de Sitter future-past surfaces & the "entanglement wedge"

K. Narayan
Chennai Mathematical Institute

- de Sitter space and dS/CFT
- dS future-past surfaces, features
- "Ghost-spins" and entanglement

2002.11950, KN; (1711.01107, '15, '16, KN) [collaborations: Dileep Jatkar, Kedar Kolekar]

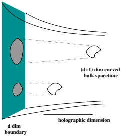
Holography and asymptotics

$\sim$ 25 yrs since AdS/CFT '97 Maldacena; '98 Gubser,Klebanov,Polyakov; Witten.

Holography: quantum gravity in $\mathcal{M} \leftrightarrow$ dual without gravity on $\partial\mathcal{M}$ ('t Hooft, Susskind).

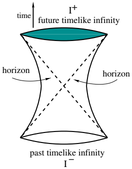
(Witten@Strings'98, '01) Gauge/gravity duality and asymptotics —

$\Lambda < 0$: AdS $\rightarrow$ asymptotics at spatial infinity.
Dual: unitary Lorentzian CFT, includes time.



$\Lambda = 0$: flat space $\rightarrow$ null infinity $\rightarrow$ S-matrix, symmetries...

$\Lambda > 0$: de Sitter space
Fascinating for various reasons. Less clear.
Boundary at future/past timelike infinity $\mathcal{I}^{\pm}$.
Dual $\rightarrow$ Euclidean CFT ...



[note: gravity dual of ordinary Euclidean CFT $\longrightarrow$ Euclidean AdS]

Cosmology, holography, entanglement . . .

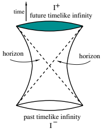
It is of great interest to understand time-dependent phenomena in string theory and holography: many questions remain with spatial holographic boundary.
Entanglement and extremal surfaces as probes: very insightful tools.

• de Sitter: very different from AdS. One might hope that studying dS provides ideas and tools for cosmology and holography more generally.

de Sitter space and holography: a natural boundary in far future $\rightarrow$ dS/CFT, Euclidean non-unitary holographic dual. Time emergent. Generalizations of Ryu-Takayanagi? de Sitter entropy as entanglement?

de Sitter space, extremal surfaces

de Sitter space and dS/CFT



dS/CFT: dual Euclidean non-unitary CFT on dS boundary at future/past timelike infinity $\mathcal{I}^{\pm}$ ('01 Strominger; Witten).

$ds^2 = \frac{R_{dS}^2}{\tau^2}(-d\tau^2 + d\vec{x}^2)$

(Maldacena '02) analytic continuation $r \to -i\tau,\ R_{AdS} \to -iR_{dS}$ from Eucl AdS

$\to$ Hartle-Hawking wavefunction of the universe $\boxed{\Psi_{dS} = Z_{CFT}}$.

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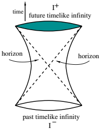

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<u>dS_4</u>: $\Psi[\varphi] \sim e^{iS_{cl}[\varphi]} \sim e^{-\int_k R_{dS}^2 k^3 \varphi_{-k}^0 \varphi_k^0 + \ldots} \longrightarrow$ dual CFT: $\langle O_k O_{k'} \rangle \sim \frac{\delta^2 Z}{\delta\varphi_k^0 \delta\varphi_{k'}^0}$

Energy-momentum $\langle TT \rangle$ 2-pt fn $\to$ $c_3 \sim -\frac{R_{dS}^2}{G_4} < 0$, ghost-CFT?

Anninos,Hartman,Strominger: higher-spin dS_4 dual to $Sp(N)$ ghost CFT_3, ...

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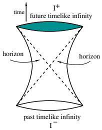

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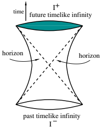
Bulk expectation values $\langle \varphi_k \varphi_{k'} \rangle \sim \int D\varphi\ \varphi_k \varphi_{k'} |\Psi|^2$

Ψ^* and Ψ in bulk vevs $\to$ dual involves two CFT copies.

