Hyperscaling violation and entanglement entropy in gauge/string theory

K. Narayan
Chennai Mathematical Institute

- Introduction, summary
- Lightcone SYM, string theory, AdS plane waves
- AdS plane waves, hyperscaling violation, entanglement entropy

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KN, Tadashi Takayanagi, Sandip Trivedi, 1212.4328.

Introduction

Interesting to explore holography with reduced symmetries.

Generalizations of AdS/CFT to nonrelativistic systems

$\rightarrow$ holographic condensed matter, ...

[Son; Balasubramanian,McGreevy; Adams et al; Herzog et al; Maldacena et al; ...]

$\rightarrow$ Phases of gauge/string theory with non-relativistic symmetries.

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$\rightarrow$ Phases of gauge/string theory with non-relativistic symmetries.

Sometimes qualitative features of some of these phases from the gravity/string descriptions can be distilled to give interesting, perhaps unexpected insights for gauge theories.

Introduction

Nonrelativistic AdS/CFT $\rightarrow$ holographic condensed matter . . .

$\rightarrow$ Phases of gauge/string theory with non-relativistic symmetries.

Sometimes qualitative features of some of these phases from the gravity/string descriptions can be distilled to give interesting, perhaps unexpected insights for gauge theories.

In the present context: entanglement entropy in various 3- and 4-dim quantum field theories exhibits a leading divergence called the area law.

Ryu-Takayanagi bulk prescription for EE using gravity dual.

- Gravity systems with hyperscaling violation exhibiting area law deviations can be realized in string theory in terms of strongly coupled CFTs with uniform lightcone momentum density T_{++}.
- Lightcone super Yang-Mills conformal field theories with $T_{++} \rightarrow$ logarithmic violation of area law for holographic entanglement entropy.
- Stronger area law deviations in some strongly coupled 3-dim CFTs.

Non-relativistic AdS/CFT

4-dim $\mathcal{N}=4$ superconformal Yang-Mills theory

dual to IIB string theory on $AdS_5 \times S^5$.

$[ds^2 = \frac{R^2}{r^2}(-2dx^+dx^- + dx_i^2 + dr^2) + R^2 d\Omega_5^2, \qquad R^4 \sim g_{YM}^2 N \alpha'^2]$

Large N limit $\rightarrow$ classical gravity description useful.

A fairly traditional way that gives non-relativistic quantum field theory is to use lightcone variables ($x^- \equiv time$), and study the system with nonzero lightcone momentum $[e.g.\ p_\mu p^\mu = 0 \ \rightarrow \ p_- = \frac{1}{2p_+} p_i^2\ ]$.

For instance, x^+-DLCQ of relativistic $\mathcal{N}=4$ SYM $\longrightarrow$

$z = 2$ nonrelativistic (Galilean) 2+1-dim system.

DLCQ $_{x^+}$ of AdS_5 in lightcone coordinates — nonrelativistic, Schrodinger (Galilean) symmetries [Goldberger, Barbon et al, Maldacena et al].

Holography and Lifshitz scaling

Lifshitz: t, x_i-translations, x_i-rotations, scaling $t \to \lambda^z t,\ x_i \to \lambda x_i$

[z: dynamical exponent]. (smaller than Galilean symmetries: $e.g.$ boosts broken)

Landau-Ginzburg action (free $z = 2$ Lifshitz): $S = \int d^3x((\partial_t\varphi)^2 - \kappa(\nabla^2\varphi)^2)$.

Lifshitz spacetime: $ds_4^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2 + dr^2}{r^2}$. Kachru,Liu,Mulligan

[$z = 1 : AdS$]

Solution to 4-dim gravity with $\Lambda < 0$ and massive gauge field $A \sim \frac{dt}{r^z}$
(or alternatively gauge field + 2-form: dualize to get A-mass) Taylor

[More general Lifshitz-attractor solutions in Einstein-Maxwell-scalar theories Trivedi et al]

[Focus on zero temperature solutions]

String realizations?

