Extremal Surfaces, Entanglement in Ghost Systems and de Sitter Entropy

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- de Sitter space, dS/CFT and extremal surfaces
- Ghost CFTs, "ghost-spins" and entanglement
- Extremal surfaces and de Sitter entropy, speculations

Refs: arXiv:1711.01107 (KN), 1706.06828, 1608.08351 (Dileep Jatkar, KN),
arXiv:1501.03019, 1504.07430, 1602.06505 (KN).
work in progress with Dileep Jatkar, Kedar Kolekar.

Partially related refs: Miyaji, Takayanagi; Sato; Dong, Silverstein, Torroba.

Holography, de Sitter space, dS/CFT

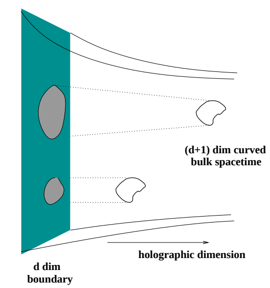
20yrs since AdS/CFT '97 Maldacena; '98 Gubser,Klebanov,Polyakov; Witten.

Holography: quantum gravity in $\mathcal{M} \leftrightarrow$ dual without gravity on $\partial\mathcal{M}$ ('t Hooft, Susskind).

(Witten@Strings'98, '01) Gauge/gravity duality and asymptotics —

$\Lambda < 0$: AdS $\rightarrow$ asymptotics at spatial infinity.

Dual: unitary Lorentzian CFT, includes time.

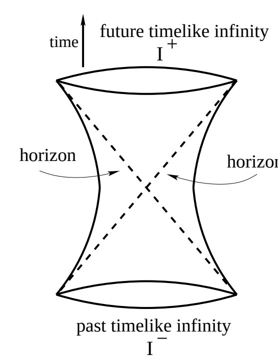


$\Lambda = 0$: flat space $\rightarrow$ null infinity $\rightarrow$ S-matrix, symmetries...

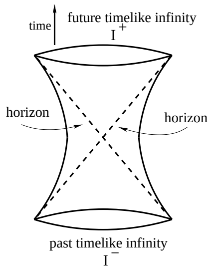
$\Lambda > 0$: de Sitter space

Fascinating for various reasons. Less clear.

Boundary at future/past timelike infinity $\mathcal{I}^{\pm}$.



de Sitter space and dS/CFT



dS/CFT: dual Euclidean non-unitary CFT on dS boundary at future/past timelike infinity $\mathcal{I}^{\pm}$ ('01 Strominger; Witten).

$ds^2 = \frac{R_{dS}^2}{\tau^2}(-d\tau^2 + d\vec{x}^2)$

(Maldacena '02) analytic continuation $r \rightarrow -i\tau,\ R_{AdS} \rightarrow -iR_{dS}$ from Eucl AdS $\rightarrow$ Hartle-Hawking wavefunction of the universe $\Psi_{dS} = Z_{CFT}$.

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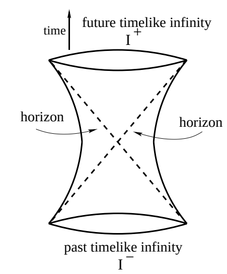

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EAdS interior regularity $\to$ Bunch-Davies dS initial conditions, $\varphi_k(\tau) \sim e^{ik\tau}$ (large $|\tau|$).

$$Z = \Psi[\varphi] \sim e^{iS_{cl}[\varphi]} \sim e^{-\int_k R_{dS}^2 k^3 \varphi_{-k}^0 \varphi_k^0 + \ldots}$$ (semiclassical)

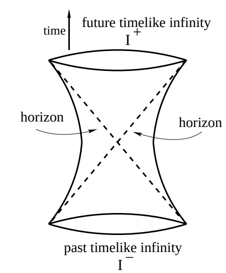
[Bulk expectation values $\langle\varphi_k\varphi_{k'}\rangle \sim \int D\varphi\, \varphi_k\varphi_{k'} |\Psi|^2$] [Dual CFT: $\langle O_k O_{k'}\rangle \sim \frac{\delta^2 Z}{\delta\varphi_k^0 \delta\varphi_{k'}^0}$]

dS_4: Energy-momentum $\langle TT\rangle$ 2-pt fn $\to$ $\mathcal{C}_3 \sim -\frac{R_{dS}^2}{G_4}$, ghost-CFT?

Dual CFT central charge $\mathcal{C}_d \sim i^{1-d}\frac{R_{dS}^{d-1}}{G_{d+1}}$, negative/imaginary, more generally.

Anninos,Hartman,Strominger: higher-spin dS_4 dual to $Sp(N)$ ghost $CFT_3, \ldots$

de Sitter space and dS/CFT



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Ψ^* and Ψ in bulk vevs $\to$ dual involves two CFT copies.

