Lifshitz-like systems
and AdS null deformations

K. Narayan
Chennai Mathematical Institute

[arXiv:1005.3291, Koushik Balasubramanian, KN,
arXiv:1103.1279, KN.]

- Lifshitz points, gravity duals
- AdS/CFT with null deformations
- x^+-compact: $z = 2$ Lifshitz systems
- x^+-noncompact: anisotropic Lifshitz systems

[see also AdS/CFT cosmo singularities: hep-th/0602107, 0610053, Das, Michelson, KN, Trivedi; arXiv:0711.2994, Awad, Das, KN, Trivedi; arXiv:0807.1517, Awad, Das, Nampuri, KN, Trivedi.]

Related work: Hartnoll,Polchinski,Silverstein,Tong; Donos,Gauntlett; Gregory,Parameswaran,Tasinato,Zavala; ...

AdS/condmat and Lifshitz systems

Interesting to explore holography with reduced symmetries.

Generalizations of AdS/CFT to nonrelativistic systems

$\rightarrow$ holographic condensed matter, ...

Son; Balasubramanian,McGreevy; Adams et al; Herzog et al; Maldacena et al; ...

Lifshitz points: arise in magnetic systems with antiferromagnetic interactions, dimer models, liquid crystals, ...

Symmetries: t, x_i-translations, x_i-rotations,

scaling $t \rightarrow \lambda^z t,\ x_i \rightarrow \lambda x_i$ [z: dynamical exponent].

Landau-Ginzburg action ($z = 2$): $S = \int d^3x((\partial_t\varphi)^2 - \kappa(\nabla^2\varphi)^2)$.

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Landau-Ginzburg action ($z = 2$): $S = \int d^3x((\partial_t\varphi)^2 - \kappa(\nabla^2\varphi)^2)$.

Lifshitz spacetime: $ds_4^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}$. Kachru,Liu,Mulligan

Scaling: $t \rightarrow \lambda^z t,\ x_i \rightarrow \lambda x_i,\ r \rightarrow \lambda r$ [$z = 1$: AdS].

[note: smaller than Galilean symmetries: $e.g.$ Galilean boosts broken]

Solution to 4-dim gravity with $\Lambda < 0$ and massive gauge field $A \sim \frac{dt}{r^z}$

(or alternatively gauge field + 2-form: dualize to get A-mass)

Lifshitz points in string theory

Previous attempts: $e.g.$ $Li_d \times M^{10-d}$ or $Li_d \times M^{11-d}$ as a solution in string or M-theory, supported by extra fluxes in the compact space (incl warping) (Li,Nishioka,Takayanagi) — various basic violations.

$z = \frac{3}{2}$ Lifshitz solutions from D3/D7-constructions (Azeyanagi,Li,Takayanagi): necessarily anistropic, require scalar that breaks scaling symmetry.

Other solutions: geometry Lifshitz but $e.g.$ scalar breaks symmetry.

Various interesting suggestions in Hartnoll,Polchinski,Silverstein,Tong.

Alternative way to realize $z = 2$ Lifshitz [Balasubramanian,KN]:
x^+-DLCQ of relativistic $\mathcal{N}=4$ SYM $\longrightarrow$
$z = 2$ nonrelativistic (Galilean) 2+1-dim system.

Gauge coupling $g^2_{YM}(x^+)$ varying in lightlike x^+-direction $\longrightarrow$ breaks x^+-shift reducing to 2+1-dim Lifshitz symmetries.
Concrete bulk realization: null deformations of $AdS_5 \times S^5$ (more generally, $AdS \times X$) sourced by lightlike scalar ($e.g.$ dilaton in IIB).