Lifshitz scaling in string theory

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String construction for $z = 2$ Lifshitz [Balasubramanian,KN]:

relativistic $\mathcal{N}$=4 SYM $\xrightarrow{x^+ - \mathrm{DLCQ}}$ $z = 2$ non-rel (Galilean) 2+1-dim system.

Gauge coupling $g_{YM}^2(x^+) = e^{\Phi(x^+)}$ varying in lightlike x^+-direction $\longrightarrow$ breaks x^+-shift reducing to 2+1-dim Lifshitz symmetries.

Bulk: AdS + $g_{++}[\sim r^0]$ $\xrightarrow{x^+ - \mathrm{dim.redn.}}$ $z = 2$ Lifshitz.

Kaluza-Klein x^+-reduction of non-normalizable null deformations of $AdS_5 \times S^5$: g_{++} sourced by lightlike scalar (dilaton $\Phi(x^+)$) [$x^- \equiv time$].

[$ds_5^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + \# r^0(dx^+)^2 \longrightarrow ds_4^2 = -\frac{(dx^-)^2}{r^4} + \frac{dx_i^2+dr^2}{r^2}$]

More general: g_{++} sourced by axion-dilaton, fluxes [Donos,Gauntlett; et al].

SYM, $T_{++} \neq 0$: AdS plane waves

[Relativistic $\mathcal{N}$=4 SYM $\xrightarrow{x^+ - \mathrm{DLCQ}}$ $z = 2$ non-rel (Galilean) 2+1-dim system.
Lightcone AdS_5 $\xrightarrow{x^+ - \mathrm{DLCQ}}$ $z = 2$ non-rel Schrodinger (Galilean) symmetries].

Consider $\mathcal{N}$=4 SYM with uniform lightcone momentum density T_{++}.
At large N, we expect the gravity dual to this excited state to be some normalizable deformation of $AdS_5 \times S^5$, with g_{++} nonzero.

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Can identify this precisely: AdS_5 plane wave [AdS + g_{++}],

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$$

Holographic boundary stress tensor using AdS/CFT rules gives

$$T_{++} = \frac{2Q}{8\pi G_5} \longrightarrow \text{uniform lightcone momentum density.}$$

SYM, $T_{++} \neq 0$: AdS plane waves

AdS_5 plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2 + R^2d\Omega_5^2$

- AdS analogs of plane-waves: AdS + g_{++}. Supersymmetric.
  Likely α'-exact string backgrounds due to lightlike nature.

- Nonzero (boundary) lightcone momentum density T_{++} :
  null energy condition $\Rightarrow$ $T_{++} \geq 0$.

- Can be realized as "zero temperature", highly boosted double scaling limit of boosted AdS Schwarzschild black branes (Singh).
  Simple excited pure states in $\mathcal{N}$=4 SYM CFT (on D3-branes):
  double scaling limit of SYM thermal state.

- Generalize to AdS_D plane wave ($e.g.$ states in M2-, M5- CFTs):
  $$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^{D-3}(dx^+)^2 + R^2d\Omega_5^2$$

  Bulk x^+-dimensional reduction?

AdS plane waves: x^+-dim. redn.

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$$

AdS plane wave: Q $\sim$ lightcone-momentum density T_{++} $[R^4 \sim g_{YM}^2 N \alpha'^2]$.

Dimensionally reduce 10-dim metric on S^5 and on x^+-circle:

4-dim Einstein metric ($x^- \equiv t$): $ds_E^2 = \frac{R^3\sqrt{Q}}{r}\left(-\frac{dt^2}{Qr^4} + dx_i^2 + dr^2\right)$,

Electric gauge field $A = -\frac{dt}{Qr^4}$, scalar $e^\phi \sim r$. Nontrivial IR scales R, Q.

Hyperscaling violation

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$$

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$$ds^2 = r^{2\theta/d}\left(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2 + dr^2}{r^2}\right).$$

d = “boundary” spatial dim (x_i).

