D-brane plane waves, hyperscaling violation and entanglement entropy

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- Introduction; gravity duals of hyperscaling violation, entanglement
- Reviewing Lifshitz in string theory; AdS plane waves
- AdS plane waves, hyperscaling violation, entanglement entropy
- Nonconformal brane plane waves, EE; NS5-branes

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1212.4328, KN, Tadashi Takayanagi, Sandip Trivedi, 1304.6697, KN.

Introduction

Interesting to explore holography with reduced symmetries.

Generalizations of AdS/CFT to nonrelativistic systems

$\rightarrow$ holographic condensed matter, ...

[Son; Balasubramanian,McGreevy; Adams et al; Herzog et al; Maldacena et al; ...]

$\rightarrow$ Phases of gauge/string theory with non-relativistic symmetries.

Introduction

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$\rightarrow$ holographic condensed matter, ...

[Son; Balasubramanian,McGreevy; Adams et al; Herzog et al; Maldacena et al; ...]

$\rightarrow$ Phases of gauge/string theory with non-relativistic symmetries.

Sometimes qualitative features of some of these phases from the gravity/string descriptions can be distilled to give interesting, perhaps unexpected insights for field theories.

Present context: entanglement entropy.

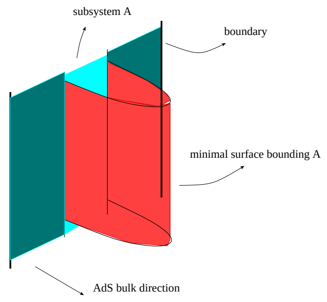
> Useful tool to characterize phases of matter.

Introduction, summary

[Nonrelativistic AdS/CFT: qualitative features of gravity phases $\rightarrow$ insights for gauge theories.]

Present context: entanglement entropy in various quantum field theories shows leading divergence $\rightarrow$ area law (Bombelli et al; Srednicki, ...).

Ryu-Takayanagi bulk prescription for EE: minimal surface area in gravity dual.



Gravity systems with "hyperscaling violation" $\rightarrow$ area law deviations.

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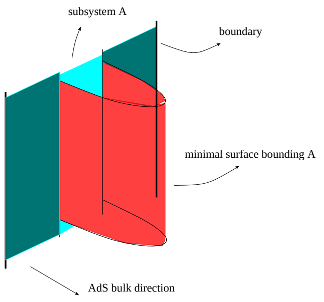

- String theory realizations $\rightarrow AdS$ plane waves, dual to strongly coupled CFTs with uniform energy-momentum density T_{++}.

- 4d Super Yang-Mills CFTs with $T_{++} \rightarrow$ logarithmic deviation from area law for holographic EE (finite part grows as $\log l$, large for fixed cutoff).
Stronger area law deviations in some strongly coupled 3-dim CFTs.

- Similarly, nonconformal brane plane wave states: strongly coupled gauge theories with T_{++}. EE^{finite} has interesting structure.

Holography and Lifshitz scaling

Lifshitz: t, x_i-translations, x_i-rotations, scaling $t \to \lambda^z t,\ x_i \to \lambda x_i$

[z: dynamical exponent]. (smaller than Galilean symmetries: $e.g.$ boosts broken)

Landau-Ginzburg action (free $z=2$ Lifshitz): $S = \int d^3x((\partial_t\varphi)^2 - \kappa(\nabla^2\varphi)^2)$.

Lifshitz spacetime: $ds_4^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}$. Kachru,Liu,Mulligan; Taylor

[$z=1 : AdS$] Solution to 4-dim gravity with $\Lambda < 0$ and massive gauge field $A \sim \frac{dt}{r^z}$

[Lifshitz-attractors, Einstein-Maxwell-scalar theories, Gubser,Rocha; Kachru,Trivedi et al, ...]

Holography, hyperscaling violation

More general gravity phases: $ds^2 = r^{2\theta/d}\big(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}\big).$

θ = hyperscaling violation exponent; d = "boundary" spatial dim (x_i).

Conformal to Lifshitz (dynamical exponent z).

Arise in effective gravity+vector+scalar (Einstein-Maxwell-scalar) theories (Gubser,Rocha; Trivedi et al; Kiritsis et al, ...)

Black holes: entropy-temperature relation is $S \sim T^{(d-\theta)/z}$.

Thermodynamics $\sim$ space dim $d_{eff} = d - \theta$: actual space is d-dim.

$\theta = d-1$: entanglement entropy $\sim \log l$, logarithmic behaviour (grows for fixed UV cutoff). Gravitational duals of hidden Fermi surfaces?

(Ogawa,Takayanagi,Ugajin; Huijse,Sachdev,Swingle)

Various aspects of holography with hyperscaling violation (Kachru et al).