AdS null deformations and Lifshitz

$$ds^2_{Einst} = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\partial_+\Phi)^2(dx^+)^2] + \frac{dw^2}{w^2} + d\Omega_5^2.$$

Lightlike dilaton: $\Phi = \Phi(x^+)$ (also 5-form). [DMNT,ADNT,ADNNT]

AdS null deformations and Lifshitz

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Lightlike dilaton: $\Phi = \Phi(x^+)$ (also 5-form). [DMNT,ADNT,ADNNT]

$\Phi = const$ $i.e.$ $g_{++} = 0$: DLCQ $_{x^+}$ of AdS in lightcone coordinates

— nonrelativistic, Schrodinger (Galilean) symmetries

[Goldberger, Barbon et al, Maldacena et al].

Regard $x^- \equiv t$ (time), $x^+ \equiv$ compact coordinate ($g_{++} \sim (\Phi')^2 > 0$).

Strictly: $x^+ = const \equiv$ null surfaces and $x^- = const$ surfaces spacelike ($g^{--} < 0$).

Symmetries: x^-, x_i-translations, x_i-rotations, $z = 2$ scaling

$x^- \equiv t \to \lambda^2 t, \quad x_i \to \lambda x_i, \quad w \to \lambda w$ (x^+ compact, no scaling).

x^+ compact $\Rightarrow$ lightlike boosts broken.

Galilean boosts $x_i \to x_i - v_i x^-, \quad x^+ \to x^+ - \frac{1}{2}(2v_i x_i - v_i^2 x^-)$:

broken by g_{++}. Also broken $z = 2$ special conformal symmetry.

Nontrivial x^+-dependence $\Rightarrow z = 2$ Galilean broken to Lifshitz.

AdS null deformations

$$ds^2_{Einst} = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\partial_+\Phi)^2(dx^+)^2] + \frac{dw^2}{w^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+).$$

- The coord. transfmn. $w = re^{-f/2}, \quad x^- = y^- - \frac{w^2 f'}{4}$, gives

$$ds^2 = \frac{1}{r^2}[e^{f(x^+)}(-2dx^+dy^- + dx_i^2) + dr^2] + d\Omega_5^2\,, \quad \Phi(x^+),$$

with the EOM constraint $R_{++} = \frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\Phi')^2$. (DMNT,ADNT)

- Function-worth of solutions, for any $\Phi(x^+)$.

Can replace $S^5 \to X^5$ (Sasaki-Einstein).

These solutions can in fact be generalized to a large family of solutions with axion-dilaton, 3-form field strength turned on (Donos,Gauntlett).

- Preserve half lightcone supersymmetry.

- There also exist $AdS_4 \times X^7$ null deformations in M-theory, with scalar arising from the G-flux on X^7: presumably dual to lightlike deformations of Chern-Simons (ABJM-like) theories arising on M2-brane stacks. Similarly AdS_7 solutions likely. Also, null deformations of AdS_3 exist in WZW models.

Dual field theory, correlators

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\Phi')^2(dx^+)^2] + \frac{dw^2}{w^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+)\,.$$

(DMNT) $d = 4\ \mathcal{N}{=}4$ super Yang-Mills theory with gauge coupling lightlike-deformed as $g_{YM}^2(x^+) = e^{\Phi(x^+)} \quad \longrightarrow \quad$ DLCQ $_{x^+}$.

Boundary metric $ds_4^2 = \lim_{w\to 0} w^2 ds_5^2$ manifestly flat. Lightlike (chiral) deformation $\Rightarrow$ no nonzero contraction exists involving metric and coupling alone (only $\partial_+\Phi$ nonvanishing) $\Rightarrow$ various physical observables ($e.g.$ trace anomaly, anomalous dims) unaffected.

2-point correlation function: operators $\mathcal{O}$ dual to massive scalars φ.

[In conformal coords $ds^2 = \frac{1}{r^2}[e^{f(x^+)}(-2dx^+dy^- + dx_i^2) + dr^2], \quad \lambda = \int e^{f(x^+)}dx^+$.