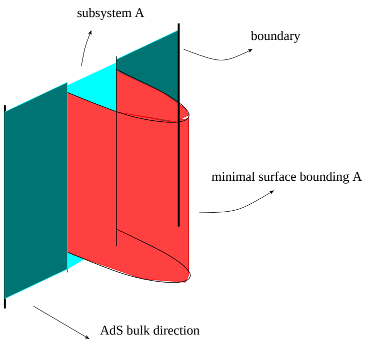
[In general $\Psi = \Psi[g^3]$, final 3-metric is g^3; sum over final boundary condns for bulk vevs.]

Entanglement as probe of dS/CFT?

Entanglement entropy: entropy of reduced density matrix of subsystem.

EE for spatial subsystem A, $S_A = -tr\rho_A \log \rho_A$, with partial trace $\rho_A = tr_B \rho$.

Ryu-Takayanagi: $EE = \frac{A_{min.surf.}}{4G}$



[$\sim$ black hole entropy] Area of codim-2 minimal surface in gravity dual.

Non-static situations: extremal surfaces (Hubeny, Rangamani, Takayanagi).

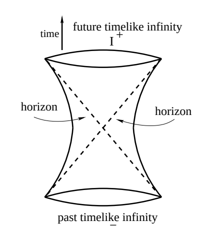
Operationally: const time slice, boundary subsystem $\rightarrow$ bulk slice, codim-2 extremal surface

A speculative generalization of Ryu-Takayanagi to de Sitter space

$\equiv$ bulk analog of setting up entanglement entropy in dual CFT $\rightarrow$

restrict to some boundary Eucl time slice $\rightarrow$ codim-2 dS surfaces

- Poincare slicing, one boundary, subregion at I^+ $\rightarrow$ area<0 surfaces
- ghost-CFTs, "ghost-spins", EE generalization to include -ve norm states.
- dS static coords, connected surfaces from I^+ to I^-: dS entropy?



de Sitter entropy as some sort of entanglement entropy?

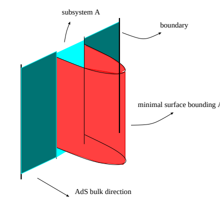
Extremal surfaces, de Sitter (Poincare)

dS extremal surfaces (Poincare)

Recall Ryu-Takayanagi: CFT ground state = empty AdS_{d+1}, $ds^2 = \frac{R^2}{r^2}(dr^2 - dt^2 + dx_i^2)$.

Strip, width $\Delta x = l$, infinitely long. Bulk surface $x(r)$. Turning point r_*.

$$S_A = \frac{V_{d-2} R^{d-1}}{4G_{d+1}} \int \frac{dr}{r^{d-1}} \sqrt{1 + (\partial_r x)^2} \quad \rightarrow \quad (\partial_r x)^2 = \frac{(r/r_*)^{2d-2}}{1-(r/r_*)^{2d-2}} .$$



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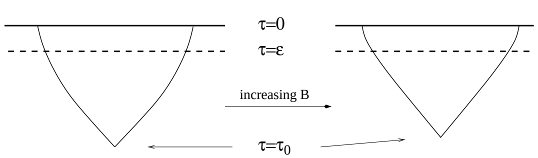
de Sitter, Poincare : $ds^2_{d+1} = \frac{R^2_{dS}}{\tau^2}(-d\tau^2 + dw^2 + dx_i^2)$ $\rightarrow$ EE in dual Eucl CFT

$\rightarrow$ bulk Eucl time slice $w = const$, subregion at I^+ $\rightarrow$ codim-2 extremal surface.

CFT central charge negative/imaginary $\rightarrow$ real surfaces will not work:

[strip] $S_{dS} = \frac{R^{d-1}_{dS} V_{d-2}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}} \sqrt{1 - (\partial_\tau x)^2}.$



$Extremize$ $\rightarrow$ $(\partial_\tau x)^2 = \frac{B^2 \tau^{2d-2}}{1+B^2 \tau^{2d-2}}$, $B^2 = const$, conserved quantity.

- Sign difference from AdS $\Rightarrow$ no real "turning point".

Join two half-extremal-surfaces with cusp $\rightarrow$ minimize area $\rightarrow$ null surface. Area vanishes.

Real codim-2 surfaces: featureless, no apparent relation to EE.

dS extremal surfaces (Poincare)

$Extremize \quad \rightarrow \quad (\partial_\tau x)^2 = \frac{-A^2\tau^{2d-2}}{1-A^2\tau^{2d-2}}$. $\qquad$ [$A^2 < 0$ is earlier real-τ solution]

dS_4/CFT_3: take $A^2 > 0$. Near $\tau \rightarrow 0$: $\dot{x}^2 \sim -A^2\tau^4 \quad i.e. \quad x(\tau) \sim \pm iA\tau^3 + x(0)$.

Spatial dirn in Eucl CFT $\Rightarrow x(\tau)$ real $\Rightarrow \tau = iT$, imaginary path. Turning point $T_* = \frac{1}{\sqrt{A}}$.

Join half-extremal-surfaces smoothly at τ_*. $\quad dS_{d+1}$, d even: $A^2 < 0,\ \tau = iT$.

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$x(\tau)$ real $\Rightarrow$ imaginary path $\tau = iT \rightarrow$ complex extremal surface
$\equiv$ analytic continuation $r \rightarrow -i\tau, R \rightarrow -iR_{dS}$ from AdS Ryu-Takayanagi.