Energy conditions: $(d-\theta)(d(z-1)-\theta) \geq 0, \quad (z-1)(d+z-\theta) \geq 0.$

$d-1 \leq \theta < d$: entanglement entropy shows area law violations.

Gauge/string realizations?

Non-relativistic AdS/CFT

4-dim $\mathcal{N}$=4 superconformal Yang-Mills theory

dual to IIB string theory on $AdS_5 \times S^5$.

$$\left[ds^2 = \frac{R^2}{r^2}(-2dx^+dx^- + dx_i^2 + dr^2) + R^2 d\Omega_5^2, \qquad R^4 \sim g_{YM}^2 N \alpha'^2\right]$$

Large N limit $\rightarrow$ classical gravity description useful.

A fairly traditional way to obtain non-relativistic quantum field theory is to use lightcone variables ($x^- \equiv time$), and study the system with nonzero lightcone momentum $\left[e.g.\ p_\mu p^\mu = 0 \ \rightarrow \ p_- = \frac{1}{2p_+} p_i^2 \ \right]$.

For instance, x^+-DLCQ of relativistic $\mathcal{N}$=4 SYM $\longrightarrow$

$z = 2$ nonrelativistic (Galilean) 2+1-dim system.

DLCQ $_{x^+}$ of AdS_5 in lightcone coordinates — nonrelativistic, Schrodinger (Galilean) symmetries [Goldberger, Barbon et al, Maldacena et al].

Lifshitz, string theory

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[$z = 1 : AdS$] Solution to 4-dim gravity with $\Lambda < 0$ and massive gauge field $A \sim \frac{dt}{r^z}$

Gauge/string construction for $z = 2$ Lifshitz [Balasubramanian,KN]:

relativistic $\mathcal{N}$=4 SYM $\xrightarrow{x^+ - \mathrm{DLCQ}}$ $z = 2$ non-rel (Galilean) 2+1-dim system.

Gauge coupling $g_{YM}^2(x^+) = e^{\Phi(x^+)}$ varying in lightlike x^+-direction $\longrightarrow$

breaks x^+-shift reducing to 2+1-dim Lifshitz symmetries.

Bulk: $AdS\ +\ g_{++}[\sim r^0]$ $\xrightarrow{x^+ - \mathrm{dim.redn.}}$ $z = 2$ Lifshitz.

Kaluza-Klein x^+-reduction of non-normalizable null deformations of

$AdS_5 \times S^5$: g_{++} sourced by lightlike scalar (dilaton $\Phi(x^+)$) [$x^- \equiv time$].

[$ds_5^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + \# r^0 (dx^+)^2 \ \longrightarrow\ ds_4^2 = -\frac{(dx^-)^2}{r^4} + \frac{dx_i^2 + dr^2}{r^2}$]

More general: g_{++} sourced by axion-dilaton, fluxes [Donos,Gauntlett; et al].

SYM, $T_{++} \neq 0$: AdS plane waves

[Relativistic $\mathcal{N}$=4 SYM $\xrightarrow{x^+ - \mathrm{DLCQ}}$ $z = 2$ non-rel (Galilean) 2+1-dim system.
Lightcone AdS_5 $\xrightarrow{x^+ - \mathrm{DLCQ}}$ $z = 2$ non-rel Schrodinger (Galilean) symmetries].

Consider $\mathcal{N}=4$ SYM with uniform lightcone momentum density T_{++}. At large N, we expect the gravity dual to this excited state to be some normalizable deformation of $AdS_5 \times S^5$, with g_{++} nonzero.

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At large N, we expect the gravity dual to this excited state to be some normalizable deformation of $AdS_5 \times S^5$, with g_{++} nonzero.

Can identify this precisely: AdS_5 plane wave [AdS + g_{++}],

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$$

Arises in gravity/5-form sector (5-dim part solves $R_{MN} = -\frac{4}{R^2} g_{MN}$).

Holographic boundary stress tensor using AdS/CFT rules gives

$T_{++} \sim Q \longrightarrow$ uniform lightcone momentum density.

SYM, $T_{++} \neq 0$: AdS plane waves

AdS_5 plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$

- AdS analogs of plane-waves: AdS + g_{++}. Supersymmetric.
  Likely α'-exact string backgrounds due to lightlike nature.

- Boundary energy-momentum density $T_{++} \geq 0$ (null energy condition).

- Can be realized as "zero temperature", highly boosted double scaling limit of boosted AdS Schwarzschild black 3-branes (Singh).

$$ds^2 = \frac{1}{r^2}\Big[-2dx^+dx^- + Qr^4\big(dx^+ + \frac{r_0^4}{2Q}dx^-\big)^2 + dx_i^2\Big] + \frac{dr^2}{r^2(1-r_0^4 r^4)}.$$

Boost $\lambda^2 = \frac{2Q}{r_0^4}$: as $r_0 \to 0$, $Q = fixed$, recover AdS_5 plane wave.

