Cosmological singularities, AdS/CFT and de Sitter deformations

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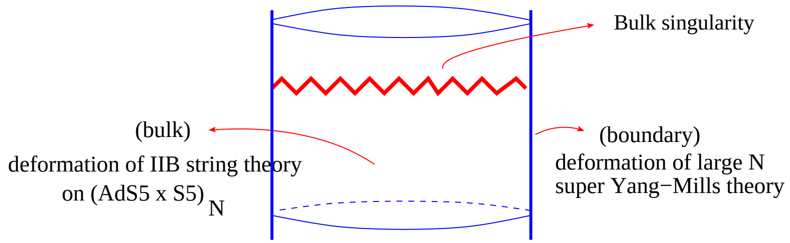
[work in progress with Sumit Das;
and arXiv:0807.1517, Adel Awad, Sumit Das, Suresh Nampuri, KN, Sandip Trivedi;
arXiv:0711.2994, Awad, Das, KN, Trivedi;
hep-th/0602107, 0610053, Das, Jeremy Michelson, KN, Trivedi.]
[see also arXiv:1012.0113, 0909.4731, KN; arXiv:0904.4532, Kallingalthodi Madhu, KN.]

- AdS/CFT with cosmological singularities: gauge theories with time-dep couplings and spacelike singularities
- de Sitter deformations, cosmological singularities and dS/CFT

Big-Bang/Crunch singularities . . .

General Relativity breaks down at singularities (spacelike, null): curvatures, tidal forces divergent, notions of spacetime break down. Want "stringy" description, eventually towards smooth quantum (stringy) completion of classical spacetime geometry.

[Previous examples: "stringy phases" in $e.g.$ 2-dim worldsheet (linear sigma model) descriptions (including time-dep versions, e.g. tachyon dynamics in (meta/)unstable vacua), dual gauge/Matrix theories, string worldsheet descriptions, . . .]



Within the AdS/CFT framework: study *time-dependent* deformations.
Bulk: deformed to contain cosmological singularity. Breaks down.
Boundary: Gauge theory dual is a sensible Hamiltonian quantum system in principle, subject to time-dependence. Response?

AdS/CFT, deformations

Bulk string theory on $AdS_5 \times S^5$ with dilaton (scalar) $\Phi = const$

and metric $\quad ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + d\Omega_5^2 \quad$ (Poincare coords),

deformed to (non-normalizable time-dep metric, dilaton defmns):

$$ds^2 = \frac{1}{z^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2) + d\Omega_5^2, \qquad \Phi = \Phi(t) \text{ or } \Phi(x^+).$$

Solution if: $\quad \tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi, \quad \tilde{\Box}\Phi = 0 \quad (\tilde{\Box} \equiv \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu).$

[solutions include $e.g.$ AdS-Kasner, -FRW, -BKL (Bianchi classification), etc]

In many cases, possible to find new coordinates such that boundary metric $ds_4^2 = \lim_{z\to 0} z^2 ds_5^2$ is flat (as expansion about $z=0$).

Then boundary 4-dim super Yang-Mills theory, with coupling $g_{YM}^2 = e^\Phi$ deformed to have external time-dependence.

[Sources approaching $e^\Phi \to 0$ at some finite point in time.
e.g. $g_{YM}^2 = e^\Phi \ \to \ t^p \ , \ p > 0$, give rise to bulk singularity $\ R_{tt} \sim \frac{1}{2}\dot{\Phi}^2 \ \sim \frac{1}{t^2}$.
Curvatures, tidal forces diverge near $t=0$.]

The gauge theory

Analyzing the gauge theory possible in various cases:
gauge theory not weakly coupled altho coupling vanishes ($g_{YM}^2 \sim t^p$).
Interactions important. Schrodinger wavefunctional near singularity ($t \sim 0$) has "wildly oscillating" phase for $p > 1$.
Also, energy expectation value generically diverges ($\langle H\rangle \sim e^{-\Phi}\langle V\rangle$).
Thus if $g_{YM}^2 = e^{\Phi} \to 0$ strictly, gauge theory response singular.

In gauge theory, deform gauge coupling so that $g_{YM}^2 = e^{\Phi}$ small but nonzero near $t = 0$. Now finite but large phase oscillation and energy production. $\dot{\Phi} \sim \frac{\dot{g}_{YM}}{g_{YM}}$ finite so bulk also nonsingular (but stringy).