Complex surfaces: no canonical action;

$$S_{AdS} = \frac{R^{d-1}V_{d-2}}{4G_{d+1}} \int \frac{dr}{r^{d-1}} \sqrt{1+x'^2} \rightarrow S_{dS} = -i\frac{R_{dS}^{d-1}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}} \frac{V_{d-2}}{\sqrt{1-(-1)^{d-1}A^2\tau^{2d-2}}}$$

- de Sitter isometry $\Rightarrow$ boundary Euclidean time direction is any symmetry direction.
- sphere: subleading log-div, conf anomaly $\leftrightarrow$ Ψ log-coeff.

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AdS/CFT: $Z_{CFT} = Z_{bulk}$; dS/CFT: $Z_{CFT} = \Psi_{dS} \Rightarrow$

EE$_{dS/CFT}$ $\neq$ bulk EE (via bulk density matrix, $\Psi^*\Psi$) (Maldacena, Pimentel)

$S_{dS_4} \sim -\frac{R_{dS}^2}{G_4} V_1 (\frac{1}{\epsilon} - c\frac{1}{l})$ $\rightarrow$ negative area in dS_4, $\mathcal{C}_3 < 0$.

Negative EE, 2-dim ghost CFTs; "Ghost-spins"

Can $EE < 0$ arise from (i) CFT: 2-dim ghost-CFTs ($c < 0$) & replica, or (ii) QM: "ghost-spins" and reduced density matrices?

bc-ghosts, $c=-2$: replica and EE

$T(w) = (\partial_w z)^2 T(z) + \frac{c}{12}\{z,w\}$, Schwarzian $\{z,w\} = \frac{2\partial_w^3 z \partial_w z - 3(\partial_w^2 z)^2}{2(\partial_w z)^2}$ $[w = x + it_E]$

Subsystem A: single interval betw $x = u, v$ on time slice $t_E = const.$ (Calabrese,Cardy)

Replica w-space $\rightarrow$ z-plane under conformal transfmn $z = (\frac{w-u}{w-v})^{1/n}$.

z-plane: $\langle T(z)\rangle_{\mathbb{C}} = 0$ $\rightarrow$ z-plane maps to $SL(2,Z)$ inv vacuum.

$$\langle T(w)\rangle_{\mathcal{R}_n} = \frac{c}{12}\{z,w\} = \frac{\langle T(w)\Phi_n(u)\Phi_{-n}(v)\rangle}{\langle \Phi_n(u)\Phi_{-n}(v)\rangle} = \frac{\int D\varphi T(w) e^{-S}}{\int D\varphi e^{-S}}$$

Twist operators at $w = u, v$ $\rightarrow$ $tr\rho_A^n \equiv \frac{Z_n}{Z_1^n}$ transforms as twist operator 2-pt fn $\rightarrow$ S_A.

bc-ghost CFTs

- $SL(2,Z)$ vacuum $|0\rangle \neq$ ghost ground state $|\downarrow\rangle$ in general.

$S \sim \int d^2z\, b\bar{\partial}c,\ (h_b, h_c) = (\lambda, 1-\lambda), \quad c = 1 - 3Q^2 < 0,$ Background Charge $Q = 1 - 2\lambda$

$b(z) = \sum \frac{b_m}{z^{m+\lambda}}, \quad c(z) = \sum \frac{c_m}{z^{m+1-\lambda}}; \quad L_0 = \sum_{n>0} n(b_{-n}c_n + c_{-n}b_n) + \frac{\lambda(1-\lambda)}{2}.$

SL(2) inv vacuum $|0\rangle: \quad T(z)|0\rangle = \sum_m \frac{L_m}{z^{m+2}}|0\rangle = regular$

$\Rightarrow \quad L_{m\geq -1}|0\rangle = 0, \quad b_{m\geq 1-\lambda}|0\rangle = 0, \quad c_{m\geq\lambda}|0\rangle = 0$ whereas $b_0|\downarrow\rangle = 0$

- $j_0^\dagger = -(j_0 + Q)$ Charge asymmetry.

$U(1)$ charge symmetry $\delta b = -i\epsilon b,\ \delta c = i\epsilon c \ \rightarrow$ ghost current $j(z) = -:bc:$

$j(z) = \sum_m \frac{j_m}{z^{m+1}}, \qquad [L_m, j_n] = -nj_{m+n} + \frac{1}{2}Qm(m+1)\delta_{m,-n}$

$[j_0, O_p] = pO_p,\ j_0|q\rangle = q|q\rangle \ \Rightarrow\ p\langle q'|O_p|q\rangle = \langle q'|[j_0, O_p]|q\rangle = (-q' - Q - q)\langle q'|O_p|q\rangle$

Corrn fn $\neq 0$ only if Bgnd Charge cancelled $i.e.\ p = -(q + q' + Q) \ \Rightarrow\ \langle -q - Q|q\rangle = 1.$

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- $\lambda = 1,\ c = -2$: $SL(2)$ vacuum $|0\rangle = |\downarrow\rangle$ ghost ground state

