost-spin $\rightarrow$ von Neumann entropy S_A.

RDM: $(\rho_A)^{ik} = \gamma_{jl}\psi^{ij}\psi^{kl*} = \gamma_{jj}\psi^{ij}\psi^{kj*}$ $(\gamma_{\pm\pm} = \pm 1)$

$$(\rho_A)^{++} = |\psi^{++}|^2 - |\psi^{+-}|^2, \qquad (\rho_A)^{+-} = \psi^{++}\psi^{-+*} - \psi^{+-}\psi^{--*},$$
$$(\rho_A)^{-+} = \psi^{-+}\psi^{++*} - \psi^{--}\psi^{+-*}, \qquad (\rho_A)^{--} = |\psi^{-+}|^2 - |\psi^{--}|^2.$$

Define $\log\rho_A$ using expansion & mixed-index RDM $(\rho_A)_i{}^k = \gamma_{ij}(\rho_A)^{jk}$.

EE: $S_A = -\gamma_{ij}(\rho_A \log\rho_A)^{ij} \rightarrow -(\rho_A)_+^+(\log\rho_A)_+^+ - (\rho_A)_-^-(\log\rho_A)_-^-$

In general, $+ve$ norm $\not\Rightarrow$ $+ve$ RDM, EE. [however, correlated ghost-spins]

Two ghost-spins

$|\psi\rangle = \sum \psi^{ij}|ij\rangle$: $\langle\psi|\psi\rangle = \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl\,*} = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 = \pm 1$

Trace over one ghost-spin $\rightarrow$ ρ_A for remaining ghost-spin $\rightarrow$ von Neumann entropy S_A.

RDM: $(\rho_A)^{ik} = \gamma_{jl}\psi^{ij}\psi^{kl\,*} = \gamma_{jj}\psi^{ij}\psi^{kj\,*}$ $(\gamma_{\pm\pm} = \pm 1)$

$$(\rho_A)^{++} = |\psi^{++}|^2 - |\psi^{+-}|^2, \qquad (\rho_A)^{+-} = \psi^{++}\psi^{-+\,*} - \psi^{+-}\psi^{--\,*},$$
$$(\rho_A)^{-+} = \psi^{-+}\psi^{++\,*} - \psi^{--}\psi^{+-\,*}, \qquad (\rho_A)^{--} = |\psi^{-+}|^2 - |\psi^{--}|^2.$$

Define $\log\rho_A$ using expansion & mixed-index RDM $(\rho_A)_i{}^k = \gamma_{ij}(\rho_A)^{jk}$.

EE: $S_A = -\gamma_{ij}(\rho_A \log\rho_A)^{ij} \rightarrow -(\rho_A)_+^+(\log\rho_A)_+^+ - (\rho_A)_-^-(\log\rho_A)_-^-$

In general, $+ve$ norm $\not\Rightarrow$ $+ve$ RDM, EE. [however, correlated ghost-spins]

Simple subfamily, diagonal ρ_A: $\log\rho_A$ diag

$$(\rho_A)_+^+ = \pm x, \quad (\rho_A)_-^- = \pm(1-x), \qquad 0 < x = \frac{|\psi^{++}|^2}{|\psi^{++}|^2 + |\psi^{--}|^2} < 1$$

$\langle\psi|\psi\rangle > 0$: $S_A^+ = -x\log x - (1-x)\log(1-x) > 0$

$+ve$ norm $\Rightarrow$ $+ve$ EE

$\langle\psi|\psi\rangle < 0$: $S_A^- = x\log(-x) + (1-x)\log(-(1-x)) = -S_A^+ + i\pi$

$-ve$ norm $\Rightarrow$ ρ^A eigenvalues $-ve$ $\Rightarrow$ $-ve$ Re(EE), const Im(EE)

Entangled ghost-spins

In general RDM shows new EE patterns.

- e.g. eigenvalues satisfy $(\rho_A)_i^k e_k = \lambda e_i \;\; i.e. \;\; (\rho_A)^{ij} e_j = \gamma^{ij} \lambda e_j .$

  -ve norm contributions $\Rightarrow \lambda$ in general complex.

- n ghost-spins: $|\psi\rangle = \psi^{++\cdots}|++\ldots\rangle + \psi^{--\cdots}|--\ldots\rangle,$

$\langle\psi|\psi\rangle = |\psi^{++\cdots}|^2 + (-1)^n|\psi^{--\cdots}|^2, \quad (\rho_A)_+^+ = |\psi^{++\cdots}|^2, \;\; (\rho_A)_-^- = (-1)^n|\psi^{--\cdots}|^2$

odd n: +ve norm $\not\Rightarrow$ +ve RDM.

