Nonsupersymmetric $\mathbb{C}^4/\mathbb{Z}_N$ singularities and closed string tachyons

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[arXiv:0912.3374, KN]

- Overview of closed string tachyons, ...
- Nonsusy 4-complex dimensional singularities:
  orbifold terminality, conifold-like singularities, phases ...
- M2-branes, nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$

Earlier work on 3-dim ($\mathbb{C}^3/\mathbb{Z}_N$, conifold-like) singularities:
hep-th/0609017, hep-th/0510104, KN;
hep-th/0412337, David Morrison, KN;
hep-th/0406039, David Morrison, KN, Ronen Plesser.

Related references:

Adams, Polchinski, Silverstein;
Vafa; Harvey, Kutasov, Martinec, Moore; $\ldots$:
closed string tachyons in (nonsusy) $\mathbb{C}/\mathbb{Z}_N$ and $\mathbb{C}^2/\mathbb{Z}_N$ (ALE) singularities.

Hanany et al; Martelli, Sparks, et al; Gauntlett et al:
various susy 3-dim and D-brane gauge theories;
various susy 4-dim (complex) singularities and ABJM-like theories.

$\ldots$

Nonsusy string/M- backgrounds

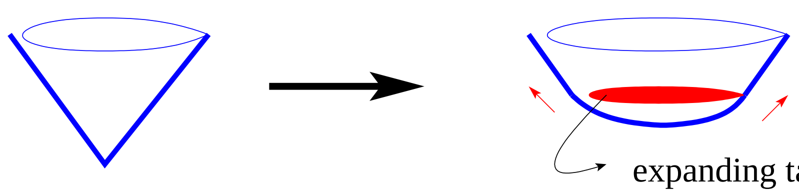
Important to study nonsusy backgrounds in string/M- theory: physical/mathematical structure and realistic model building aspects. Often, broken susy implies spacetime instabilities. String spectra in such backgrounds, when tractable, often contain tachyons ($m^2 < 0$ spacetime fields): $e.g.$ open string tachyons in p-$\bar{p}$ systems (Sen, $\ldots$).

Direct time evolution in spacetime: often difficult to study.
String worldsheet renormalization group flows: useful tool in understanding phase structure of the decay of such unstable systems.
There exist RG flows from 2-dim conformal fixed points representing unstable spacetimes to new, more stable endpoints. If CFT description exists, RG flows induced by tachyons in the string spectrum.
If $worldsheet$ susy unbroken, RG flows under control, can be tracked.
RG flow matches qualitative properties of on-shell time evolution, $e.g.$ flow directions and fixed points.

Nonsusy singularities, closed string tachyons

Closed string tachyons complicated: gravity involved. In general, hard to understand "decay of spacetime" (unlike p-$\bar{p}$ decay).

Study systems with localized (noncompact) instabilities in an otherwise stable spacetime background.



Consider spacetimes with conical singularities of the form $\mathbb{R}^{7,1} \times \mathbb{C}/\mathbb{Z}_N$ or $\mathbb{R}^{5,1} \times \mathbb{C}^2/\mathbb{Z}_N$ (APS, Harvey et al, $\ldots$): tachyon condensation corresponds to blowing up cycles (compact subspaces) initially collapsed at the singularity.

Singularity fully resolved eventually: $e.g.$ canonical (Hirzebruch-Jung) "minimal" resolution in $\mathbb{C}^2/\mathbb{Z}_N$ (ALE) case.

3-dimensional ($\mathbb{C}^3/\mathbb{Z}_N$) orbifolds ?

Morrison,KN,Plesser; Morrison,KN

Full spacetime: $\mathbb{R}^{3,1}[x^{0123}] \times \mathcal{M}[\{x^{4,5}, x^{6,7}, x^{8,9}\} \equiv z^{1,2,3}]$.

Orbifold action: $\mathbb{C}^3/\mathbb{Z}_N(k_1, k_2, k_3),\ z^i \to e^{2\pi i k_i/N} z^i$.