$b_{m\geq 0}|0\rangle = 0,\ c_{m\geq 1}|0\rangle = 0;\quad Q = -1$: $\langle +1|0\rangle = \langle 0|c_0|0\rangle = 1 \ \leftarrow$ zero mode insertion

$\langle b(z)c(w)\rangle_0 \equiv \langle 0|c_0 \sum_{m,n} \frac{b_m}{z^{m+1}}\frac{c_n}{w^n}|0\rangle = \langle 0|c_0 \sum_{m=0}^{\infty} \frac{w^m}{z^{m+1}} b_m c_{-m}|0\rangle = \frac{1}{z-w}\langle 0|c_0|0\rangle$

whereas $\langle 0|b(z)c(w)|0\rangle = \frac{1}{z-w}\langle 0|0\rangle = 0.$ Plethora of negative norm states

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- Replica argument is applicable for the ghost ground state if it is the $SL(2)$ vacuum: $c = -2$ bc-ghost CFT $\to |\downarrow\rangle = |0\rangle$ with $L_0 = 0$.
- Regularity condition $\langle T(z)\rangle_{\mathbb{C}} = 0$ $\to$ $\langle -Q|\, T(z)\, |0\rangle = 0$

Incorporate background charge, or $\langle T(z)\rangle = 0$ trivially from zero modes. $[c = -2 \to Q = -1]$

Replica formulation formally applies now: $c < 0 \Rightarrow S_A < 0$.

$\mathbb{Z}_N$ bc-orbifold CFTs (Saleur, Kausch, Flohr, … '90s) confirm negative conf dims of twist ops $[l \equiv v - u]$

$$tr\rho_A^n = \prod_{k=1}^{n-1} \langle 0|\sigma_{k/N}^{-}(v)\sigma_{k/N}^{+}(u)|0\rangle = l^{\frac{1}{3}(n-1/n)} \to S_A = -\lim_{n\to 1}\partial_n\, tr\rho_A^n = -\frac{2}{3}\log\frac{l}{\epsilon}$$

“Ghost-spins”

Abstract away from technicalities of ghost CFTs, replica subtleties: simple QM toy models of ghost-like systems with negative norm states $\rightarrow$ reduced density matrix (RDM) after partial trace $\rightarrow$ EE.

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Recall ordinary spin: $\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1, \qquad \langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0$

$$|\psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle \;\; \rightarrow \;\; \langle\psi|\psi\rangle = |c_1|^2 + |c_2|^2 > 0$$

cook up

“Ghost-spin” $\rightarrow$ 2-state spin variable with indefinite norm. $\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 0, \qquad \langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 1$

$$|\psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle \;\; \rightarrow \;\; \langle\psi|\psi\rangle = c_1 c_2^* + c_2 c_1^* \not> 0. \;\; e.g. \; |\uparrow\rangle - |\downarrow\rangle \text{ has norm } -2.$$

$|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle); \;\; \langle+|+\rangle = \gamma_{++} = 1, \;\; \langle-|-\rangle = \gamma_{--} = -1, \;\; \langle+|-\rangle = \langle-|+\rangle = 0$

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$|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$; $\ \langle +|+\rangle = \gamma_{++} = 1,\ \langle -|-\rangle = \gamma_{--} = -1,\ \ \langle +|-\rangle = \langle -|+\rangle = 0$

Two ghost-spins: $|\psi\rangle = \sum \psi^{ij}|ij\rangle$, adjoint: $\langle\psi| = \sum \langle ij|\psi^{ij*}$,

$$\langle\psi|\psi\rangle = \langle k|i\rangle\langle l|j\rangle\psi^{ij}\psi^{kl*} \equiv \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl*} = \gamma_{ii}\gamma_{jj}|\psi^{ij}|^2.$$

$\rho = |\psi\rangle\langle\psi| \ \rightarrow$ trace over one ghost-spin $\rightarrow$ reduced density matrix for remaining ghost-spin $\rightarrow$ von Neumann entropy.

Even number of ghost-spins $\rightarrow$ calculations, interpretation sensible.

Two ghost-spins

$$|\psi\rangle = \sum \psi^{ij}|ij\rangle, \quad \gamma_{\pm\pm} = \pm 1 \,.$$

$$\langle\psi|\psi\rangle = \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl*} = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 = \pm 1$$

Trace over one ghost-spin $\rightarrow$ ρ_A for remaining ghost-spin $\rightarrow$ von Neumann entropy S_A.

RDM: $\rho_A = tr_B\rho \equiv (\rho_A)^{ik}|i\rangle\langle k| \,, \quad (\rho_A)^{ik} = \gamma_{jl}\psi^{ij}\psi^{kl*} = \gamma_{jj}\psi^{ij}\psi^{kj*}$

$$(\rho_A)^{++} = |\psi^{++}|^2 - |\psi^{+-}|^2, \qquad (\rho_A)^{+-} = \psi^{++}\psi^{-+*} - \psi^{+-}\psi^{--*},$$
$$(\rho_A)^{-+} = \psi^{-+}\psi^{++*} - \psi^{--}\psi^{+-*}, \quad (\rho_A)^{--} = |\psi^{-+}|^2 - |\psi^{--}|^2.$$

Define $\log\rho_A$ using expansion, using mixed-index RDM $(\rho_A)_i{}^k = \gamma_{ij}(\rho_A)^{jk}$.