$S \sim \int d^3x d^3x' dx^+ dx^{+\prime}\ e^{3f(x^+)/2} e^{3f(x^{+\prime})/2}\ \varphi(x^+,\vec{x})\varphi(x^{+\prime},\vec{x}')\ (\frac{\Delta\lambda}{\Delta x^+})^{1-\Delta} \frac{1}{[(\Delta\vec{x})^2]^\Delta}$,

$(\Delta\vec{x})^2 = -2(\Delta x^+)(\Delta x^-) + \sum_{i=1,2}(\Delta x_i)^2, \ \ \Delta = 2 + \sqrt{4+m^2}\,.$

$\Delta x^+ \ll \Delta x^-, \Delta x_i$: approximate $\frac{\Delta\lambda}{\Delta x^+} \sim \frac{d\lambda}{dx^+} = e^f,\ e^{f(x^+)} \sim 1$.]

$$\langle\mathcal{O}(x_i)\mathcal{O}(x_i')\rangle \ \sim\ \frac{1}{[\sum_i(\Delta x_i)^2]^\Delta}\,, \quad \text{and} \quad \langle\mathcal{O}(t)\mathcal{O}(t')\rangle \sim \frac{1}{(\Delta x^-)^\Delta}\,.$$

Agrees with equal-time 2-pt fn of $2+1$-dim Lifshitz theory (KLM).

Equal time correlators: $2+1$-Lifshitz $\equiv$ 2D Eucl. CFT (Ardonne,Fendley,Fradkin)

Calculation difficult in form with $g_{++} \neq 0$: scalar wave eqn not straightforward to solve.

Bulk $z = 2$ Lif$_4$ from dim'nal redux

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\Phi')^2(dx^+)^2] + \frac{dw^2}{w^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+)\,.$$

Naive, standard Kaluza-Klein reduction along x^+ as:

$$ds^2 = g_{mn}dx^m dx^n = G_{\mu\nu}dx^\mu dx^\nu + G_{dd}(x^d + A_\mu dx^\mu)^2 :$$

$$\longrightarrow \frac{1}{w^2}[\frac{1}{4}w^2(\Phi')^2(dx^+)^2 - 2dx^+dx^-] = \frac{1}{4}(\Phi')^2[dx^+ - \frac{4dx^-}{w^2(\Phi')^2}]^2 - \frac{4(dx^-)^2}{w^4(\Phi')^2}.$$

Long-wavelength bulk 4-dim metric: $ds^2 = -\frac{4(dx^-)^2}{w^4(\Phi')^2} + \frac{dx_i^2}{w^2} + \frac{dw^2}{w^2}$.

Remnant $(\Phi')^2$, nontrivial x^+-dependence: not standard KK-redux.

"Minimal" off-shell metric ansatz containing above metric

$$ds^2 = -N^2(x^+)K^2(s^i)dt^2 + \frac{1}{N^2(x^+)}(dx^+ + N^2(x^+)A)^2 + \frac{1}{w^2}(ds^i)^2$$

$N(x^+)$ controls g_{++} component, $x^- \equiv t$ and $s^i = x^i, w$.

Kaluza-Klein gauge field $A \equiv A_0 K dt$ (purely electric, $A_i = 0$).

On-shell solution: $g_{tt} = -N^2K^2(1 - A_0^2) = 0, \;\; N = \frac{2}{\Phi'}\,, \;\; K = \frac{1}{w^2}$.

Want off-shell lower dim eff. action: calculate 5d Ricci scalar, action for scalar Φ, retain $K(s^i), A_0(s^i)$ independently (separate lower dim gauge field from metric). Eff action $\rightarrow$ lower dim metric.

Bulk $z = 2$ Lif$_4$ from dim'nal redux

$$ds^2 = -N^2(x^+)K^2(s^i)dt^2 + \frac{1}{N^2(x^+)}(dx^+ + N^2(x^+)A)^2 + \frac{1}{w^2}(ds^i)^2$$

$\Rightarrow\ R^{(5)} = R^{(4)}\ - 2(NN'' + (N')^2)\ \ + \frac{1}{8}F_{0i}^2 + 2(NN'' + (N')^2)A_0^2$,

with $\quad R^{(4)} = -\frac{2}{K}(w^2\partial_i^2 K - w\partial_w K + 3K)$.