[In general $\Psi = \Psi[g^3]$, final 3-metric is g^3; sum over final boundary condns for bulk vevs.]

de Sitter, dS/CFT, extremal surfaces

$ds^2 = -(1-\frac{r^2}{l^2})dt^2 + \frac{dr^2}{1-\frac{r^2}{l^2}} + r^2 d\Omega_{d-1}^2$. N, S $(0 \leq r < l)$: static patches.



t is time: translation symmetry. Event horizons for observers in N, S.

de Sitter entropy = area of cosmological horizon. (Gibbons,Hawking)

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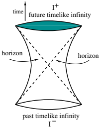

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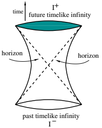

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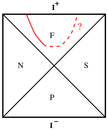

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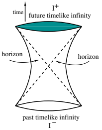

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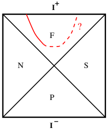

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τ is bulk time now.

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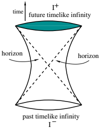

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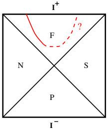

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- No real $I^+ \rightarrow I^+$ turning point, surfaces do not return to I^+ : maybe end at I^-?

  Real extremal surfaces stretching from I^+ to I^-? [bulk $\rightarrow \Psi^*\Psi \rightarrow$ two boundaries]

Future-past surfaces, de Sitter entropy

KN

$ds^2_{d+1} = \frac{l^2}{\tau^2}(-\frac{d\tau^2}{f} + f dw^2 + d\Omega^2_{d-1})$, $[f = 1-\tau^2]$; Area $S = l^{d-1} V_{S^{d-2}} \int \frac{d\tau}{\tau^{d-1}} \sqrt{\frac{1}{f} - f(w')^2}$

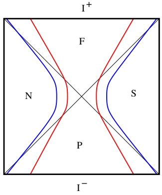
Boundary Eucl time slice: $S^{d-2} \in S^{d-1}$; codim-2 surfaces wrap S^{d-2} [all S^{d-1} equatorial planes equivalent]

Extremize $\rightarrow$ $\dot{w}^2 \equiv (1-\tau^2)^2 (w')^2 = \frac{B^2 \tau^{2d-2}}{1-\tau^2+B^2\tau^{2d-2}}$

B=const, $S = \frac{2l^{d-1} V_{S^{d-2}}}{4G_{d+1}} \int_\epsilon^{\tau_*} \frac{d\tau}{\tau^{d-1}} \frac{1}{\sqrt{1-\tau^2+B^2\tau^{2d-2}}}$

Future-past surfaces stretching from I^+ to I^-

Hartman-Maldacena surfaces (AdS bh) rotated.



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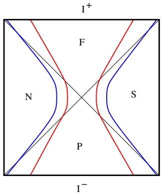

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$\left[dy = \frac{d\tau}{1-\tau^2};\right.$ real turning point τ_* at $|\dot{w}| \rightarrow \infty$: $\left. 1 - \tau_*^2 + B^2 \tau_*^{2d-2} = 0.\right]$

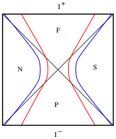
Limiting surface as $\Delta w \rightarrow \infty$, whole space at $I^{\pm}$ $\left(dS_4 :\ B \rightarrow \frac{1}{2} :\ \tau_* \rightarrow \sqrt{2}. \right)$

Area law divergence $S^{div} \sim \frac{\pi l^2}{G_4} \frac{l}{\epsilon_c}$; Finite part $S^{fin} \sim \frac{\pi l^2}{G_4} \Delta w$

Scaling: de Sitter entropy $\rightarrow$ akin to number of degrees of freedom in dual CFT.

[recall AdS_4 BH RT-EE $\sim \frac{R^2}{G_4}(\frac{V}{\epsilon} + \# T^2 V l)$]

de Sitter future-past surfaces, entanglement



de Sitter future-past extremal surfaces stretching between $I^{\pm}$.