θ = hyperscaling violation exponent.

Conformal to Lifshitz (dynamical exponent z).

Arise in effective gravity+vector+scalar theories (Trivedi et al, Kiritsis et al)

Thermodynamics reflects effective space dim $(d-\theta)$ $[e.g.\ S \sim T^{(d-\theta)/z}]$.

Above: $\quad d = 2,\ \ z = 3,\ \ \theta = 1$ $\quad(d_{eff} = d - \theta = 1)$.

Hyperscaling violation

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^{D-3}(dx^+)^2$$

AdS_D plane wave, $D = d+3$, Q lightcone-momentum density, $(D-1)$-dim.

Dim'nal redn: $$ds_E^2 = \frac{R^2(R^2Q)^{1/(D-3)}}{r}\left(-\frac{dt^2}{Qr^{D-1}} + dx_i^2 + dr^2\right).$$

"boundary" spatial dimension $d = D-3$, $z = \frac{d}{2}+2\,,\quad \theta = \frac{d}{2}\,.$

In particular, for the conformal branes of M-theory:

$M2$-brane stacks $\rightarrow$ AdS_4 deformations, $d=1,\ z=\frac{5}{2},\ \theta=\frac{1}{2}$.

$M5$-brane stacks $\rightarrow$ AdS_7 deformations, $d=4,\ z=4,\ \theta=2$.

[General dim'nal reduction: $\int d^Dx\sqrt{-g^{(D)}}\,(R^{(D)} - 2\Lambda)\ =$

$\int dx^+ d^{D-1}x\sqrt{-g^{(D-1)}}\,(R^{(D-1)} - \#\Lambda e^{-2\phi/(D-3)} - \#(\partial\phi)^2 - \# e^{2(D-2)\phi/(D-3)} F_{\mu\nu}^2)$

(recall effective gravity+vector+scalar theories, $e.g.$ Trivedi et al, Kiritsis et al, Takayanagi et al)]

Hyperscaling violation, holog. EE

Entanglement entropy for subsystem A, $S_A = -tr\rho_A \log \rho_A$,

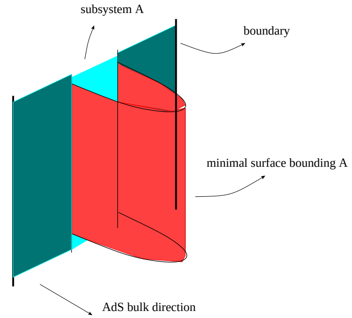
partial trace over A-complement, $\rho_A = tr_B \rho$.

- Ryu-Takayanagi bulk prescription:

find bulk surface bounding A with minimal area.

Holographic entanglement entropy S_A for subsystem A

$\propto$ area of bulk minimal surface bounding A.



E.g.: SYM CFT ground state, strip with width l, bndry area L^2:

ordinary time-t slice in bulk $AdS_5 \longrightarrow S_A \sim \frac{R^3}{G_5}\left(\frac{L^2}{\epsilon^2} - \#\frac{L^2}{l^2}\right)$,

with leading divergence reflecting area law (Bombelli et al; Srednicki, ...)

[ϵ = SYM UV cutoff], and finite part.

- Hyperscaling violating bulk metrics $ds^2 = r^{2\theta/d}\left(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}\right)$

with $\theta = d-1 \longrightarrow$ logarithmic violation of area law.

Gravitational dual of hidden Fermi surfaces? Takayanagi et al, Sachdev et al

AdS plane waves, holog. EE

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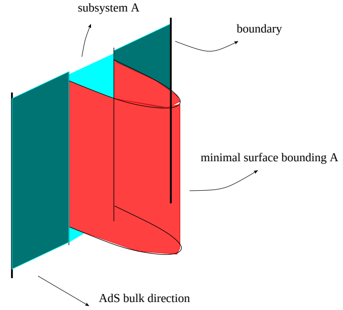
holographic entanglement entropy S_E for subsystem A
$\propto$ area of bulk minimal surface bounding A.