(susy if $\mathbb{Z}_N \in SU(3),\ i.e.\ \sum_i k_i = 0(mod\ 2N)$.)

Singularities in complex dimension 3 are particularly interesting: rich connections of tachyon physics with resolution theory in algebraic geometry. Toric singularities are special, described by combinatorial data. Quotient singularities: no complex structure deformations.

- (geometric) Terminal singularities: no Kahler blowup modes (tachyons or moduli), arising from chiral ring twisted sector states.
- No canonical resolution: resolutions of distinct topology related by flop (moduli) and flip (tachyonic) transitions.

$\mathbb{C}^3/\mathbb{Z}_N$ orbifolds: CFT and geometry

Unlike point particles which would see a singular space, string modes can "wind" around singularity: these localized twisted sector excitations (in Conformal Field Theory) map precisely to blowup modes governing the resolution of the singularity.

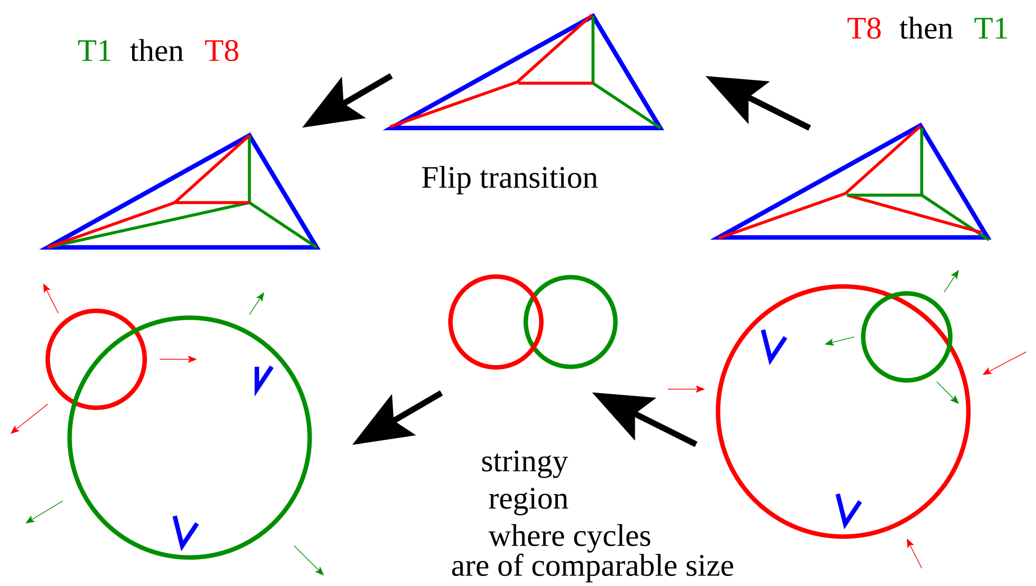
Analyzing all metric blowup modes (all twisted states, from all chiral and antichiral rings) shows:

- Combinatoric proof shows no terminal singularities in (Type II string) theories with spacetime fermions, $i.e.$ singularity always resolved.

- In (Type 0) string theories with no spacetime fermions but containing a delocalized tachyon, there is in fact a terminal singularity $\mathbb{C}^3/\mathbb{Z}_2(1,1,1)$, which arises as a generic endpoint – this singularity has no tachyons and no moduli.

Topology change: flip transitions

Orbifolds: generically there are multiple tachyons: if a more dominant tachyon condenses during the condensation of some tachyon, there is a blowdown of one cycle and a topologically distinct cycle blows up, mediating mild dynamical topology change.



Inherent time dependence here: geometry dynamically evolves towards less singular resolution.