Akin to rotated Hartman-Maldacena surfaces in AdS black hole.

Red curve, generic subregion.

Blue curve, limiting surface as subregion $\rightarrow$ whole space.

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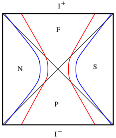

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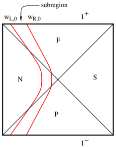

Future-past surfaces, generic subregion: "top-bottom symmetry".

Lie on some S^2 equatorial plane, endpoints $w_{L,0}, w_{R,0}$ at left & right edge (disconnected). Area $S[\mathcal{A}] = S[w_{L,0}] + S[w_{R,0}]$.

Given subregion Δw at $I^{\pm}$, there is a unique future-past surface

$w(y) = w_0 \mp \int_0^y dy\, \dot{w}(B) \Rightarrow w_0 = \pm \int_0^{y_*} dy\, \dot{w}(B) \lessgtr 0$ (no $I^+ \rightarrow I^+$ turning point)

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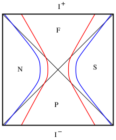

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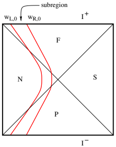

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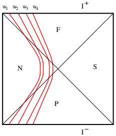


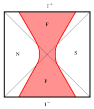
Two disjoint subregions $A \equiv (w_1, w_2)$ and $B \equiv (w_3, w_4)$ at $I^{\pm}$ and their future-past surfaces. Strong subadditivity saturated.
$S[\mathcal{A} \cup \mathcal{B}] = S[w_1] + S[w_2] + S[w_3] + S[w_4] = S[\mathcal{A}] + S[\mathcal{B}]$
so mutual information vanishes: reminiscent of well-separated subsystems in finite temperature system (dS temperature).

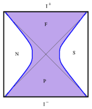
Future-past surfaces, "entanglement wedge"

de Sitter isometries $\Rightarrow$ all S^2 equatorial plane slices equivalent.

Union of codim-2 surfaces $\rightarrow$ codim-1 "envelope" surface.

(Below, mostly from geometric intuition: deeper understanding via dS/CFT?)





Subregions at $I^{\pm}$ and their future-past surfaces suggest "entanglement wedge" : codim-0 bulk region enclosed betw extremal surface and boundary subregion at $I^{\pm}$. Interior of codim-1 "envelope" surface.

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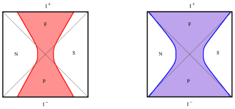

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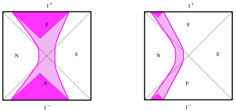

Generic subregions and "entanglement wedge" : bigger than causal wedge (= domain of dependence = past lightcone wedge).

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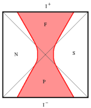

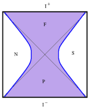

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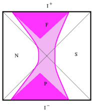

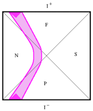

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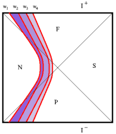


Multiple subregions at $I^{\pm}$, corresponding top-bottom symmetric future-past surfaces $\rightarrow$ analog of subregion duality:

bulk subregion $\leftrightarrow$ boundary subregion.

e.g. middle boundary subregion $[w_2, w_3]$ dual to middle bulk "entanglement wedge".

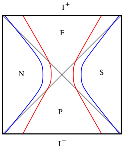
(other features also fit)

dS future-past surfaces; ghost entanglement

Real connected future-past surfaces stretching from I^+ to I^-.

(akin to Hartman-Maldacena surfaces (AdS bh) rotated)

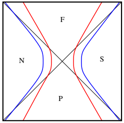
Suggests future-past entanglement (betw $I^\pm$). Meaning?



Speculation: dS_4 approximately dual to $CFT_F \times CFT_P$ in thermofield-double-like entangled state $|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^P} |i_n^F\rangle |i_n^P\rangle$?

KN; see also Arias,Diaz,Sundell

dS future-past surfaces; ghost entanglement



Real connected future-past surfaces stretching from I^+ to I^-.