Eventual gauge theory endpoint depends on details of energy production. On long timescales, expect gauge theory thermalizes: then late-time bulk is possibly AdS-Schwarzschild black hole.

[see also arXiv:0906.3275, Awad, Das, Ghosh, Oh, Trivedi: slowly varying dilaton cosmologies and their gauge theory duals.]

Null singularities, gauge theory

$$g_{YM}^2 = e^{\Phi(x^+)}, \qquad ds^2 = \frac{1}{z^2}[e^{f(x^+)} dx_\mu dx^\mu + dz^2] + ds_{S^5} \ .$$

There exist gauge theory variables where interaction terms unimportant near $e^\Phi \to 0$. Near singularity lightcone Schrodinger wavefunctional suggests weakly coupled Yang-Mills theory [e.g. near $x^+ = 0$, $e^\Phi \sim (x^+)^p$]. These variables appear to be dual to stringy objects in bulk.

This suggests that while classical bulk supergravity variables are bad, lightcone Hamiltonian time evolution of the gauge theory is sensible.

[**Renormalization effects:** introduce "short-time" (momentum) cutoff. Sufficiently high frequency modes in gauge theory (relative to $\dot{\Phi}$) might give nontrivial contributions to gauge theory effective action/Hamiltonian, so previous arguments might be modified.]

An aside on AdS/CMT: These solutions can be recast as $ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\partial_+\Phi)^2(dx^+)^2] + \frac{dw^2}{w^2}$. In this form, natural to define $x^- \equiv time$. Then (with Φ regular) these provide concrete string constructions (Koushik Balasubramanian, KN) of Lifshitz points.

Null singularities, free strings

Bulk: Expect stringy effects (beyond GR) are important.
AdS string technically difficult.

Possible to construct simpler toy models with no (RR) fluxes or dilaton, where the singularity is $purely\ gravitational$: can analyse worldsheet string theory in detail. These contain null Kasner-like cosmological singularities. Equivalent to anisotropic plane-waves.

Worldsheet string propagation: nonsingular time-dependence in lightcone string Schrodinger wavefunctional for certain singularities in Rosen-Kasner variables, but observables generically divergent.
Also, on near-singularity cutoff (const null time) surface, various string oscillator states light relative to local curvature scale.

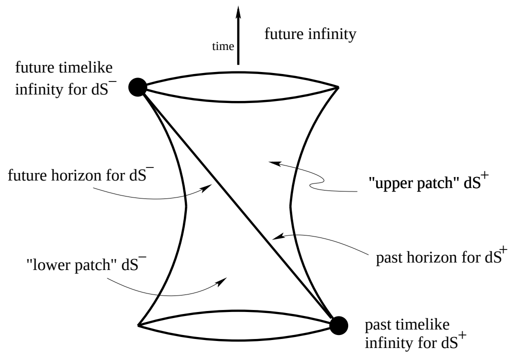
Thus large proliferation of light string oscillator states in near singularity region: string highly excited in vicinity of singularity.
String interactions possibly important.

de Sitter deformations, singularities

de Sitter space dS_{d+1}: planar coords, $ds^2 = \frac{1}{\tau^2}\left[-d\tau^2 + \delta_{ij}dx^i dx^j\right] \to$

$ds^2 = \frac{1}{\tau^2}\left[-d\tau^2 + \tilde{g}_{ij}(x^i)dx^i dx^j\right]$: solution if $\tilde{R}_{ij} = 0$.

With scalar sources ϕ, solution if $\tilde{R}_{ij} = \frac{1}{2}\partial_i\phi\partial_j\phi, \quad \tilde{\Box}\phi = 0$.



$R^{ABCD}R_{ABCD}$ diverges as $|\tau| \to \infty$ if $\tilde{R}^{ijab}\tilde{R}_{ijab} \neq 0$.

In some cases, past/future horizon nonsingular, past/future timelike infinity singular $\to$ spacelike singularity. Else null singularity.

[d>3: purely gravitational solutions exist, $e.g.$ $\tilde{g}_{ij}$ being ALE spaces etc;
$d = 3$, scalar (or other) sources required for nontrivial solution.]

de Sitter deformations, singularities

Initial value (ADM) formulation $ds^2 = -N^2 dt^2 + h_{ij} dx^i dx^j$.