EE: $S_A = -\gamma_{ij}(\rho_A \log\rho_A)^{ij} \rightarrow -(\rho_A)_+^+(\log\rho_A)_+^+ - (\rho_A)_-^-(\log\rho_A)_-^-$

In general, $+ve$ norm $\not\Rightarrow$ $+ve$ RDM, EE. [however, correlated ghost-spins]

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Simple subfamily, diagonal ρ_A: $\rho_A^{++} = \pm x,\ \rho_A^{--} = \mp(1-x)$. [$\log\rho_A$ diag]

$$(\rho_A)_+^+ = \pm x, \quad (\rho_A)_-^- = \pm(1-x), \qquad 0 < x = \frac{|\psi^{++}|^2}{|\psi^{++}|^2+|\psi^{--}|^2} < 1$$

$\langle\psi|\psi\rangle > 0$: $S_A = -x\log x - (1-x)\log(1-x) > 0$

$+ve$ norm $\Rightarrow$ $+ve$ EE.

$\langle\psi|\psi\rangle < 0$: $S_A = x\log(-x) + (1-x)\log(-(1-x)) = x\log x + (1-x)\log(1-x) + i\pi$

$-ve$ norm $\Rightarrow$ some ρ^A eigenvalues $-ve$ $\Rightarrow$ $-ve$ Re(EE), const Im(EE).

Entangled ghost-spins and spins

- Entangled ghost-spins: in general RDM shows new EE patterns.

  n ghost-spins: $|\psi\rangle = \psi^{++\cdots}|++\ldots\rangle + \psi^{--\cdots}|--\ldots\rangle$,

$\langle\psi|\psi\rangle = |\psi^{++\cdots}|^2 + (-1)^n|\psi^{--\cdots}|^2$, $(\rho_A)^+_+ = |\psi^{++\cdots}|^2$, $(\rho_A)^-_- = (-1)^n|\psi^{--\cdots}|^2$

e.g. 2 ghost-spins: $|\psi\rangle = \psi^{++}|+\rangle|+\rangle + \psi^{--}|-\rangle|-\rangle$ $\longrightarrow$ Correlated ghost-spins.

Finite ghost-spin chains: nearest neighbour interactions $\rightarrow$ organize EE.

Even ghost-spins: ground states $+ve$ norm, $+ve$ EE.

Add small $-ve$ norm, still $+ve$ EE.

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Finite ghost-spin chains: nearest neighbour interactions $\rightarrow$ organize EE.

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Add small $-ve$ norm, still $+ve$ EE.

- Disentangled ghost-spins and spins $\Rightarrow$ product states $|\psi\rangle = |\psi_s\rangle\,|\psi_{gs}\rangle$

Ghost-spins: $\gamma_{++} = 1, \;\; \gamma_{--} = -1$; Spin metric $+ve$ definite: $g_{ij} = \delta_{ij}$.

$\langle\psi|\psi\rangle = \langle\psi_s|\psi_s\rangle\langle\psi_{gs}|\psi_{gs}\rangle, \quad \langle\psi_s|\psi_s\rangle > 0, \; \langle\psi_{gs}|\psi_{gs}\rangle = \gamma_{i_1j_1}..(\psi_{gs})^{i_1i_2\cdots}(\psi^*_{gs})^{j_1j_2\cdots}.$

Normalize $\pm$ norm states to norm ± 1: $\langle\psi_{gs}|\psi_{gs}\rangle \gtrless 0 \rightarrow \langle\psi|\psi\rangle = \pm 1$

Trace over all ghost-spins $\rightarrow$ RDM $\rho^s_A = tr_{gs}\rho$ for spins alone $\rightarrow tr\rho^s_A = \pm 1 \;\longrightarrow$

$+ve$ norm: $S_A = -\sum_i \lambda_i \log\lambda_i > 0$; $\quad -ve$ norm: $S_A = \sum_i \lambda_i \log\lambda_i + i\pi, \; ReS_A < 0$

Entangled ghost-spins and spins: more intricate, new EE patterns.

Ghost-spin chains $\rightarrow$ bc-ghost CFTs

Infinite ghost-spin chains $\rightarrow$ continuum limit $\rightarrow$ ghost-CFTs?

Ghost-spins as microscopic building blocks of ghost/nonunitary CFTs?

Recall: Ising model at critical point is a CFT of free massless fermions.