Even n: $\prod^n |-\rangle$ is $+ve$ norm.

2 ghost-spins: $|\psi\rangle = \psi^{++}|+\rangle|+\rangle + \psi^{--}|-\rangle|-\rangle \xrightarrow{+ve}$ **Correlated ghost-spins**

Entangle identical ghost-spins from each copy $\rightarrow$ $+ve$ norm, RDM, EE

Also true for 2 copies of ghost-spin ensembles or chains $\{|\sigma_n\rangle\}$

$\mathcal{GC}_1 \times \mathcal{GC}_2$: $\quad |\psi\rangle = \sum_{|\sigma_n\rangle} \psi^{\sigma_n,\sigma_n}|\sigma_n\rangle|\sigma_n\rangle \,, \qquad \langle\psi|\psi\rangle = \sum_{|\sigma_n\rangle} |\psi^{\sigma_n,\sigma_n}|^2 > 0$

Entangled ghost-spins and spins

- Disentangled ghost-spins and spins $\Rightarrow$ product states $|\psi\rangle = |\psi_s\rangle\,|\psi_{gs}\rangle$

Ghost-spins: $\gamma_{++} = 1,\ \ \gamma_{--} = -1$; Spin metric $+ve$ definite: $g_{ij} = \delta_{ij}$.

$\langle\psi|\psi\rangle = \langle\psi_s|\psi_s\rangle\langle\psi_{gs}|\psi_{gs}\rangle,\quad \langle\psi_s|\psi_s\rangle > 0;\qquad \langle\psi_{gs}|\psi_{gs}\rangle \gtrless 0 \to \langle\psi|\psi\rangle = \pm 1$ (normalize)

$\longrightarrow$ RDM $\rho_A^s = tr_{gs}\,\rho$ for spins alone $\to$ $tr\rho_A^s = \pm 1$ $\longrightarrow$

$+ve$ norm: $S_A = -\sum_i \lambda_i \log\lambda_i > 0;\qquad -ve$ norm: $S_A = \sum_i \lambda_i \log\lambda_i + i\pi,\ \ ReS_A < 0$

- Entangled ghost-spins and spins: more intricate, new EE patterns.

Ghost-spin chains → bc-ghost CFTs

Infinite ghost-spin chains → continuum limit → ghost-CFTs?

Ghost-spins as microscopic building blocks of ghost/nonunitary CFTs?

Recall: Ising model at critical point is a CFT of free massless fermions.

Hamiltonian $H = J\sum_n \left(\sigma_{b(n)}\sigma_{c(n+1)} + \sigma_{b(n)}\sigma_{c(n-1)}\right)$

Spin variables: $\{\sigma_{bn},\ \sigma_{cn}\} = 1\ ,\quad [\sigma_{bn}, \sigma_{bn'}] = [\sigma_{cn}, \sigma_{cn'}] = [\sigma_{bn}, \sigma_{cn'}] = 0.$

$\sigma_{bn}^\dagger = \sigma_{bn},\ \ \sigma_{cn}^\dagger = \sigma_{cn}\ ;\qquad \sigma_b|\downarrow\rangle = 0,\quad \sigma_b|\uparrow\rangle = |\downarrow\rangle,\quad \sigma_c|\uparrow\rangle = 0,\quad \sigma_c|\downarrow\rangle = |\uparrow\rangle.$

Like b_n, c_n ops of bc-CFT, $\{b_n, c_m\} = \delta_{n,-m}$: but σ_{bn}, σ_{cn} bosonic (distinct sites, commute).

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Jordan-Wigner: $a_{bn} = \prod_{k=1}^{n-1} i(1-2\sigma_{ck}\sigma_{bk})\sigma_{bn}, \quad a_{cn} = \prod_{k=1}^{n-1}(-i)(1-2\sigma_{ck}\sigma_{bk})\sigma_{cn}$

→ Fermionic gh.sp. variables: $\{a_{bi}, a_{cj}\} = \delta_{ij}\ , \quad \{a_{bi}, a_{bj}\} = 0\ , \quad \{a_{ci}, a_{cj}\} = 0\ .$

$H = J\sum_n(\sigma_{b(n)}\sigma_{c(n+1)} + \sigma_{b(n)}\sigma_{c(n-1)}) \ \rightarrow\ iJa_{bn}(a_{c(n+1)} - a_{c(n-1)}) \sim -b\partial c$

→ lattice discretization of bc-ghost CFT.