Suggests lower dim spacetime is: $\quad ds^2 = -K^2(s^i)dt^2 + \frac{1}{w^2}ds^{i\,2}$.

On-shell solution: $N = \frac{2}{\Phi'}\ ,\quad K = \frac{1}{w^2}\ ,\qquad \Rightarrow\ Lif_4\colon\quad ds^2 = -\frac{dt^2}{w^4} + \frac{dx_i^2 + dw^2}{w^2}$.

Gauge field mass from scalar kinetic terms agrees with that from lower dim system: $\quad -\frac{1}{2}g^{++}(\partial_+\Phi)^2 \to -\frac{1}{2}N^2(1 - A_0^2)(\Phi')^2 + \ldots \to \frac{1}{2}N^2(\Phi')^2 A_0^2$.

Consistency: 5-d system is on-shell solution if $(\partial_i A_0 + A_0\frac{\partial_i K}{K})^2 = \frac{4}{w^2}$,

[from [00]-component], admitting solution $K = \frac{1}{w^2},\ A = -\frac{dt}{w^2}$.

Naive dim redux has Φ', disappears here — perhaps surprising.

Nontrivial x^+-dependence complicates Wilsonian separation-of-scales argument: mixing with other modes?

$e.g.$ turn on KK vector potential $A_i dx^i$. No conclusive result here for consistent dim'nal reduction: then $R^{(5)}$ has extraneous factors of $N(x^+)$ appearing in analogous calculation.

Harder to interpret lower dim system.

Scalar probes of Lifshitz

Long wavelength geometry seen by bulk supergravity scalar?

Scalar action $S = \frac{1}{G_5} \int d^5x \sqrt{-g}\, g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$: restrict to modes with no x^+-dependence ($i.e.\ \partial_+ \varphi = 0$) $\Rightarrow$

$$S \to \frac{1}{G_5} \int \frac{d^4x}{w^5} \left[-w^4 \left(\frac{\int dx^+ (\Phi')^2}{4} \right) (\partial_- \varphi)^2 + w^2 L (\partial_i \varphi)^2 + w^2 L (\partial_w \varphi)^2 \right]$$
$$= \frac{1}{G_4} \int \frac{d^4x}{w^5} [-w^4 (\partial'_- \varphi)^2 + w^2 (\partial_i \varphi)^2 + w^2 (\partial_w \varphi)^2] \,,$$

L: compactification size. $G_4 = \frac{G_5}{L}$, 4-dim Newton const.

Rescale $x^- \to x^{-\prime} = \frac{L}{\int dx^+ (\Phi')^2} x^- \ \rightarrow$ scalar action in 4-dim $z = 2$

Lifshitz background $ds^2 = -\frac{dt^2}{w^4} + \frac{dx^{i\,2}}{w^2} + \frac{dw^2}{w^2}$.

Eqn of motion: 4-dim Lifshitz geometry arises on scales large compared with the typical scale of variation (compactification size), $i.e.$ effectively setting $\Phi' \sim const.$

Recap: x^+-compact, $z = 2$ Lifshitz

Lightlike deformation $\rightarrow$ break Galilean symmetries $\rightarrow$ Lifshitz.

$AdS_5 \times S^5 \rightarrow$ null deformations with lightlike dilaton $\rightarrow$ DLCQ $_{x^+}$:

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\Phi')^2(dx^+)^2] + \frac{dw^2}{w^2} + d\Omega_5^2, \quad \Phi = \Phi(x^+)\ .$$

Symmetries: x^-, x_i-translations, x_i-rotations,

$z = 2$ scaling $\quad x^- \equiv t \rightarrow \lambda^2 t,\ \ x_i \rightarrow \lambda x_i,\ \ w \rightarrow \lambda w \quad$ (x^+ compact, no scaling).

$d = 4\ \mathcal{N}{=}4$ super Yang-Mills theory with gauge coupling lightlike-deformed as $g_{YM}^2(x^+) = e^{\Phi(x^+)} \longrightarrow$ DLCQ $_{x^+}$.