(akin to Hartman-Maldacena surfaces (AdS bh) rotated)

Suggests future-past entanglement (betw $I^\pm$). Meaning?

Speculation: dS_4 approximately dual to $CFT_F \times CFT_P$ in thermofield-double-like entangled state $|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^P} |i_n^F\rangle |i_n^P\rangle$?

KN; see also Arias,Diaz,Sundell

(Witten, Strominger '01) bulk time evolution maps I^- to I^+.

$|i_n^P\rangle \rightarrow |i_n^F\rangle \;\; \Rightarrow \;\; |\psi^{tfd}\rangle$ unitarily equivalent to $|\psi^{tfd}\rangle = \sum \psi^{i_n^F, i_n^F} |i_n^F\rangle |i_n^F\rangle$

i.e. TFD-like state in two dual CFT copies at I^+.

dS_4/CFT_3: dual is ghost-like CFT (-ve central charge, Maldacena '02).

Entanglement in ghost CFTs and ghost-like quantum mechanical systems?

Negative norm states: positive subsectors of entanglement entropy?

Entanglement in ghost theories

Entanglement in ghost theories: "ghost-spins"

KN; Jatkar,KN

- Replica arguments (Calabrese, Cardy) can be generalized to $c = -2$ ghost CFTs: twist operator 2-pt fn $\rightarrow$ $c < 0 \Rightarrow S < 0$. ▸ rep $[|\downarrow\rangle = |0\rangle; \;\; \langle -Q|T(z)|0\rangle = 0]$

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- "Ghost-spin" $\rightarrow$ 2-state spin variable with indefinite norm.
$$\langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 1, \qquad \langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 0$$
[ordinary spin: $\langle\uparrow|\uparrow\rangle=1=\langle\downarrow|\downarrow\rangle$]

$|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$; $\langle\pm|\pm\rangle = \gamma_{\pm\pm} = \pm 1$, $\langle+|-\rangle = \langle-|+\rangle = 0$

Infinite ghost-spin chains, $\langle nn\rangle$-intns $\rightarrow$ continuum limit $\rightarrow$ bc-ghost CFT. ▸ gsp

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Infinite ghost-spin chains, $\langle nn\rangle$-intns $\rightarrow$ continuum limit $\rightarrow$ bc-ghost CFT. gsp

- Two ghost-spins: $|\psi\rangle = \sum \psi^{ij}|i\rangle|j\rangle \ \rightarrow \rho = |\psi\rangle\langle\psi| \ \rightarrow$ partial trace $\rightarrow$ RDM for remaining ghost-spin $\rightarrow$ von Neumann entropy. gsEE

$$\langle\psi|\psi\rangle = \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl*} = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 = \pm 1$$

RDM: $(\rho_A)^{ik} = \gamma_{jl}\psi^{ij}\psi^{kl*}$; EE: $S_A = -\gamma_{ij}(\rho_A \log \rho_A)^{ij}$

In general: (i) $+ve$ norm $|\psi\rangle \not\Rightarrow +ve$ RDM, EE. (ii) new entanglement patterns.

e.g. $(\rho_A)_i^k e_k = \lambda e_i$ *i.e.* $(\rho_A)^{ij} e_j = \gamma^{ij}\lambda e_j$. -ve norm $\Rightarrow$ eigenvalues λ in general complex.

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- Entangle identical ghost-spins from each copy $\rightarrow$ $+ve$ norm, RDM, EE

$|\psi\rangle = \psi^{++}|+\rangle|+\rangle + \psi^{--}|-\rangle|-\rangle \Rightarrow \langle\psi|\psi\rangle > 0 \longrightarrow$ correlated ghost-spins

Also true for 2 copies of ghost-spin ensembles: $|\psi\rangle = \sum_{|\sigma_n\rangle} \psi^{\sigma_n,\sigma_n}|\sigma_n\rangle|\sigma_n\rangle, \ \langle\psi|\psi\rangle > 0$

Conclusions, questions

- de Sitter future-past extremal surfaces appear to be a way to organize bulk entanglement. Akin to rotated versions of Hartman,Maldacena.
  Various features, *e.g.* limiting surface, vanishing mutual information, "entanglement wedge", subregion duality, ...
  Suggest a TFD-like entangled dual of two CFT copies at future boundary.