Momentum variables, continuum limit $H \xrightarrow{J\sim 1/2a} \sum_{k>0} k\,(b_{-k}c_k + c_{-k}b_k) + zpe$

Conf symm: $a \to \xi^{-1}a\ , \ \ H \to \xi H\ , \ \ \sigma_{b(n)} \to \xi^{\lambda}\sigma_{b(n)}\ , \ \ \sigma_{c(n+1)} \to \xi^{1-\lambda}\sigma_{c(n+1)}$

$\mathrm{CFT}_3^{Sp(N)}$, symplectic fermions ?

N-level ghost-spins

- $O(N)$ symmetry flavour generalization of the bc-ghost system:

$\langle\downarrow^A|\uparrow^B\rangle = \delta^{AB} = \langle\uparrow^A|\downarrow^B\rangle, \quad \langle\downarrow^A|\downarrow^B\rangle = \langle\uparrow^A|\uparrow^B\rangle = 0,$ A,B=1,...,N.

Essentially N copies of the 2-level ghost-spin system.

Ghost-spin chains at criticality lead to bc-CFTs with $O(N)$ flavour symmetry, $\int b^A \partial c^A$.

- N-levels with symplectic-like structure:

$\langle\uparrow^A|\downarrow^B\rangle = i\,\Omega^{AB}, \quad \langle\downarrow^A|\uparrow^B\rangle = i\,\Omega^{AB}, \quad \langle\uparrow^A|\uparrow^B\rangle = 0 = \langle\downarrow^A|\downarrow^B\rangle\,,$ A,B=1,...,2N.

Symplectic structure built into the inner product; in part motivated by 3-dim ghost-CFTs of symplectic fermions (higher spin dS_4/CFT_3, Anninos,Hartman,Strominger).

- N irreducible levels: *i.e.* generalize states $|\uparrow\rangle, |\downarrow\rangle$ to $|e_1\rangle, \ldots, |e_N\rangle$ such that

$$\langle e_i|e_i\rangle = 0; \quad \langle e_i|e_j\rangle = 1 \quad \text{for } i \neq j, \quad i,j = 1,2,\ldots,N.$$

Elemental ghost-spins not 2-level anymore (with flavour indices), but irreducibly N-level.

Two copies of ghost-spin ensembles: correlated states $\rightarrow +ve$ norm, $EE > 0$.

$$\mathcal{GC}_1 \times \mathcal{GC}_2: \ |\psi\rangle = \sum_{|\sigma\rangle} \psi^{\sigma,\sigma} |\sigma\rangle|\sigma\rangle\,, \qquad \langle\psi|\psi\rangle = \sum_{|\sigma\rangle} |\psi^{\sigma,\sigma}|^2 > 0$$

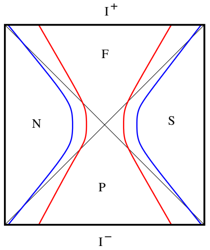
$$\xrightarrow{ground\ states} \{|\sigma\rangle\}\ 2^N\text{-dim} \ \rightarrow\ \text{maximal EE} \ \rightarrow\ S_A = N\log 2$$

dS_4 entropy & entangled TFD-type ghost-CFT states $|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^P} |i_n^F\rangle |i_n^P\rangle$

de Sitter entropy as entanglement

Real connected surfaces, I^+ to I^-, limiting surface as $\Delta w \to \infty$ (Hartman-Maldacena AdS bh, rotated)

$$ds^2 = \frac{l^2}{\tau^2}\Big(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2\Big) \to S^{div} = \frac{\pi l^2}{G_4}\frac{l}{\epsilon_c} \quad \text{(area law)}$$



Bndry Eucl time slice: any S^{d-1} equatorial plane or $w = const$ slice.

Area law coefficient $\sim$ dS entropy $\sim$ number of degrees of freedom in dual CFT.

Existence of such future/past connected surfaces suggests $\to$

Speculation: dS_4 approximately dual to $CFT_F \times CFT_P$ in thermofield-double-like entangled state $|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^P} |i_n^F\rangle |i_n^P\rangle$?