2-point correlation function: operator $\mathcal{O}$ dual to massless scalar φ.

$$\langle \mathcal{O}(x_i)\mathcal{O}(x_i')\rangle \sim \frac{1}{[\sum_i(\Delta x_i)^2]^\Delta}\ , \quad \text{and } \langle \mathcal{O}(t)\mathcal{O}(t')\rangle \sim \frac{1}{(\Delta x^-)^\Delta}\ .$$

Agrees with equal-time 2-pt fn of $2+1$-dim Lifshitz theory (KLM).

x^+-noncompact, anisotropic Lifshitz

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\Phi')^2(dx^+)^2] + \frac{dw^2}{w^2} + d\Omega_5^2,$$

$\Phi = \Phi(Qx^+)$, $\qquad$ Q parameter of mass dim one.

Symmetries: time x^--translations, spatial x_i-translations/rotations (x^+-translations broken by nontrivial x^+-dependence), and anisotropic Lifshitz scaling $\quad w \to \lambda w, \quad x_i \to \lambda x_i, \quad x^- \to \lambda^2 x^-, \quad x^+ \to \lambda^0 x^+$,

$$i.e. \quad z = 2 \ \ \{x_i, x^-\}: \quad x^- \to \lambda^2 x^-, \quad x_i \to \lambda x_i \ ,$$
$$z = \infty \ \ \{x^+, x^-\}: \quad x^- \to \lambda^2 x^-, \quad x^+ \to \lambda^0 x^+.$$

In addition, dilaton $\Phi(Qx^+)$ acts as spatial x^+-potential.

[Recall $z = 0$ Schrodinger systems: $ds^2 = -dt^2 + \frac{dx_i^2 + dt d\xi + dr^2}{r^2}$.
Different from present context: $x^- \equiv$ time since const-x^- surface is spacelike $\longrightarrow$ crucial for $z = \infty$ scaling, and broken Galilean boosts.]

Also recall D3-D7 anisotropic Lifshitz systems + radial scalars (Azeyanagi,Li,Takayanagi), and dilatonic Lifshitz-like black branes (Goldstein,Kachru,Prakash,Trivedi).

Linear dilaton, anisotropic Lifshitz

$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2(dx^+)^2] + \frac{dw^2}{w^2}, \quad \Phi = \Phi_0 + 2Qx^+.$

Now metric also has x^+-translations: linear dilaton acts as spatial x^+-potential. Reminiscent of Liouville-like walls in $c = 1$ string theory.

$z = 2$ Lifshitz-like induced metric on *e.g.* D5-brane probe wrapping const-x^+ hypersurface (and $S^2 \in S^5$)? Difficult to realize since const-x^+ hypersurface is null. Might be interesting to study D-brane probes in these geometries.

Dual gauge theory: d=4 $\mathcal{N}=4$ SYM theory living on flat spacetime (flat boundary metric), with gauge coupling
$g_{YM}^2(x^+) = e^{\Phi(x^+)} \equiv g_s e^{2Qx^+}$ (with $g_s = e^{\Phi_0}$).
$x^+ \to -\infty$: weakly coupled gauge theory.
Consistent: string frame metric $ds_{str}^2 = e^{\Phi/2} ds^2$ degenerates here, highly curved.

Linear dilaton, Lifshitz, correlators

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2(dx^+)^2] + \frac{dw^2}{w^2}, \quad \Phi = \Phi_0 + 2Qx^+.$$

2-point correlation function, operators dual to bulk scalars.

Scalar mode functions $\varphi(x) = e^{ik_-x^- + ik_ix^i + ik_+x^+} R(w)$, with

radial equation $w^3\partial_w\left(\frac{1}{w^3}\partial_w R(w)\right) - \left(k^2 + \frac{m^2}{w^2} - w^2Q^2k_-^2\right) R(w) = 0\,,$

$$k^2 = -2k_+k_- + k_i^2, \qquad \alpha = -\frac{iQk_-}{2}\,, \qquad \Delta = 2 + \sqrt{4+m^2} = 2 + \nu.$$

Can be solved in terms of conf.hypergeom.fns: pick growing solution, demanding regularity in interior. Also note for $Q = 0$, this should reduce to AdS lightcone calculation ($w^2K_\nu(kw)$).