Hamiltonian $H = J \sum_n \left(\sigma_{b(n)}\sigma_{c(n+1)} + \sigma_{b(n)}\sigma_{c(n-1)}\right)$

Spin variables: $\{\sigma_{bn},\ \sigma_{cn}\} = 1\ ,\quad [\sigma_{bn}, \sigma_{bn'}] = [\sigma_{cn}, \sigma_{cn'}] = [\sigma_{bn}, \sigma_{cn'}] = 0.$

$\sigma_{bn}^\dagger = \sigma_{bn},\ \ \sigma_{cn}^\dagger = \sigma_{cn}\ ;\qquad \sigma_b|\downarrow\rangle = 0,\quad \sigma_b|\uparrow\rangle = |\downarrow\rangle,\quad \sigma_c|\uparrow\rangle = 0,\quad \sigma_c|\downarrow\rangle = |\uparrow\rangle.$

Like b_n, c_n ops of bc-CFT, $\{b_n, c_m\} = \delta_{n,-m}$: but σ_{bn}, σ_{cn} bosonic (distinct sites, commute).

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Like b_n, c_n ops of bc-CFT, $\{b_n, c_m\} = \delta_{n,-m}$: but σ_{bn}, σ_{cn} bosonic (distinct sites, commute).

Construct Jordan-Wigner transform $\rightarrow$ fermionic gh.sp. variables.

$$a_{bn} = \prod_{k=1}^{n-1} i(1-2\sigma_{ck}\sigma_{bk})\sigma_{bn}\ , \qquad a_{cn} = \prod_{k=1}^{n-1}(-i)(1-2\sigma_{ck}\sigma_{bk})\sigma_{cn}.$$

This gives: $\{a_{bi}, a_{cj}\} = \delta_{ij}\ , \quad \{a_{bi}, a_{bj}\} = 0\ , \quad \{a_{ci}, a_{cj}\} = 0\ .$

Inner products: $|\underrightarrow{\uparrow\uparrow}\rangle = a_{c1}a_{c2}|\downarrow\downarrow\rangle; \quad \langle\underleftarrow{\downarrow\downarrow}| = \langle\uparrow\uparrow|a_{b2}a_{b1}; \quad \langle\underleftarrow{\downarrow\downarrow}|\underrightarrow{\uparrow\uparrow}\rangle = \langle\uparrow\uparrow|\downarrow\downarrow\rangle = 1$ etc

$\psi_1|\downarrow\downarrow\rangle + \psi_2|\underrightarrow{\uparrow\uparrow}\rangle \ \rightarrow\ \psi_1^*\psi_2\langle\uparrow\uparrow|a_{b_2}a_{b1}\ a_{c1}a_{c2}|\downarrow\downarrow\rangle + \psi_2^*\psi_1\langle\uparrow\uparrow|\downarrow\downarrow\rangle = \psi_1^*\psi_2 + \psi_2^*\psi_1.$

Ghost-spin chains → bc-ghost CFTs

Jordan Wigner: $a_{bn} = \prod_{k=1}^{n-1} i(1-2\sigma_{ck}\sigma_{bk})\sigma_{bn}, \quad a_{cn} = \prod_{k=1}^{n-1}(-i)(1-2\sigma_{ck}\sigma_{bk})\sigma_{cn}.$

fermionic ghost-spin variables: $\{a_{bi}, a_{cj}\} = \delta_{ij}, \quad \{a_{bi}, a_{bj}\} = 0, \ \{a_{ci}, a_{cj}\} = 0.$

$$H = J\sum_n (\sigma_{b(n)}\sigma_{c(n+1)} + \sigma_{b(n)}\sigma_{c(n-1)}) \ \rightarrow \ iJa_{bn}(a_{c(n+1)} - a_{c(n-1)}) \sim -b\partial c$$

→ lattice discretization of bc-ghost CFT.

Momentum space variables: $b_k = \frac{1}{\sqrt{N}}\sum_n e^{ikn} a_{bn}, \quad c_k = \frac{1}{\sqrt{N}}\sum_n e^{ikn} a_{cn}$

→ $\{b_k, c_{k'}\} = \delta_{k+k',0}, \quad \{b_k, b_{k'}\} = 0 = \{c_k, c_{k'}\}$ [regulated with N odd].

→ continuum limit (reinstate lattice spacing a):

$$H = 2J\sum_k \sin(k'a)\, b_k c_{k'}\, \delta_{k+k',0} \ \xrightarrow{J\sim 1/2a} \ \sum_{k>0} k\,(b_{-k}c_k + c_{-k}b_k) + zpe$$

Symmetries: $\sigma_{b(n)} \rightarrow e^{i\alpha}\sigma_{b(n)}, \ \sigma_{c(n+1)} \rightarrow e^{-i\alpha}\sigma_{c(n+1)} \rightarrow U(1)$ symmetry in bc-CFT.

$$a \rightarrow \xi^{-1}a\,, \quad H \rightarrow \xi H\,, \qquad \sigma_{b(n)} \rightarrow \xi^{\lambda}\sigma_{b(n)}\,, \qquad \sigma_{c(n+1)} \rightarrow \xi^{1-\lambda}\sigma_{c(n+1)}$$

global scaling symmetry → continuum conformal symmetry, weights $(h_b, h_c) = (\lambda, 1-\lambda)$.

Ghost-spins: microscopic building blocks of general ghost-CFTs?