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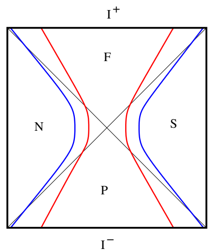

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- (Witten, Strominger '01) bulk time evolution: $|i_n^P\rangle \to |i_n^F\rangle \Rightarrow$

$|\psi^{tfd}\rangle$ unitarily equivalent to $|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^F} |i_n^F\rangle |i_n^F\rangle$ with both ghost CFT copies at I^+.

- 3-dim N-level ghost-spin chains in universality class of ghost CFT$_3$ dual to dS_4: $N \sim \frac{l^2}{G_4}$.

  Two copies of ghost-CFTs in $|\psi^{tfd}\rangle \to +ve$ norm, $+ve$ EE $\sim N \to dS_4$ entropy.

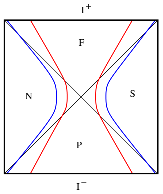
- Near $I^\pm$, $\tau \to 0$: $ds^2 \sim \frac{l^2}{\tau^2}(-d\tau^2 + dw^2 + d\Omega_{d-1}^2) \to$ Eucl CFT$_{F,P}$ on $R_w \times S^{d-1}$.

  Global $dS \to$ dual $CFT_F \times CFT_F$ on $(R_w \times S^{d-1})^2$.

  dS_4 temperature? Casimir energy for ghost-CFT on $R_w \times S^{d-1} \sim \frac{1}{l}$?

Conclusions, questions

- de Sitter: future/past connected surfaces, I^+ to I^-



limiting surface as subregion becomes whole space, coefficient dS_4 entropy.

- Ghost-spins: toy QM models, $-ve$ norm states, $Re(EE) < 0$.

  Two copies of ghost-spin ensembles: correlated states $|\psi\rangle = \sum_{|\sigma\rangle} \psi^{\sigma,\sigma}|\sigma\rangle|\sigma\rangle$

  $\rightarrow\ +ve$ norm, $EE > 0$.

??? Subregions? Entanglement wedge?

??? Interpreting these extremal surface areas as entanglement?

Lewkowycz,Maldacena, …?

??? $dS_4 \leftrightarrow CFT_F \times CFT_F$ in entangled thermofield-double-like states

$|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^F} |i_n^F\rangle|i_n^F\rangle$? Horizons, HKLL/Papadodimas,Raju? Replica for $Z_F^* \times Z_F$?

??? Ghost-spins as microscopic building blocks for ghost-CFTs …

3-dim ghost CFTs? Ghost-spin glasses? Models for $CFT_3^{ghost} \leftrightarrow dS_4$: emergence of time?

??? Ghosts and worldsheet entanglement?

Conceptual issues with Bell pairs of spins and ghost-spins etc …?

dS extremal surfaces (Poincare)

Extremize $\rightarrow$ $(\partial_\tau x)^2 = \frac{-A^2\tau^{2d-2}}{1-A^2\tau^{2d-2}}$. [$A^2 < 0$ is earlier real-τ solution]

dS_4/CFT_3: take $A^2 > 0$. Near $\tau \to 0$: $\dot{x}^2 \sim -A^2\tau^4$ $i.e.$ $x(\tau) \sim \pm iA\tau^3 + x(0)$.

Spatial dirn in Eucl CFT $\Rightarrow x(\tau)$ real $\Rightarrow$ $\tau = iT$, imaginary path. Turning point $T_* = \frac{1}{\sqrt{A}}$.

Join half-extremal-surfaces smoothly at τ_*. dS_{d+1}, d even: $A^2 < 0$, $\tau = iT$.

> $x(\tau)$ real $\Rightarrow$ imaginary path $\tau = iT \rightarrow$ complex extremal surface
> $\equiv$ analytic continuation $r \to -i\tau$, $R \to -iR_{dS}$ from AdS Ryu-Takayanagi.

Complex surfaces: no canonical action;

$$S_{AdS} = \frac{R^{d-1}V_{d-2}}{4G_{d+1}}\int \frac{dr}{r^{d-1}}\sqrt{1+x'^2} \rightarrow S_{dS} = -i\frac{R_{dS}^{d-1}}{4G_{d+1}}\int \frac{d\tau}{\tau^{d-1}}\frac{V_{d-2}}{\sqrt{1-(-1)^{d-1}A^2\tau^{2d-2}}}$$

- de Sitter isometry $\Rightarrow$ boundary Euclidean time direction is any symmetry direction.
- sphere: subleading log-div, conf anomaly $\leftrightarrow$ Ψ log-coeff.