Simple excited pure states in $\mathcal{N}=4$ SYM CFT (no entropy density):
limit of thermal state, $\lambda \to \infty$, $T \to 0$, with $Q \sim \lambda^2 T^4$ fixed.

- Generalize to AdS_D plane wave ($e.g.$ states in M2-, M5- CFTs):

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^{D-3}(dx^+)^2 + R^2 d\Omega^2$$

Bulk x^+-dimensional reduction?

AdS plane wave, hyperscaling violation

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2 + R^2 d\Omega_5^2$$

AdS plane wave: Q $\sim$ lightcone-momentum density T_{++} $[R^4 \sim g_{YM}^2 N \alpha'^2]$.

Dimensionally reduce 10-dim metric on S^5 and on x^+-circle:

4-dim Einstein metric ($x^- \equiv t$): $ds_E^2 = \frac{R^3\sqrt{Q}}{r}\Big(-\frac{dt^2}{Qr^4} + dx_i^2 + dr^2\Big)$,

Electric gauge field $A = -\frac{dt}{Qr^4}$, scalar $e^\phi \sim r$. Nontrivial IR scales R, Q.

$$ds^2 = r^{2\theta/d}\Big(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2 + dr^2}{r^2}\Big).$$

d = "boundary" spatial dim (x_i).

θ = hyperscaling violation exponent.

Hyperscaling violating spacetime (effective Einstein-Maxwell-dilaton theories)

Thermodynamics reflects effective space dim $(d-\theta)$ [$e.g.\ S \sim T^{(d-\theta)/z}$].

Above: $d = 2,\ \theta = 1,\ z = 3$ ($d_{eff} = d - \theta = 1$).

[Dp-branes, S^{8-p} reduction $\rightarrow$ $d = p,\ z = 1,\ \theta \neq 0$ (Dong,Harrison,Kachru,Torroba,Wang)]

AdS plane wave, hyperscaling violation

$$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^{D-3}(dx^+)^2 + R^2 d\Omega^2$$

AdS_D plane wave, $D = d+3$, Q lightcone-momentum density, $(D-1)$-dim.

Dim'nal redn: $ds_E^2 = \frac{R^2(R^2Q)^{1/(D-3)}}{r}\left(-\frac{dt^2}{Qr^{D-1}} + dx_i^2 + dr^2\right)$.

"boundary" spatial dimension $d = D-3$, $\theta = \frac{d}{2}$, $z = \frac{d}{2} + 2$.

In particular, for the conformal branes of M-theory:

$M2$-brane stacks $\rightarrow$ AdS_4 deformations, $d=1$, $z=\frac{5}{2}$, $\theta=\frac{1}{2}$.

$M5$-brane stacks $\rightarrow$ AdS_7 deformations, $d=4$, $z=4$, $\theta=2$.

[General dim'nal reduction: $\int d^Dx\sqrt{-g^{(D)}}\,(R^{(D)} - 2\Lambda) =$

$\int dx^+ d^{D-1}x\sqrt{-g^{(D-1)}}\,(R^{(D-1)} - \#\Lambda e^{-2\phi/(D-3)} - \#(\partial\phi)^2 - \#e^{2(D-2)\phi/(D-3)}F_{\mu\nu}^2)$

(recall effective gravity+vector+scalar theories, $e.g.$ Trivedi et al, Kiritsis et al, Takayanagi et al)]

Summary so far: AdS plane waves dual to simple pure excited CFT states. x^+-dim. redn $\rightarrow$ gravity backgrounds, hyperscaling violation. AdS_5 plane wave $\rightarrow \theta = d-1$.

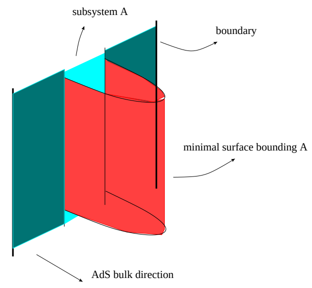
Entanglement, hyperscaling violation

Entanglement entropy for subsystem A, $S_A = -tr\rho_A \log\rho_A$, with partial trace $\rho_A = tr_B\rho$.

- Ryu-Takayanagi bulk prescription:

find bulk surface bounding A with minimal area.

EE$_A \propto$ area of bulk minimal surface bounding A.