Bulk-boundary propagator: $a = \frac{\Delta-1}{2} - \frac{k^2}{8\alpha} = \frac{\nu+1}{2} + \frac{k^2}{4iQk_-}, \quad c = \Delta - 1 = \nu + 1\,.$

$$G(k_i, k_+, k_-, w) = \mathcal{N}(k)\, e^{ik_ix^i + ik_-x^- + ik_+x^+}\, w^\Delta e^{\alpha w^2} U(a, c, -2\alpha w^2)\,.$$

Momentum space 2-point correlation fns:

$$(\nu \notin \mathbb{Z}) \quad \langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim\ -\nu 2^\nu \alpha^\nu \frac{\Gamma(-\nu)}{\Gamma(\nu)} \frac{\Gamma(\frac{1+\nu}{2} + \frac{k^2}{4iQk_-})}{\Gamma(\frac{1-\nu}{2} + \frac{k^2}{4iQk_-})}\,.$$

$$\nu = 2\ \ (\Delta = 4): \quad \langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \ \sim$$
$$((k_i^2 - 2k_+k_-)^2 + 4Q^2k_-^2)\,(\log(iQk_-) + \psi(\tfrac{3}{2} + \frac{k_i^2 - 2k_+k_-}{4iQk_-})).$$

Reminiscent of 2-pt correlation fns in Kachru,Liu,Mulligan, but also differences $\longrightarrow$

Linear dilaton, Lifshitz, correlators

$\Delta = 4$: $\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim ((k_i^2 - 2k_+k_-)^2 + 4Q^2k_-^2)\,(\log(iQk_-) + \psi(\frac{3}{2} + \frac{k_i^2 - 2k_+k_-}{4iQk_-})).$

[Consistent with Lifshitz scaling: $(x^-, x^+, x_i) \rightarrow (\lambda^2 x^-, \lambda^0 x^+, \lambda x_i)$.]

[For $Q \rightarrow 0$, $\psi() \rightarrow \log()$: recover AdS 2-point correlation fn $k^4 \log k^2$.]

To gain some insight, note:

$(k_i^2 - 2k_+k_-)^2 + 4Q^2k_-^2 = (k_i^2 - 2(k_+ - iQ)k_-)(k_i^2 - 2(k_+ + iQ)k_-)$

Effective x^+-momentum shift: $k_+ \rightarrow k_+ \pm iQ$ $(e^{ik_+x^+} \rightarrow e^{ik_+x^+}e^{\pm Qx^+})$

— reminiscent of Liouville-like wall in $c = 1$ string theory.

Also, for $k_i = 0$: $(k_i^2 - 2k_+k_-)^2 + 4Q^2k_-^2 \rightarrow (k_+^2 + Q^2)k_-^2$,

$i.e.$ effective mass-gap in x^+-direction.

Free SYM: $S = \int \frac{d^4x}{g_{YM}^2(x^+)} TrF^2 \rightarrow \int d^4x\, e^{-\Phi(x^+)}[(\partial_j A_i)^2 - 2(\partial_+ A_i)(\partial_- A_i)]$.

Wave modes $e^{ik_+x^+ + ik_-x^- + ik_ix^i} \rightarrow k_i^2 + 2(k_+ + iQ)k_- = 0$, $i.e.$

$k_+ = -\frac{k_i^2}{2k_-} - iQ$. For generic k_i, k_-, x^+-momentum k_+ nonzero, $i.e.$ generic waves move along x^+-direction due to dilaton x^+-potential.