Deeper understanding and interpretation? Quantum extremal surfaces?

- With ordinary spatial (AdS-like) boundary, a spacelike RT/HRT surface gives real area. Then as the holographic boundary is rotated to timelike infinity, we acquire a relative minus sign, suggesting complex areas for timelike extremal surfaces. (KN '15, analytic continuation from AdS RT)

In this sense, future-past surfaces have overall i which we are removing.
(a bit like calling the length of a timelike geodesic as time, rather than i·space)

So perhaps this is a new object, "temporal entanglement"?

Holographic entanglement entropy

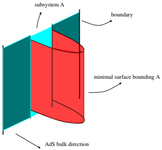
Entanglement entropy: entropy of reduced density matrix of subsystem.

EE for spatial subsystem A, $S_A = -tr\rho_A \log \rho_A$, with partial trace $\rho_A = tr_B \rho$.

Ryu-Takayanagi: $EE = \frac{A_{min.surf.}}{4G}$

[$\sim$ black hole entropy] Area of codim-2 minimal surface in gravity dual.

Non-static situations: extremal surfaces (Hubeny, Rangamani, Takayanagi).



Operationally: const time slice, boundary subsystem $\rightarrow$ bulk slice, codim-2 extremal surface

Example: CFT$_d$ ground state = empty AdS_{d+1}, $ds^2 = \frac{R^2}{r^2}(dr^2 - dt^2 + dx_i^2)$.

Strip, width $\Delta x = l$, infinitely long. Bulk surface $x(r)$. Turning point r_*.

$$S_A = \frac{V_{d-2} R^{d-1}}{4G_{d+1}} \int \frac{dr}{r^{d-1}} \sqrt{1 + (\partial_r x)^2} \quad \rightarrow \quad (\partial_r x)^2 = \frac{(r/r_*)^{2d-2}}{1-(r/r_*)^{2d-2}} , \quad \frac{l}{2} = \int_0^{r_*} dr \partial_r x.$$

$$S_A = \frac{V_{d-2} R^{d-1}}{4G_{d+1}} \int_\epsilon^{r_*} \frac{dr}{r^{d-1}} \frac{2}{\sqrt{1-(r/r_*)^{2d-2}}} \quad \longrightarrow \quad S_A = \frac{R}{2G_3} \log \frac{l}{\epsilon} , \quad \frac{3R}{2G_3} = c \quad [2d].$$

$$S_A \sim \frac{R^{d-1}}{G_{d+1}} \left(\frac{V_{d-2}}{\epsilon^{d-2}} - c_d \frac{V_{d-2}}{l^{d-2}} \right) , \qquad \frac{R^3}{G_5} \sim N^2 \quad [4d], \quad \frac{R^2}{G_4} \sim N^{3/2} \quad [3d].$$

CFT thermal state (AdS black brane): minimal surface wraps horizon. $S^{fin} \sim N^2 T^3 V_2 l$

Back

bc-ghosts, $c=-2$: replica and EE

KN

$T(w) = (\partial_w z)^2 T(z) + \frac{c}{12}\{z,w\}$, Schwarzian $\{z,w\} = \frac{z'''}{z'} - \frac{3}{2}(\frac{z''}{z'})^2$ (Calabrese,Cardy)

Subsystem A: interval betw $x = u, v$; replica w-space $\rightarrow$ $z = (\frac{w-u}{w-v})^{1/n}$ $\rightarrow$ z-plane.

z-plane: $\langle T(z)\rangle_{\mathbb{C}} = 0$ $\rightarrow$ z-plane maps to $SL(2,Z)$ inv vacuum.