AdS/CFT: $Z_{CFT} = Z_{bulk}$; dS/CFT: $Z_{CFT} = \Psi_{dS} \Rightarrow$

> $\text{EE}_{dS/CFT} \neq$ bulk EE (via bulk density matrix, $\Psi^*\Psi$)

(Maldacena, Pimentel)

$S_{dS_4} \sim -\frac{R_{dS}^2}{G_4}V_1(\frac{1}{\epsilon} - c\frac{1}{l})$ $\rightarrow$ negative area in dS_4, $\mathcal{C}_3 < 0$.

bc-ghost CFTs

- $SL(2,Z)$ vacuum $|0\rangle \neq$ ghost ground state $|\downarrow\rangle$ in general.

$S \sim \int d^2z\, b\bar{\partial}c$, $(h_b, h_c) = (\lambda, 1-\lambda)$, $c = 1 - 3Q^2 < 0$, Background Charge $Q = 1 - 2\lambda$

$b(z) = \sum \frac{b_m}{z^{m+\lambda}}$, $c(z) = \sum \frac{c_m}{z^{m+1-\lambda}}$; $L_0 = \sum_{n>0} n(b_{-n}c_n + c_{-n}b_n) + \frac{\lambda(1-\lambda)}{2}$.

SL(2) inv vacuum $|0\rangle$: $T(z)|0\rangle = \sum_m \frac{L_m}{z^{m+2}}|0\rangle = regular$

$\Rightarrow$ $L_{m\geq -1}|0\rangle = 0$, $b_{m\geq 1-\lambda}|0\rangle = 0$, $c_{m\geq\lambda}|0\rangle = 0$ whereas $b_0|\downarrow\rangle = 0$

- $j_0^\dagger = -(j_0 + Q)$ Charge asymmetry.

$U(1)$ charge symmetry $\delta b = -i\epsilon b$, $\delta c = i\epsilon c$ $\rightarrow$ ghost current $j(z) = -:bc:$

$j(z) = \sum_m \frac{j_m}{z^{m+1}}$, $[L_m, j_n] = -nj_{m+n} + \frac{1}{2}Qm(m+1)\delta_{m,-n}$

$[j_0, O_p] = pO_p$, $j_0|q\rangle = q|q\rangle$ $\Rightarrow$ $p\langle q'|O_p|q\rangle = \langle q'|[j_0, O_p]|q\rangle = (-q' - Q - q)\langle q'|O_p|q\rangle$

Corrn fn $\neq 0$ only if <u>Bgnd Charge cancelled</u> $i.e.$ $p = -(q + q' + Q)$ $\Rightarrow$ $\langle -q - Q|q\rangle = 1$.

- $\lambda = 1$, $c = -2$: $SL(2)$ vacuum $|0\rangle = |\downarrow\rangle$ ghost ground state

$b_{m\geq 0}|0\rangle = 0$, $c_{m\geq 1}|0\rangle = 0$; $Q = -1$: $\langle +1|0\rangle = \langle 0|c_0|0\rangle = 1$ $\leftarrow$ zero mode insertion

$\langle b(z)c(w)\rangle_0 \equiv \langle 0|c_0 \sum_{m,n} \frac{b_m}{z^{m+1}} \frac{c_n}{w^n}|0\rangle = \langle 0|c_0 \sum_{m=0}^{\infty} \frac{w^m}{z^{m+1}} b_m c_{-m}|0\rangle = \frac{1}{z-w}\langle 0|c_0|0\rangle$

whereas $\langle 0|b(z)c(w)|0\rangle = \frac{1}{z-w}\langle 0|0\rangle = 0$. Plethora of negative norm states

Entangled ghost-spins

- In general RDM shows new EE patterns.

e.g. eigenvalues satisfy $(\rho_A)_i^k e_k = \lambda e_i$ $i.e.$ $(\rho_A)^{ij} e_j = \gamma^{ij} \lambda e_j$ $\Rightarrow$

$\pm$ norm: $\lambda^2 - (\mathrm{tr}\rho_A)\lambda - \det\rho_A^{ik} = (\lambda \mp \frac{1}{2})^2 - \frac{1}{4} - \det\rho_A^{ik} = 0$.

2 gs state: $\det\rho_A^{ik} = -|\psi^{++}\psi^{--} - \psi^{+-}\psi^{-+}|^2$ can be large, -ve: λ in general complex.