Aspects of some solutions

$$\Phi' = 2Q\tanh(Qx^+),\ i.e.\ \ \Phi = \Phi_0 + 2\log\cosh(Qx^+),$$
$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2\tanh^2(Qx^+)(dx^+)^2 + dw^2].$$
$$x^+ \to \pm\infty: \ \ g_{++} \sim (\Phi')^2 \to Q^2, \qquad x^+ \to 0: \ \ g_{++} \sim 0.$$

Akin to "junction" of two Lifshitz-like systems joined together with $\frac{1}{Q}$-sized AdS-Schrodinger-like core about $x^+ = 0$.

Q large, core shrinks, "junction" becomes sharper.

SYM gauge coupling: $g_{YM}^2 = e^\Phi = g_s\cosh^2(Qx^+)$, x^+-potential for charged dyonic states (varying D-probe tension).

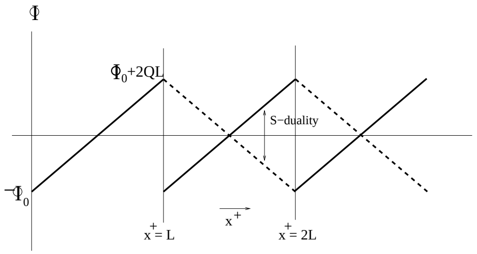
$$\Phi' = \frac{Q}{\cosh^2(Qx^+)}, \qquad \Phi = \Phi_0 + \tanh Qx^+,$$
$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2\frac{Q^2(dx^+)^2}{\cosh^4(Qx^+)} + dw^2].$$

Large x^+ : $g_{++} \to 0$, Schrodinger-like.

$x^+ = 0$: earlier linear dilatonic system, Lifshitz-like behaviour.

Interpolating solutions with radial w-dependence?

Linear dilaton, non-geom DLCQ



Compactify x^+-direction? Linear dilaton not x^+-periodic.

Piecewise linear dilaton: $\Phi = \Phi_0 + 2Qx^+ , \quad x^+ \in [0, L]$,

$$\Phi = \Phi(L) - 2Q(x^+ - L), \quad x^+ \in [L, 2L], \quad \ldots$$

[Einstein metric $ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2(dx^+)^2 + dw^2]$ smooth.]

$\Phi(x^+)$ continuous, not periodic: note S-duality symmetry, $\tau \rightarrow -\frac{1}{\tau}$, $i.e.\ \Phi \rightarrow -\Phi$.

Φ periodic upto S-duality if $\Phi(x^+ + L) = -\Phi(x^+)$, $i.e.$

$$\Phi(L) = -\Phi(0) \quad \Rightarrow \quad \Phi_0 + 2QL = -\Phi_0 \ , \quad i.e. \quad g_s = e^{\Phi_0} = e^{-QL}.$$

Asymptotic string coupling fixed by this non-geometric construction: large string corrections?

Lightlike deformations of $AdS_5 \times S^5$: no nonzero contractions (only $\partial_+\Phi$ nonzero, $g^{++} = 0$), likely no higher derivative corrections. Also preserves half susy.

Also note: dilaton bounded, no singularities anywhere. Contrast with $e.g.$ F-theory, elliptic fiber degenerations, 7-brane sources, ...

Lower dim Lifshitz symmetries $\rightarrow$ lightlike structure in 5-dim, possibly controlled corrections.

$\Rightarrow$ non-geometric construction for dual $\mathcal{N}$=4 SYM too.

Solutions with lightlike axion

Lightlike axion: $c_0 = c_0(x^+)$.

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2(\partial_+ c_0)^2(dx^+)^2 + dw^2] + ds_5^2.$$

Linear axion configuration:

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2 Q^2(dx^+)^2 + dw^2] + ds_5^2\ ,$$

$$c_0 = c_0^0 + 2Qx^+.$$

With x^+-direction compact: $c_0 \to c_0 + 2QL$ as $x^+ \to x^+ + L$: equivalent to $\tau \to \tau + 1$ shift if $QL = \frac{1}{2}$.

D3-D7 interpretation? Axion $\to$ θ-angle in dual gauge theory.