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with $\theta = d-1 \quad \longrightarrow$ logarithmic violation of area law.

• As we have seen, AdS plane waves give microscopic (top-down) descriptions for some of these: $e.g.$

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2 + R^2d\Omega_5^2,$$

AdS_5 plane wave: $\xrightarrow{x^+-\mathrm{dim.redn.}}$ $d=2,\ \ z=3,\ \ \theta=1$

• Concrete gauge/string example: $\mathcal{N}$=4 SYM in pure excited state with uniform lightcone momentum density T_{++} $\rightarrow$ logarithmic behaviour (rather than area law) of entanglement entropy holographically.

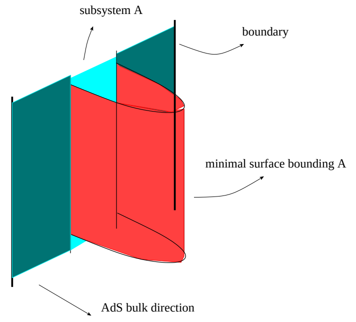
AdS plane waves, holog. EE

AdS_5 plane wave: $\quad ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2.$

- AdS_5 plane wave $\xrightarrow{x^+-\mathrm{dim.redn.}}$

hyperscaling violation $ds^2 = r^{2\theta/d}\left(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}\right)$

with $d = 2, \quad z = 3, \quad \theta = 1$.



4-dim $\mathcal{N}$=4 SYM CFT in these pure excited states ($T_{++} \sim Q$) exhibits logarithmic behaviour of holog. entanglement entropy (not area law).

Holographic EE S_E = area of bulk minimal surface bounding A.
Subsystem A = strip in x_i-plane, width l (possibly wrapping x^+-direction), lying on const-x^- slice ($\equiv$ constant time-t slice (4-dim)).

Logarithmic behaviour of EE: $\quad S_A = \frac{R^3\sqrt{Q}}{2G_5} L_+ L \log\frac{l}{\epsilon}.$

[This uses only 5-d spacetime: also applies to various $\mathcal{N}$=1 SYM CFTs.]

Explore further ... Weak coupling field theory?

AdS plane waves, holo. EE

AdS_5 plane wave: $\quad ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2$

[EE for subsystem A, $S_A = -tr\rho_A \log \rho_A$, partial trace over A-complement, $\rho_A = tr_B\rho$.]

Want to de-mystify entanglement entropy in null slicing:

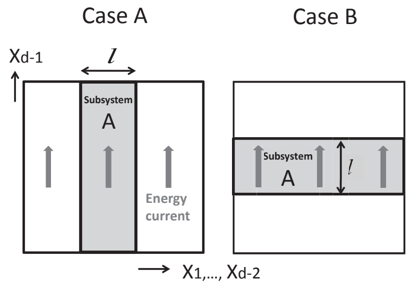
consider spacelike subsystems ($\Delta x^+ > 0 > \Delta x^-$, strip, width l).

Non-static spacetime: use covariant HEE (Hubeny,Rangamani,Takayanagi).

HEE $\sim$ area of bulk extremal surface bounding A

(stationary point of area functional; if several surfaces exist, choose minimal area).

Two choices for subsystem depending on energy flux T_{++} direction:

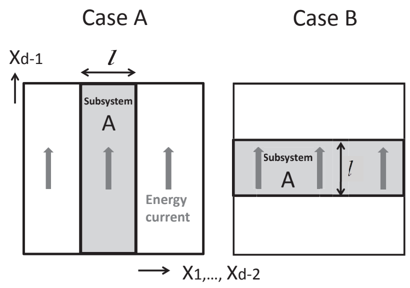


(spacelike subsystem: leading divergence is area law $\sim \frac{V_2}{\epsilon^2}$).