[E.g.: SYM CFT ground state, strip with width l, bndry area L^2:
ordinary time-t slice in bulk AdS_5 $\longrightarrow S_A \sim \frac{R^3}{G_5}\big(\frac{L^2}{\epsilon^2} - \#\frac{L^2}{l^2}\big)$,
with leading divergence reflecting area law (Bombelli et al; Srednicki, . . .)
[ϵ = SYM UV cutoff], and finite cutoff-independent part.]

- Hyperscaling violating bulk metrics $ds^2 = r^{2\theta/d}\big(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}\big)$

with $\theta = d-1$ $\longrightarrow$ logarithmic violation of area law.

Gravitational dual of Fermi surfaces? (Ogawa,Takayanagi,Ugajin; Huijse,Sachdev,Swingle)

- AdS plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2 + R^2d\Omega_5^2$.

Dual to CFT pure excited state, energy-momentum density T_{++}.

AdS_5 plane wave: $\rightarrow x^+ -$ dim.redn. $\rightarrow$ $d=2,\ \theta=1,\ z=3.$

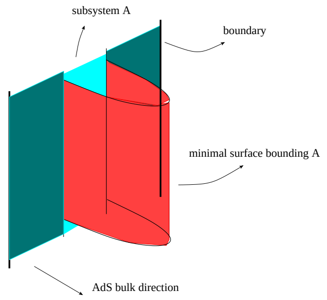
AdS plane waves, holog. EE

AdS_5 plane wave: $\quad ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2 + R^2d\Omega_5^2.$

- AdS_5 plane wave $\xrightarrow{x^+-\mathrm{dim.redn.}}$

hyperscaling violation $ds^2 = r^{2\theta/d}(-\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2})$

with $d = 2, \ \theta = 1, \ z = 3$.



4-dim $\mathcal{N}$=4 SYM CFT in these pure excited states ($T_{++} \sim Q$) exhibits logarithmic behaviour of holog. entanglement entropy (not area law).

Holographic EE S_E = area of bulk minimal surface bounding A.
Subsystem A = strip in x_i-plane, width l (possibly wrapping x^+-direction), lying on const-x^- slice (≡ constant time-t slice (4-dim)).

Logarithmic behaviour of EE: $\quad S_A = \frac{R^3\sqrt{Q}}{2G_5} L_+ L \log\frac{l}{\epsilon}.$

[This uses only 5-d spacetime: also applies to various $\mathcal{N}$=1 SYM CFTs.]
Other AdS_D plane waves, entanglement different (not log).

Null entanglement entropy: perhaps somewhat mysterious per se.

AdS plane waves, holo. EE

AdS_5 plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2$

[EE for subsystem A, $S_A = -tr\rho_A \log \rho_A$, partial trace over A-complement, $\rho_A = tr_B\rho$.]

Consider entanglement entropy with spacelike (not null) slicing:

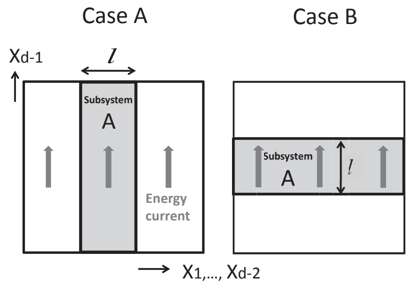
consider spacelike subsystems ($\Delta x^+ > 0 > \Delta x^-$, strip, width l).

Non-static spacetime: use covariant HEE (Hubeny,Rangamani,Takayanagi).

HEE $\sim$ area of bulk extremal surface bounding A

(stationary point of area functional; if several surfaces exist, choose minimal area).

Two choices for subsystem depending on energy flux T_{++} direction:

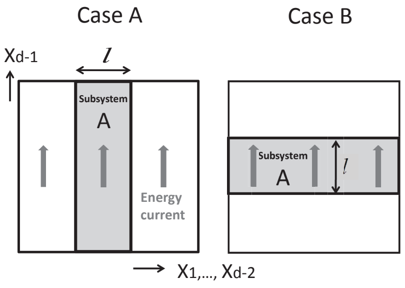


(spacelike subsystem: leading divergence is area law $\sim \frac{V_2}{\epsilon^2}$).

AdS plane waves, holo. EE

AdS_5 plane wave: $$ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2Qr^2(dx^+)^2$$

Simple excited pure states in $\mathcal{N}=4$ SYM CFT: $T_{++} = const$.



spacelike subsystem:
leading divergence
is area law $\sim \frac{V_2}{\epsilon^2}$.

Case A: width direction is x_i. Strip along energy flux.

Finite piece $N^2V_2\sqrt{Q}\log(lQ^{1/4})$, grows with size.

Less than $N^2T^3V_2l$ (thermal entropy), larger than $-N^2\frac{V_2}{l^2}$ (CFT ground state).

Heuristically, size increases, entanglement increases.