Possibly solutions with nontrivial axion-dilaton exist too.

AdS_4 Lifshitz and M-theory

$ds^2 = \frac{1}{r^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dr^2) + 4d\Omega^2_{X^7}, \quad G_4 = 6\text{vol}(M_4) + Cd\Phi \wedge \Omega_3.$

C normalization const, X^7 Sasaki-Einstein 7-manifold admitting harmonic 3-form Ω_3. Scalar $\Phi(x^\mu)$: no natural interpretation in 11-dim, arises from 4-form flux after compactification.

$\Phi = const, \ \ \tilde{g}_{\mu\nu} = \eta_{\mu\nu} \ \Rightarrow$ usual $AdS_4 \times X^7$ solution.

Effective 3-dim system: $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi, \quad \Box\Phi = 0$.

3-dim gravity $\tilde{g}_{MN}$ trivial: dynamics driven by scalar.

Null: $ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{2}w^2(\Phi')^2(dx^+)^2] + \frac{dw^2}{w^2}$.

Dim'nal reduction $\Rightarrow$ $2+1$-dim bulk $z=2$ Lifshitz spacetime

$$ds^2 = -\frac{dt^2}{w^4} + \frac{dx^2}{w^2} + \frac{dw^2}{w^2} \ .$$

Field theory dual: $1+1$-dim strongly coupled theory, dim'nal redux of null deformation of Chern-Simons theories on M2-branes at conical (CY 4-fold) singularities. ABJM generalizations ... Explore further.

Other AdS Lifshitz-like solutions

- AdS_5 null deformation: lifts to (11-dim) M-theory. Similar to earlier solution: $ds^2 = \frac{-2dx^+dx^- + d\vec{x}^2 + dw^2}{w^2} + (dx^+)^2,\ \ \varphi(x^+) = \sqrt{\frac{2}{\ell^2}}\frac{e^{i\ell x^+}}{R}$.
Note: no x^+-dep in metric, KK-reduction standard, gives $z = 2\ Li_4$ and gauge field.
M-theory lift: $AdS_5^{null} \times \mathbb{CP}^2 \times S^1 \times S^1$ supported by G-flux.
Boundary theory: unclear, possibly M5-brane dual.

- Anisotropic radial Kasner-like solutions
$$ds^2 = \frac{1}{r^2}[dr^2 - r^{2p_0}dt^2 + \sum_i r^{2p_i}(dx^i)^2] + d\Omega_5^2,\ \ e^\Phi = r^\alpha,$$
$$p_0 + \sum_i p_i = 0,\ p_0^2 + \sum_i p_i^2 = \frac{\alpha^2}{2}.$$
With nontrivial scalar Φ ($\alpha \neq 0$): static solutions with spatially anistropic Lifshitz-like scaling, except scalar breaks scaling symmetry ($\alpha = 0\ \Rightarrow\ p_0, p_i = 0$).

- Time-dep AdS (cosmological) solutions with Lifshitz-like symmetries.

Conclusions, open questions

• AdS null deformations $\rightarrow$ dim'nal redux $\rightarrow$ $z = 2$ Lifshitz (for AdS_5) dual to DLCQ of $\mathcal{N}$=4 SYM with varying coupling.

• x^+-noncompact $\rightarrow$ spatially anisotropic Lifshitz-like systems with dilaton x^+-potential. Interesting structure in holographic correlators.

• Finite temperature? Fermions?

• Interesting radial dependence of scalar/metric: holographic RG flows between AdS-lightcone (Schrodinger), Lifshitz, or $Li_{z_1} \rightarrow Li_{z_2}$? Holographic superfluid ground states?

• Further exploration of field theory dual to Lifshitz: $\mathcal{N}$=4 SYM with lightlike varying coupling, DLCQ ...

• Further exploration of AdS_4 solutions: $1 + 1$-dim field theory dual possibly deformation of Chern-Simons theories on M2-branes at singularities (generalizations of ABJM ...).