AdS plane waves, holo. EE

AdS_5 plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2$

Simple excited pure states in $\mathcal{N}{=}4$ SYM CFT: $T_{++} = const.$



spacelike subsystem: leading divergence is area law $\sim \frac{V_2}{\epsilon^2}$.

Case A: width direction is x_i.
Finite piece $V_2\sqrt{Q}\log(lQ^{1/4})$, grows with size.
Smaller than $\sim l$ (thermal entropy), larger than $\sim l^{-2}$ (CFT ground state).
Strip along energy flux: (heuristically) size increases, entanglement increases.

Case B: width along x_3 (and flux).
Phase transition: $S_A \sim$ constant (saturated) beyond $l_c \sim Q^{-1/4}$
(no connected surface, $\Delta x^+ > 0 > \Delta x^-$; disconnected surfaces for large l).
Strip orthogonal to energy flux: correlation length $\sim Q^{-1/4}$.
Large l: entanglement saturation.

AdS plane waves, holo. EE

AdS_5 plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2$

To understand phase transition better, consider regulated AdS_5 plane wave: boosted black D3-brane (Singh).

$$ds^2 = \frac{1}{r^2}\Big[-2dx^+dx^- + Qr^4\big(dx^+ + \frac{r_0^4}{2Q}dx^-\big)^2 + dx_i^2\Big] + \frac{dr^2}{r^2(1-r_0^4 r^4)}.$$

Boost $\lambda^2 = \frac{2Q}{r_0^4}$: as $r_0 \to 0,\ Q = fixed,$ recover AdS_5 plane wave.

For small r_0, the scale Q dominates, so phase transition persists (no connected extremal surface for large size with spacelike subsystem).

Connected extremal surface for Case B can be found by scaling towards horizon, and towards double zero of surface equation (which gives large size) [becomes disconnected surface in AdS plane wave limit].

Hyperscaling violation cont'd

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^{D-3}(dx^+)^2$$

AdS_D plane wave, $D = d+3$, Q lightcone-momentum density, $(D-1)$-dim.

Dim'nal redn: $\quad ds_E^2 = \frac{R^2(R^2Q)^{1/(D-3)}}{r}\left(-\frac{dt^2}{Qr^{D-1}} + dx_i^2 + dr^2\right).$

"boundary" spatial dimension $d = D-3$, $\quad z = \frac{d}{2}+2\,, \quad \theta = \frac{d}{2}\,.$

In particular, for the conformal branes of M-theory:

$M2$-brane stacks $\to$ AdS_4 deformations, $\quad d=1,\ \ z=\frac{5}{2},\ \ \theta=\frac{1}{2}$.

$M5$-brane stacks $\to$ AdS_7 deformations, $\quad d=4,\ \ z=4,\ \ \theta=2$.

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(recall effective gravity+vector+scalar theories, $e.g.$ Trivedi et al, Kiritsis et al, Takayanagi et al)]

M2-brane CFT: super 3-dim Chern-Simons theory with matter.

Deviations from area law stronger than logarithmic.

Entanglement (finite piece) grows with subsystem size as $L_+\sqrt{Ql}$.

Hyperscaling violation

(Recall Dp-brane phases, Itzhaki,Maldacena,Sonnenschein,Yankielowicz)

Conformal M-theory brane stack $ds^2 = ds^2_{AdS\times S} + g_{++}(dx^+)^2$, for g_{++} (non-)normalizable deformations, gives rise in appropriate IIA string/sugra phase to hyperscaling violation and new z, θ with $ds^2_E = r^{2\theta/d}(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2})$.

[ds^2_{M-th} $\rightarrow$ 10-d ds^2_{st} + dilaton Φ $\rightarrow$ ds^2_E $\rightarrow$ dim.redn.$^{\mathrm{sphere}S}$ $\rightarrow$
null-deformed solution, non-compact x^+ $\rightarrow$ dim.redn.$^{x^+}$ $\longrightarrow$ z, θ .
Note: 10-d solutions can be checked independently in IIA supergravity.]