AdS plane waves, holo. EE

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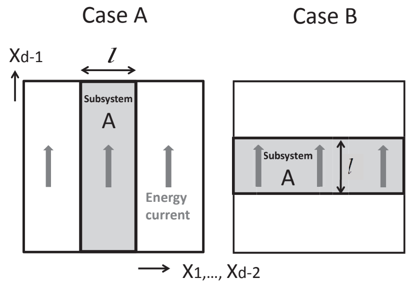

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Less than $N^2 T^3 V_2 l$ (thermal entropy), larger than $-N^2 \frac{V_2}{l^2}$ (CFT ground state).

Heuristically, size increases, entanglement increases.

Case B: width along x_3 (and flux). Strip orthogonal to energy flux.

Phase transition: $S_A \sim$ constant (saturated) beyond $l_c \sim Q^{-1/4}$

(no connected surface, $\Delta x^+ > 0 > \Delta x^-$; disconnected surfaces for large l).

correlation length $\sim Q^{-1/4}$. Large l: entanglement saturation.

(Analysing regulated AdS plane wave (with horizon) vindicates this.)

AdS plane waves, holo. EE

AdS_5 plane wave: $ds^2 = \frac{R^2}{r^2}[-2dx^+dx^- + dx_i^2 + dr^2] + R^2 Q r^2 (dx^+)^2$

To understand phase transition better, consider regulated AdS_5 plane wave: boosted black D3-brane (Singh).

$$ds^2 = \frac{1}{r^2}\Big[-2dx^+dx^- + Qr^4\big(dx^+ + \frac{r_0^4}{2Q}dx^-\big)^2 + dx_i^2\Big] + \frac{dr^2}{r^2(1-r_0^4 r^4)}.$$

Boost $\lambda^2 = \frac{2Q}{r_0^4}$: as $r_0 \to 0,\ Q = fixed,$ recover AdS_5 plane wave.

For small r_0, the scale Q dominates, so phase transition persists (no connected extremal surface for large size with spacelike subsystem).

Connected extremal surface for Case B can be found by scaling towards horizon, and towards double zero of surface equation (which gives large size) [becomes disconnected surface in AdS plane wave limit].

AdS_{d+1} plane waves, EE

Uniformize notation with nonconformal case: redefine $Q \to Q\frac{G_{d+1}}{R^{d-1}}$.

$\left[Q \to \frac{Q}{N^2}\ (D3),\ \ Q \to \frac{Q}{N^{3/2}}\ (M2),\ \ Q \to \frac{Q}{N^3}\ (M5)\right]$

$$ds^2 = \frac{R^2}{r^2}(-2dx^+dx^- + dx_i^2 + dr^2) + \frac{G_{d+1}Q}{R^{d-3}} r^{d-2}(dx^+)^2 + R^2 d\Omega^2$$

Plane wave excited states: EE^{finite} (strip along flux direction):

$$\pm\sqrt{Q}V_{d-2}l^{2-\frac{d}{2}}\sqrt{\frac{R^{d-1}}{G_{d+1}}} \qquad [+:\ d<4,\quad -:\ d>4];$$

$\sqrt{Q}V_2N\ \log(lQ^{1/4})$ (D3), $\sqrt{Q}L\sqrt{l}\sqrt{N^{3/2}}$ (M2), $-\sqrt{Q}\frac{V_4}{l}\sqrt{N^3}$ (M5).

3d, 4d: finite entanglement grows with width l (large for fixed cutoff).

[spacelike strip subsystem: leading divergence is area law, $\frac{V_2}{\epsilon^2}\ (4d),\ \ \frac{V_1}{\epsilon}\ (3d)$]

[Strip $\perp$ flux: phase transition.]

[EE^{fin} scaling estimates $\leftarrow$ approximate r_*, S^{fin} for large Q, l from EE area functional]

[Ground state EE: $S_A \sim \frac{R^{d-1}}{G_{d+1}}\left(\frac{V_{d-2}}{\epsilon^{d-2}} - c_d\frac{V_{d-2}}{l^{d-2}}\right)$]

[Temperature parameters: $r_0^4 \sim G_{10}\varepsilon_4\ (D3),\ \ r_0^6 \sim G_{11}\varepsilon_3\ (M2),\ \ r_0^3 \sim G_{11}\varepsilon_6\ (M5)$

$\lambda \to \infty,\ \varepsilon_{p+1} \to 0,$ with $\frac{\lambda^2\varepsilon_{p+1}}{2} \equiv Q =$ fixed. Boundary $T_{++} = Q$.]