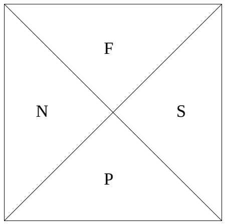
$e.g.$ appropriate 3-dim chain of N-level ghost-spins $\longrightarrow$ $\text{CFT}_3^{Sp(N)}$ (symplectic fermions)?

Extremal surfaces, de Sitter entropy

de Sitter, static coordinatization

dS_{d+1}: $ds^2 = -(1 - \frac{r^2}{l^2})dt^2 + \frac{dr^2}{1-\frac{r^2}{l^2}} + r^2 d\Omega_{d-1}^2$.

N, S $(0 \le r < l)$: static patches. t is time $\rightarrow$ translations are symmetries. Event horizons for observers in N, S.



de Sitter entropy = area of cosmological horizon. (Gibbons,Hawking)

Euclidean continuation $t \rightarrow -it_E$ is sphere (no boundary): Eucl action $I_E = -\log Z = \beta F$.

de Sitter entropy $S_{dS_{d+1}} = -I_E = \frac{l^{d-1} V_{S^{d-1}}}{4G_{d+1}} \quad \rightarrow \quad \frac{\pi l^2}{G_4}$ $[dS_4]$.

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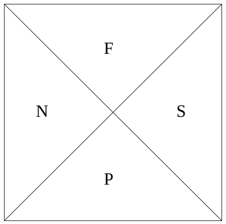

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Future/past universes F, P: $1 \le \frac{r}{l} \le \infty \ \rightarrow\ dS/CFT$, extremal surfaces?

Recast as $ds^2 = \frac{l^2}{\tau^2}\big(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2\big) \qquad [\tau = \frac{l}{r},\ \ w = \frac{t}{l}]$

τ is bulk time direction now: near $I^\pm$, $\tau \rightarrow 0$, $\quad ds^2 \sim \frac{l^2}{\tau^2}(-d\tau^2 + dw^2 + d\Omega_{d-1}^2)$.

Small patches at $I^\pm$ $\sim$ Poincare: Eucl CFT$_{F,P}$ on $R_w \times S^{d-1}$ at I^+ or I^-.

Global dS $\sim$ dual to $CFT_P \times CFT_F$ on $(R_w \times S^{d-1})^2$.

Real extremal surfaces from I^+ to I^-? Bulk physics $\rightarrow \Psi^*\Psi \rightarrow$ two boundaries?

Extremal surfaces, de Sitter entropy

$$ds^2 = \frac{l^2}{\tau^2}\Big(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2\Big) \qquad [\tau = \frac{l}{r},\ \ w = \frac{t}{l}]$$

Boundary Euclidean time slice $\rightarrow$ codim-2 surfaces, area $\sim \frac{l^{d-1}}{G_{d+1}}$

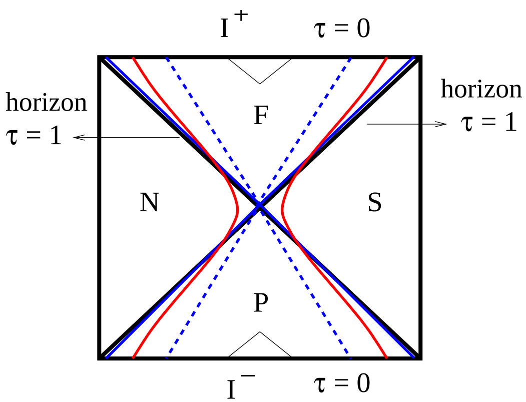
All S^{d-1} equatorial planes equivalent: take $\theta = \frac{\pi}{2}$ for convenience.

Area $S = l^{d-1} V_{S^{d-2}} \int \frac{d\tau}{\tau^{d-1}} \sqrt{\frac{1}{1-\tau^2} - (1-\tau^2)(w')^2}$

Extremize $\longrightarrow$ $\dot{w}^2 \equiv (1-\tau^2)^2(w')^2 = \frac{B^2\tau^{2d-2}}{1-\tau^2+B^2\tau^{2d-2}}$,

B^2=const, $S = 2l^{d-1} V_{S^{d-2}} \int_\epsilon^{\tau_*} \frac{d\tau}{\tau^{d-1}} \frac{1}{\sqrt{1-\tau^2+B^2\tau^{2d-2}}}$.

Hartman-Maldacena surfaces (AdS bh) rotated.