$$\langle T(w)\rangle_{\mathcal{R}_n} = \frac{c}{12}\{z,w\} = \frac{\langle T(w)\Phi_n(u)\Phi_{-n}(v)\rangle}{\langle\Phi_n(u)\Phi_{-n}(v)\rangle} \rightarrow tr\rho_A^n \equiv \frac{Z_n}{Z_1^n} \sim \text{twist op 2-pt fn.}$$

- Replica argument is applicable for the ghost ground state if it is the $SL(2)$ vacuum: $c=-2$ bc-ghost CFT $\rightarrow |\downarrow\rangle = |0\rangle$ with $L_0 = 0$.
- Regularity condition $\langle T(z)\rangle_{\mathbb{C}} = 0$ $\rightarrow$ $\langle -Q|\, T(z)\, |0\rangle = 0$

Incorporate background charge, or $\langle T(z)\rangle = 0$ trivially from zero modes. [$c=-2 \rightarrow Q=-1$]

Replica formulation formally applies now: $\boxed{c<0 \;\Rightarrow\; S_A<0}$

$\mathbb{Z}_N$ bc-orbifold CFTs (Saleur, Kausch, Flohr, … '90s) confirm negative conf dims of twist ops [$l \equiv v-u$]

$$tr\rho_A^n = \prod_{k=1}^{n-1}\langle 0|\sigma^-_{k/N}(v)\sigma^+_{k/N}(u)|0\rangle = l^{\frac{1}{3}(n-1/n)} \rightarrow S_A = -\lim_{n\rightarrow 1}\partial_n\, tr\rho_A^n = -\frac{2}{3}\log\frac{l}{\epsilon}$$

Back

Two ghost-spins

KN

$|\psi\rangle = \sum \psi^{ij}|ij\rangle$: $\langle\psi|\psi\rangle = \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl*} = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 = \pm 1$

Trace over one ghost-spin $\rightarrow$ ρ_A for remaining ghost-spin $\rightarrow$ von Neumann entropy S_A.

RDM: $(\rho_A)^{ik} = \gamma_{jl}\psi^{ij}\psi^{kl*} = \gamma_{jj}\psi^{ij}\psi^{kj*}$ $\quad(\gamma_{\pm\pm} = \pm 1)$

$$(\rho_A)^{++} = |\psi^{++}|^2 - |\psi^{+-}|^2, \qquad (\rho_A)^{+-} = \psi^{++}\psi^{-+*} - \psi^{+-}\psi^{--*},$$
$$(\rho_A)^{-+} = \psi^{-+}\psi^{++*} - \psi^{--}\psi^{+-*}, \qquad (\rho_A)^{--} = |\psi^{-+}|^2 - |\psi^{--}|^2.$$

Define $\log\rho_A$ using expansion & mixed-index RDM $(\rho_A)_i{}^k = \gamma_{ij}(\rho_A)^{jk}$.

EE: $S_A = -\gamma_{ij}(\rho_A \log\rho_A)^{ij} \rightarrow -(\rho_A)_+^+(\log\rho_A)_+^+ - (\rho_A)_-^-(\log\rho_A)_-^-$

In general, $+ve$ norm $\not\Rightarrow$ $+ve$ RDM, EE. [however, correlated ghost-spins]

Simple subfamily, diagonal ρ_A: $\log\rho_A$ diag

$(\rho_A)_+^+ = \pm x, \quad (\rho_A)_-^- = \pm(1-x), \qquad 0 < x = \frac{|\psi^{++}|^2}{|\psi^{++}|^2 + |\psi^{--}|^2} < 1$

$\langle\psi|\psi\rangle > 0$: $S_A^+ = -x\log x - (1-x)\log(1-x) > 0$ $\quad$ $+ve$ norm $\Rightarrow$ $+ve$ EE

$\langle\psi|\psi\rangle < 0$: $S_A^- = x\log(-x) + (1-x)\log(-(1-x)) = -S_A^+ + i\pi$

$-ve$ norm $\Rightarrow$ ρ^A eigenvalues $-ve$ $\Rightarrow$ $-ve$ Re(EE), const Im(EE)

Back

Ghost-spin chains → bc-ghost CFTs

Jatkar,KN

Infinite ghost-spin chains → continuum limit → ghost-CFTs?