[$G_5 \sim G_{10}R_{D3}^5, G_{4,7} \sim G_{11}R_{M2,M5}^{7,4}$, with $R_{D3}^4 \sim g_sNl_s^4, R_{M2}^6 \sim Nl_P^6, R_{M5}^3 \sim Nl_P^3$]

Nonconformal brane plane waves

(Recall Dp-brane phases, Itzhaki,Maldacena,Sonnenschein,Yankielowicz)

$$ds^2_{st} = \frac{r^{(7-p)/2}}{R_p^{(7-p)/2}} dx_\parallel^2 + \frac{G_{10} Q_p}{R_p^{(7-p)/2}} \frac{(dx^+)^2}{r^{(7-p)/2}} + R_p^{(7-p)/2} \frac{dr^2}{r^{(7-p)/2}} + R_p^{(7-p)/2} r^{(p-3)/2} d\Omega_{8-p}^2$$

$$e^\Phi = g_s \left(\frac{R_p^{7-p}}{r^{7-p}} \right)^{\frac{3-p}{4}}, \quad g_{YM}^2 \sim g_s \alpha'^{(p-3)/2}, \quad R_p^{7-p} \sim g_{YM}^2 N \alpha'^{5-p} \sim g_s N \alpha'^{(7-p)/2}.$$

[g_{++}-deformation obtained from double scaling limit of boosted black Dp-branes
$r_0^{7-p} = (U_0 \alpha')^{7-p} \sim G_{10} \varepsilon_{p+1}; \quad \lambda \to \infty, r_0 \to 0$, with $\frac{\lambda^2 \varepsilon_{p+1}}{2} \equiv Q_p$ fixed.]

Strongly coupled Yang-Mills theories with constant energy flux T_{++}.

Dimensionally reducing on S^{8-p} and x^+, Einstein metric $ds_E^2 = e^{-\Phi/2} ds_{st}^2$ gives hyperscaling violating metrics with $\theta = \frac{p^2-6p+7}{p-5}, \quad z = \frac{2(p-6)}{p-5}$ (Singh).

D-brane plane waves, EE

$$ds^2_{st} = \frac{r^{(7-p)/2}}{R_p^{(7-p)/2}} dx_{\|}^2 + \frac{G_{10} Q_p}{R_p^{(7-p)/2}} \frac{(dx^+)^2}{r^{(7-p)/2}} + R_p^{(7-p)/2} \frac{dr^2}{r^{(7-p)/2}} + R_p^{(7-p)/2} r^{(p-3)/2} d\Omega_{8-p}^2$$

$$e^{\Phi} = g_s \left(\frac{R_p^{7-p}}{r^{7-p}}\right)^{\frac{3-p}{4}}, \quad g_{YM}^2 \sim g_s \alpha'^{(p-3)/2}, \quad R_p^{7-p} \sim g_{YM}^2 N \alpha'^{5-p} \sim g_s N \alpha'^{(7-p)/2}.$$

[$r_0^{7-p} = (U_0\alpha')^{7-p} \sim G_{10}\varepsilon_{p+1}; \quad \lambda \to \infty, r_0 \to 0$, with $\frac{\lambda^2 \varepsilon_{p+1}}{2} \equiv Q_p$ fixed.

Strongly coupled Yang-Mills theories with constant energy flux T_{++}.]

Ground state: Ryu-Takayanagi, Barbon-Fuertes

$$S_A = N_{eff}(\epsilon)\frac{V_{d-2}}{\epsilon^{d-2}} - c_d N_{eff}(l)\frac{V_{d-2}}{l^{d-2}}, \qquad N_{eff}(\epsilon) = N^2 \left(\frac{g_{YM}^2 N}{\epsilon^{p-3}}\right)^{\frac{p-3}{5-p}}$$

Plane wave excited states: leading divergence as above.

Scaling estimates from entanglement entropy area functional:

$$l \sim \frac{R_p^{\frac{7-p}{2}}}{r_*^{\frac{5-p}{2}}}, \qquad S_A^{finite} \sim \frac{V_{p-1}\sqrt{Q}}{(3-p)\sqrt{G_{10}}} \frac{R_p^{7-p}}{r_*^{(3-p)/2}} \qquad \text{(strip along flux)}$$

EE^{finite}: $\displaystyle \frac{1}{3-p} \frac{V_{p-1}\sqrt{Q}}{l^{(p-3)/2}} N \left(\frac{g_{YM}^2 N}{l^{p-3}}\right)^{\frac{p-3}{2(5-p)}} = \frac{\sqrt{N_{eff}(l)}}{3-p} \frac{V_{p-1}\sqrt{Q}}{l^{(p-3)/2}}$

[involves dimensionless combination $\frac{V_{p-1}\sqrt{Q}}{l^{(p-3)/2}}$ and $N_{eff}(l)$]

D-brane plane waves, EE

Plane wave excited states: leading divergence $N_{eff}(\epsilon)\frac{V_{d-2}}{\epsilon^{d-2}}$ as for ground states (area law).