Turning point τ_* at $|\dot{w}| \rightarrow \infty$: $1 - \tau_*^2 + B^2\tau_*^{2d-2} = 0$. Width $\Delta w \sim \log\frac{2}{\tau_* - 1}$

Extremal surfaces, de Sitter entropy

$$ds^2 = \frac{l^2}{\tau^2}\left(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2\right) \qquad [\tau = \frac{l}{r},\ \ w = \frac{t}{l}]$$

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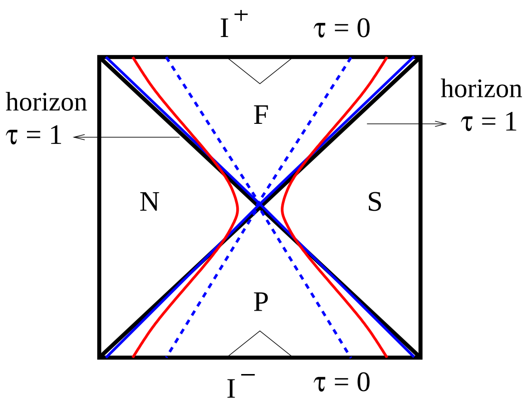


Turning point τ_* at $|\dot{w}| \rightarrow \infty$: $\ 1 - \tau_*^2 + B^2 \tau_*^{2d-2} \ = \ 0$. $\quad$ Width $\Delta w \sim \log \frac{2}{\tau_* - 1}$

dS_4: Well-defined for $0 < B < \frac{1}{2}$: as $B^2 \rightarrow 0$, turning point near $\tau_* \sim 1 + \frac{B^2}{2}$.

Real connected surfaces from I^+ to I^-, near bifurcation region ($\tau_* \sim 1$).

$$\text{Area} \quad \frac{S}{4G_{d+1}} = \frac{2 l^{d-1} V_{S^{d-2}}}{4G_{d+1}} \int_\epsilon^1 \frac{d\tau}{\tau^{d-1}} \frac{1}{\sqrt{1-\tau^2}} \ \rightarrow \ \frac{S}{4G_4} = \frac{\pi l^2}{G_4} \frac{l}{\epsilon_c}$$

Area law divergence coefficient: dS_4 entropy. Excludes N, S.

$w = const$: tricky, but similar surfaces, same area.

Extremal surfaces, de Sitter entropy

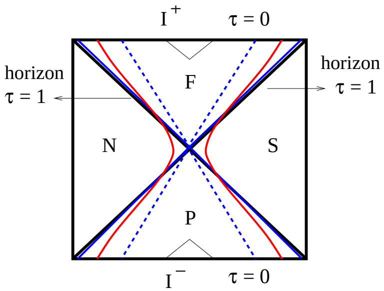
$$ds^2 = \frac{l^2}{\tau^2}\Big(-\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2\Big) \qquad [\tau = \frac{l}{r},\ \ w = \frac{t}{l}]$$

Boundary Euclidean time slice: any S^{d-1} equatorial plane or $w = const$ slice.

Real connected surfaces from I^+ to I^-, near bifurcation region ($\tau_* = 1$).

S^{d-1} equator: like rotated versions of Hartman-Maldacena surfaces (AdS bh): passing near bifurcation region, min area $\frac{S}{4G_4} = \frac{\pi l^2}{G_4}\frac{l}{\epsilon_c}$

$w = const$: more tricky, but can find similar surfaces, same area.



Interpreted in CFT: this is just area law divergence; rescaling cutoff modifies coefficient.

Speculations: dS_4 approximately dual to $CFT_F \times CFT_P$ in thermofield-double-like entangled state $|\psi\rangle = \sum \psi^{i_n^F, i_n^P} |i_n^F\rangle |i_n^P\rangle$?

Consider 3-dim N-level ghost-spin chains in universality class of ghost CFT$_3$ dual to dS_4:
two copies of ghost-CFTs, at I^+ and I^- $\rightarrow$ effectively even ghost-spins (correlated)
$\rightarrow$ $+ve$ norm, $+ve$ entanglement entropy $\rightarrow$ emergence of bulk physics?

Bulk time evolution: $|i_n^P\rangle \rightarrow |i_n^F\rangle \Rightarrow$ unitarily equivalent to $|\psi\rangle = \sum \psi^{i_n^F, i_n^F} |i_n^F\rangle |i_n^F\rangle$ at I^+.

Bulk time evolution $\neq$ boundary Eucl time evoln; ER=EPR (Maldacena,Susskind), explore ...

Conclusions, questions

- Complex codim-2 extremal surfaces in de Sitter space (Poincare)
  $\rightarrow$ analytic continuation from AdS Ryu-Takayanagi.
  EE$_{dS/CFT}$ with $Z_{CFT} = \Psi_{dS}$. dS_4: area$< 0 \rightarrow EE < 0 \leftarrow c < 0$.
- Ghost-spins: toy QM models, negative norm states, $Re(EE) < 0$.
  Entangled ghost-spins & spins. Subsectors: $+ve$ norm $\rightarrow +ve$ EE.
- de Sitter, static coordinates: real connected surfaces from I^+ to I^-,
  passing near bifurcation region in a limit, coefficient dS_4 entropy.

??? $dS_4 \leftrightarrow CFT_F \times CFT_P$ in entangled states? $+ve$ norm subspace?

EE as probe of dS/CFT? Bulk EE vs EE$_{dS/CFT}$?

??? Ghost-spin chains as microscopic building blocks for ghost-CFTs

$\ldots$ N-level? Ghost-spin glasses? Models for $CFT_3^{ghost} \leftrightarrow dS_4$: emergence of time?

??? Conceptual issues with Bell pairs of spins and ghost-spins etc $\ldots$?