Ghost-spins as microscopic building blocks of ghost/nonunitary CFTs?

Recall: Ising model at critical point is a CFT of free massless fermions.

Hamiltonian $H = J\sum_n \left(\sigma_{b(n)}\sigma_{c(n+1)} + \sigma_{b(n)}\sigma_{c(n-1)}\right)$

Spin variables: $\{\sigma_{bn}, \sigma_{cn}\} = 1$, $[\sigma_{bn}, \sigma_{bn'}] = [\sigma_{cn}, \sigma_{cn'}] = [\sigma_{bn}, \sigma_{cn'}] = 0.$

$\sigma_{bn}^\dagger = \sigma_{bn}$, $\sigma_{cn}^\dagger = \sigma_{cn}$; $\sigma_b|\downarrow\rangle = 0$, $\sigma_b|\uparrow\rangle = |\downarrow\rangle$, $\sigma_c|\uparrow\rangle = 0$, $\sigma_c|\downarrow\rangle = |\uparrow\rangle$.

Like b_n, c_n ops of bc-CFT, $\{b_n, c_m\} = \delta_{n,-m}$: but σ_{bn}, σ_{cn} bosonic (distinct sites, commute).

Jordan-Wigner: $a_{bn} = \prod_{k=1}^{n-1} i(1-2\sigma_{ck}\sigma_{bk})\sigma_{bn}$, $a_{cn} = \prod_{k=1}^{n-1}(-i)(1-2\sigma_{ck}\sigma_{bk})\sigma_{cn}$

→ Fermionic gh.sp. variables: $\{a_{bi}, a_{cj}\} = \delta_{ij}$, $\{a_{bi}, a_{bj}\} = 0$, $\{a_{ci}, a_{cj}\} = 0$.

$H = J\sum_n(\sigma_{b(n)}\sigma_{c(n+1)} + \sigma_{b(n)}\sigma_{c(n-1)}) \rightarrow iJa_{bn}(a_{c(n+1)} - a_{c(n-1)}) \sim -b\partial c$

→ lattice discretization of bc-ghost CFT.

Momentum variables, continuum limit $H \xrightarrow{J\sim 1/2a} \sum_{k>0} k\left(b_{-k}c_k + c_{-k}b_k\right) + zpe$

Conf symm: $a \to \xi^{-1}a$, $H \to \xi H$, $\sigma_{b(n)} \to \xi^{\lambda}\sigma_{b(n)}$, $\sigma_{c(n+1)} \to \xi^{1-\lambda}\sigma_{c(n+1)}$

$\mathrm{CFT}_3^{Sp(N)}$, symplectic fermions ?

Back

(DJ,KN,Kolekar) N-level ghost-spins: • $O(N)$ flavour generalization: $\langle\downarrow^A|\uparrow^B\rangle = \delta^{AB} = \langle\uparrow^A|\downarrow^B\rangle$

• N-levels, symplectic-like structure: $\langle\uparrow^A|\downarrow^B\rangle = i\,\Omega^{AB} = \langle\downarrow^A|\uparrow^B\rangle$ (towards symplectic fermions), . . .

Two ensemble copies $\mathcal{GC}_1 \times \mathcal{GC}_2$: $|\psi\rangle = \sum_{|\sigma\rangle}\psi^{\sigma,\sigma}|\sigma\rangle|\sigma\rangle$, $\langle\psi|\psi\rangle = \sum_{|\sigma\rangle}|\psi^{\sigma,\sigma}|^2 > 0$

$\xrightarrow{ground\ states}$ $\{|\sigma\rangle\}$ 2^N-dim → maximal EE → $S_A = N\log 2$