Especially useful from the point of view of lower patch dS^-:

h_{ij} provides initial data for metric to evolve.

Singularity in the future (Big-Crunch).

[action $S_{bulk} = \int d^4x \sqrt{h} N \, [K_{ij}K^{ij} - K^2 + \tilde{R}^{(3)} - 2\Lambda + \frac{1}{2} N^{-2} \dot{\phi}^2 - \frac{1}{2} h^{ij} \partial_i \phi \partial_j \phi]$,
boundary contribution $S_B \sim \int d^3x \sqrt{\tilde{g}} K$.]

Action for fluctuations $h_{ij} = a^2(t)(\delta_{ij} + \gamma_{ij})$ shows that above solutions do not arise from Bunch-Davies-type solutions.

Instead, above solutions arise if time-dependence of spatial slice h_{ij} constrained to be as in pure de Sitter $(h_{ij} = \frac{1}{\tau^2} \tilde{g}_{ij})$.

Action evaluated on-shell for these solutions, non-singular

$\Big(S = \int d^{d+1}x \sqrt{h} N [R^{(3)} - 2\Lambda - \frac{1}{2} \tau^2 \tilde{g}^{ij} \partial_i \phi \partial_j \phi] \sim \int d^{d+1}x \frac{\sqrt{\tilde{g}}}{\tau^{d+1}}\Big)$.

de Sitter deformations, dS/CFT

dS/CFT (Strominger, Witten, Maldacena, ...) defined by symmetries, causal structure etc: analytic continuation $z \to -i\tau, \quad R_{AdS} \to -iR_{dS}$, from AdS/CFT. *Euclidean* CFT dual to de Sitter space.
Not yet well-developed.

For above solutions, expect sources for metric and scalar turned on.
Futuristic question: is dual CFT nonsingular despite bulk singularity?

Holographic stress tensor can be calculated for generic metric+scalar deformations, defining action+counterterms such that divergences at small $|\tau| \sim 0$ cancel

$$T^{ij} = \frac{1}{8\pi G_{d+1}}\left[K^{ij} - Kh^{ij} + (d-1)h^{ij} + \frac{1}{2}G^{ij} - \frac{1}{4}\partial^i\phi\partial^j\phi + \frac{1}{8}h^{ij}(\partial\phi)^2\right]$$

For above solutions, this can be shown to vanish identically.

de Sitter deformations, dS/CFT

From the point of view of deforming the lower patch dS^-:
specific initial conditions/state for (Eucl.) CFT $\rightarrow$ bulk Big-Crunch.

• Generically expect nonzero 1-point function for dual operator if source turned on: in this case, expect $\langle T_{ij} \rangle \neq 0$, since boundary metric turned on. However we find stress tensor vanishes.
Requiring vanishing $\langle T_{ij} \rangle$ in fact leads to above solutions.

• Fefferman-Graham expansion for asymptotically (locally) de Sitter metric: $ds^2 = -\frac{d\tau^2}{\tau^2} + \frac{1}{\tau^2} \left[g^0_{ij}(x^i) + \tau^2 g^2_{ij}(x^i) + \ldots \right] dx^i dx^j$.
Turning on a source g^0_{ij} generically leads to nonvanishing g^n_{ij}: these subleading parts of the boundary metric in fact contain information about the state of the dual CFT. In our case, only $g^0_{ij} \neq 0$.
Requiring $g^n_{ij} = 0$ in fact leads to above solutions.

These dS-deformations containing Big-Crunch/Bang singularity thus appear to be dual to special fine-tuned states for the Euclidean CFT.

Conclusions, questions

AdS-cosmologies: gauge theory dual amenable to analysis in various cases. Suggests spacelike singularities might be pathological in dual gauge theory too, while null ones are more promising.
Free string analysis suggests proliferation of light string oscillator states near simpler purely gravitational singularities, with possibly large backreaction due to highly excited strings.

de Sitter deformations: deformations containing cosmological singularities arise if time-dependence of spatial slices are constrained in specific ways. These are dual to sources for spatial metric (and possibly scalar) in a dS/CFT perspective, and additionally might correspond to special fine-tuned states for the dual Euclidean CFT.

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