Dp-branes $\rightarrow$ $d = p,\ \ z = 1,\ \ \theta \neq 0$. (Dong,Harrison,Kachru,Torroba,Wang)

M2 $\rightarrow$ D2 null non-normalizable: $d = 1,\ \ z = \frac{7}{3},\ \ \theta = 0$.

M2 $\rightarrow$ D2 null normalizable: $d = 1,\ \ z = 3,\ \ \theta = \frac{2}{3}$.

M5 $\rightarrow$ D4 null non-normalizable: $d = 3,\ \ z = 2,\ \ \theta = -1$.

M5 $\rightarrow$ D4 null normalizable: $d = 3,\ \ z = 4,\ \ \theta = \frac{1}{3}$.

Note: some of these (with $d = 1$) have $d - 1 \leq \theta \leq d$ $\rightarrow$
Violation of area law of entanglement entropy. Field theory duals nontrivial: explore?

Hyperscaling violation

(Following Dp-brane phase structure of Itzhaki,Maldacena,Sonnenschein,Yankielowicz):

These solutions are of the form of null-deformed Dp-brane systems, with rich phase structure. z, θ-values flow.

Null-deformed D2-brane phases: flow from DLCQ$_{x^+}$ of 2+1-dim perturbative SYM (UV) regime $\rightarrow$ IIA supergravity region with null-deformed D2-solution (valid in some intermediate regime where x^+-circle large relative to 11-th circle) $\rightarrow$ 11-dim AdS_4 null-deformed M2-brane IR phase (dual to DLCQ of null-defmn of Chern-Simons ABJM-like theory).

Null-deformed D4-brane phases: flow from AdS_7 null-deformed M5-brane UV phase (dual to null-deformation of $(2, 0)$ M5-theory) $\rightarrow$ intermediate IIA supergravity region with null-deformed D4-solution $\rightarrow$ DLCQ$_{x^+}$ of 4+1-dim perturbative SYM IR phase.

AdS null defmns + inhomogeneities

Most general family of (static) AdS plane waves:

$$ds^2 = \frac{1}{r^2}(-2dx^+dx^- + dx_i^2) + g_{++}[r, x_i](dx^+)^2 + \frac{dr^2}{r^2} + d\Omega_5^2$$

- AdS analogs of plane-waves: AdS + g_{++}. Supersymmetric.

Likely α'-exact string backgrounds due to lightlike nature.

- Restricting to normalizable modes (near boundary), these are general lightcone states in SYM with inhomogeneities: nonzero lightcone momentum density T_{++} varying spatially (frozen in lightcone time).

- Static normalizable backgrounds: generically, g_{++} vanishes at specific locii in the interior, even if positive definite near boundary ($i.e.$ $T_{++} \geq 0$). Effectively a $horizon$: time-like Killing vector $\partial_- \rightarrow$ null.

- On x^+-dimensional reduction, this means string modes winding around x^+-circle become light in the vicinity of $g_{++} = 0$ locii. New stringy physics beyond the gravity approximation: new operators in SYM with low anomalous dimensions.

Similar inhomogenous solutions exist for asymptotically Lifshitz solutions too.

Conclusions, open questions

- AdS plane waves $\rightarrow$ dim'nal redux

  $\rightarrow$ Lifshitz scaling, hyperscaling violation.

Dual to 4-d SYM CFT (or more exotic 3d CFT) excited state with T_{++}.

- Some of these lead to logarithmic (or stronger) deviations of entanglement entropy holographically relative to area law.
- AdS plane waves: simple excited pure states. Entanglement entropy can be studied explicitly for strip-shaped subsystems, results depend on strip direction w.r.t. energy flux. For strip orthogonal to flux, phase transition.

More general AdS null deformations with inhomogeneities.

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Entanglement entropy area law deviations from field theory?

What are these materials?

$\ldots$ $\ldots$