EE^{finite}: $\frac{\sqrt{N_{eff}(l)}}{3-p}\frac{V_{p-1}\sqrt{Q}}{l^{(p-3)/2}}$, $\quad N_{eff}(l) = N^2\left(\frac{g_{YM}^2 N}{l^{p-3}}\right)^{\frac{p-3}{5-p}}$

D2-M2: $V_1\sqrt{l}\sqrt{Q}\sqrt{\frac{N^2}{(g_{YM}^2 Nl)^{1/3}}}\,(D2);\quad V_1\sqrt{l}\sqrt{Q}\sqrt{N^{3/2}}\,(M2).$

IIA regime of validity (IMSY) for turning point r_* gives

$1 \ll g_{YM}^2 Nl_{D2} \ll N^{6/5}$, and so $N^{3/2} \ll \frac{N^2}{(g_{YM}^2 Nl)^{1/3}} \ll N^2$. Thus

$S_A^{D2,sugra}$ betw free 3d SYM (UV) and M2 (IR) (RG-consistent).

D4-M5: $-\frac{V_3\sqrt{Q}}{\sqrt{l}}\sqrt{N^2\frac{g_{YM}^2 N}{l}}\,(D4);\quad -\sqrt{Q}\frac{V_4}{l}\sqrt{N^3}\,(M5).$

The finite parts for D4-sugra and M5-phases are actually same expression: D4 is wrapped M5 ($R_{11} = g_s l_s = g_{YM}^2$) and $V_4 = V_3 R_{11},\ Q_{D4} = Q_{M5} R_{11}$. IIA: $1 \ll \frac{g_{YM}^2 N}{l} \ll N^{2/3}$.

D1: $l\sqrt{Q}\sqrt{\frac{N^2}{(g_{YM}^2 Nl^2)^{1/2}}}$

Strip orthogonal to flux: indications of phase transitions, constrained however by IIA regime of validity.

NS5-brane plane waves

NS5-branes in certain decoupling limits dual to nonlocal 6-dim "little string" theories (Seiberg)

Plane wave excited states in little string theories (as for D-branes previously).

$$ds^2_{st} = -2dx^+dx^- + \frac{Q\alpha'^4}{r^2}(dx^+)^2 + dy_i^2 + N\alpha'\frac{dr^2}{r^2} + N\alpha' d\Omega_3^2 \ ,$$

Dilaton $e^{2\Phi} = g_s^2 \frac{N\alpha'}{r^2}$ unchanged.

g_{++}-deformation lightlike, likely α'-exact, supersymmetric.

[Finite temp NS5-branes (incl asymptotic flat space)

$$ds^2 = -(1-\frac{r_0^2}{r^2})dt^2 + \left(1+\frac{N\alpha'}{r^2}\right)\left(\frac{dr^2}{1-r_0^2/r^2} + r^2 d\Omega_3^2\right) + \sum_{i=1}^5 dy_i^2, \ \ e^{2\Phi} = g_s^2\left(1+\frac{N\alpha'}{r^2}\right)$$

Define lightcone coordinates $t = \frac{x^+ + x^-}{\sqrt{2}}, \ x_5 = \frac{x^+ - x^-}{\sqrt{2}}$, and lightlike boost $x^\pm \rightarrow \lambda^{\pm 1} x^\pm$ with $r_0^2 = G_{10}\mu$. Then take $\lambda \rightarrow \infty, \ g_s \rightarrow 0$, with $\frac{\lambda^2 g_s^2 \mu}{2} \equiv Q$ fixed.]

[NS5-plane wave solution can be checked independently from NS-NS eqn of motion.]

[Dim.redn. on S^3, x^+, gives metric distinct from hyperscaling violating family]

Appears distinct from Hagedorn temperature limit (Maldacena,Strominger) and other double scaling limits ($e.g.$ Giveon,Kutasov).

Explore?

AdS null defmns + inhomogeneities

Most general family of (static) AdS plane waves:

$$ds^2 = \frac{1}{r^2}(-2dx^+dx^- + dx_i^2) + g_{++}[r, x_i](dx^+)^2 + \frac{dr^2}{r^2} + d\Omega_5^2$$

- AdS analogs of plane-waves: AdS + g_{++}. Supersymmetric. Likely α'-exact string backgrounds due to lightlike nature.
- Restricting to normalizable modes (near boundary), these are general lightcone states in SYM with inhomogeneities: nonzero lightcone momentum density T_{++} varying spatially (frozen in lightcone time).
- Static normalizable backgrounds: generically, g_{++} vanishes at specific locii in the interior, even if positive definite near boundary ($i.e.$ $T_{++} \geq 0$). Effectively a $horizon$: time-like Killing vector $\partial_- \rightarrow$ null.
- On x^+-dimensional reduction, this means string modes winding around x^+-circle become light in the vicinity of $g_{++} = 0$ locii. New stringy physics beyond the gravity approximation: new operators in SYM with low anomalous dimensions.

Similar inhomogenous solutions exist for asymptotically Lifshitz solutions too.

Lifshitz singularities, string theory

Mild singularities present in Lifshitz spacetimes $ds_4^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2+dr^2}{r^2}$: curvature invariants finite, diverging tidal forces as $r \to \infty$ (interior). Expect that these are zero temperature limits of Lifshitz black holes with regular horizons: however zero temperature limit singular.

Singularities also reflected in above string constructions exhibiting exact Lifshitz symmetries (Horowitz,Way): origins? resolution?

Silverstein et al: in the Lifshitz-string constructions of Hartnoll,Polchinski,Silverstein,Tong, metric tidal force calculation in IR of string construction possibly incomplete: sources modify string scattering.]

String constructions involving AdS null deformations?

$$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2] + \#r^0(dx^+)^2 + \frac{dr^2}{r^2} + d\Omega_5^2, \quad \Phi(x^+).$$

Balasubramanian,KN; Donos Gauntlett; et al

Lifshitz singularities, string theory

$ds^2 = \frac{1}{r^2}[-2dx^+dx^- + dx_i^2] + \frac{1}{4}(\partial_+\Phi)^2(dx^+)^2 + \frac{dr^2}{r^2} + d\Omega_5^2, \quad \Phi(x^+)$ dilaton.

- Holographic stress tensor (Awad,Das,KN,Trivedi) vanishes identically:

$$T^{\mu\nu} \sim K^{\mu\nu} - Kh^{\mu\nu} - 3h^{\mu\nu} + \frac{1}{2}G^{\mu\nu} - \frac{1}{4}\partial^\mu\Phi\partial^\nu\Phi + \frac{1}{8}h^{\mu\nu}(\partial\Phi)^2$$

Dual CFT deformed by $\int \Phi(x^+)O(x)$. Assuming modifications arise due to $\Phi(x^+)$ variation, $\langle T_{\mu\nu}\rangle = 0$ uncorrected: lightlike $\Rightarrow$ no nonzero contractions involving $\partial_+\Phi$.

Confusing: source for bulk field $\Rightarrow$ generically expect response.

- Coord transfm: $ds^2 = \frac{1}{w^2}[e^{f(x^+)}(-2dx^+dy^- + dx_i^2) + dw^2]; \quad \frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\Phi')^2$.

This lies in general family of AdS-deformations (also 5-form)

$$ds^2_{Einst} = \frac{R^2}{w^2}\left(\tilde{g}_{\mu\nu}(x^\mu)dx^\mu dx^\nu + dw^2\right) + R^2 d\Omega_5^2, \quad \Phi = \Phi(x^\mu),$$

solutions if $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi, \quad \tilde{\Box}\Phi \equiv \partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0.$ (Das,KN,Trivedi et al)

Potential singularities (Poincare horizon $w \to \infty$): $R_{ABCD}R^{ABCD}$ diverges.

Null solutions: invariants vanish, no nonzero contraction. Diverging tidal forces as $w \to \infty$.

Fefferman-Graham expansion: $ds^2 = \frac{dw^2}{w^2} + \frac{1}{w^2}[g^0_{\mu\nu}(x^\mu) + w^2 g^2_{\mu\nu}(x^\mu) + \ldots]dx^\mu dx^\nu$

Above solutions constrained: $g^n_{\mu\nu} = 0,\ n > 0$, consistent with holographic RG (Skenderis et al).

- Maybe mild bulk singularity required to encode Lifshitz boundary conditions in these AdS/CFT-based constructions?

(IR singularities in Lifshitz field theory?)

Conclusions, questions

- AdS plane waves $\rightarrow$ dim'nal redux $\rightarrow$ hyperscaling violation.

Dual to 4-d SYM CFT (or more exotic 3d CFT) excited state with T_{++}.

- Some of these lead to logarithmic (or stronger) deviations of entanglement entropy holographically relative to area law.

- AdS plane waves: simple excited pure states. Entanglement entropy can be studied explicitly for strip subsystems, results depend on strip direction w.r.t. energy flux. Strip orthogonal to flux: phase transition.

- Nonconformal Dp-brane plane waves, dual to strongly coupled SYM states with T_{++}. Finite part of entanglement consistent with checks.

- NS5-brane plane waves, little string excited states.

Entanglement entropy area law deviations from field theory?

What are these materials? . . . . . .