Nonsusy $\mathbb{C}^4/\mathbb{Z}_N$ singularities

KN

Bgnd (10-dim): $\mathbb{R}^{1,1}[x^{01}] \times \mathcal{M}[\{x^{2,3}, x^{4,5}, x^{6,7}, x^{8,9}\} \equiv z^{1,2,3,4}]$;

(11-dim): $\mathbb{R}^{2,1}[x^{0,1,10}] \times \mathcal{M}[\{x^{2,3}, x^{4,5}, x^{6,7}, x^{8,9}\} \equiv z^{1,2,3,4}]$.

Orbifold action: $\mathbb{C}^4/\mathbb{Z}_N(k_1, k_2, k_3, k_4),\ z^i \rightarrow e^{2\pi i k_i/N} z^i$.

(susy if $\mathbb{Z}_N \in SU(4),\ i.e.\ \sum_i k_i = 0(mod\ 2N)$: Calabi-Yau.)

As in 3-dim, no canonical resolution. Resolutions of distinct topology related by 4-dim flips/flops.

New features: There exist susy terminal singularities.
Combinatorics much more complicated (CFT and toric geometry): need to use numerics (Maple program).
Algebro-geometric structure more intricate, harder to visualise: $e.g.$ toric cones in (real) 4-dim.
Topology change slightly different (dimensionality of cycles changes).

$\mathbb{C}^4/\mathbb{Z}_N$ generalities

M-theory: metric, 3-form modes compactified on $\mathbb{C}^4/\mathbb{Z}_N$ give classical Kahler parameter and scalar dual to $U(1)$ gauge field in $\mathbb{R}^{2,1}$ (somewhat like CY 4-fold (Gukov,Vafa,Witten)): these complexified scalars govern blowup modes of singularity.

In Type IIA string obtained by compactifying $\mathbb{R}^{2,1} \rightarrow \mathbb{R}^{1,1}$, these map to (NS-NS) metric, B-field modes: complexified Kahler parameters of various collapsed cycles at singularity.

In either string/M- theory, the geometric resolution structure of the singularity itself governed by toric geometry or equivalently the corresponding gauged linear sigma models.

Beautiful correspondence (as in lower dim): physical orbifold CFT twisted sector spectrum $\leftrightarrow$ toric geometry of singularity.

We use this to gain insight into the structure of $\mathbb{C}^4/\mathbb{Z}_N$ singularities.

Perhaps gives insight into possible direct M-theory analysis.

$\mathbb{C}^4/\mathbb{Z}_N$ CFT

Focus on $\mathbb{C}^4/\mathbb{Z}_N(k_1, k_2, k_3, k_4) = \mathbb{C}^4/\mathbb{Z}_N(1, p, q, r)$.
Consider rotation generator $R = \exp\,[\frac{2\pi i}{N}(J_{23} + pJ_{45} + qJ_{67} + rJ_{89})]$
and spinor states $\{s_{ij}\} = \{\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\} \equiv |\pm\pm\pm\pm\rangle$
$[\sum s_{ij} = even]$.

Orbifold nonsusy if action of R does not preserve any spinor states,
$i.e.\quad \pm 1 \pm p \pm q \pm r \neq 0 (mod\, 2N)$.

Type II: $R^N = 1$, preserving fermions, removing bulk tachyon $\Rightarrow$
$\pm 1 \pm p \pm q \pm r = even \;\Rightarrow\; 1 + p + q + r = \sum_i k_i = even$.

This Type II GSO projection can also be found from modular invariance of RNS partition function.

Also gives the GSO projections for twisted sector states.

$\mathbb{C}^4/\mathbb{Z}_N$ CFT, twisted sector states

Product structure of orbifold CFT (with (2,2) worldsheet susy) means twisted spectrum of string states organized into sixteen chiral/antichiral rings in eight conjugate pairs (one from each complex plane, either chiral (c_{X_i}) or antichiral (a_{X_i}): chiral rings labelled $(c_{X_1}, c_{X_2}, c_{X_3}, c_{X_4}), (c_{X_1}, c_{X_2}, c_{X_3}, a_{X_4}), \ldots \equiv (cccc), (ccca), \ldots$.

States described by chiral ring twist field (vertex) operators $X_j = \prod_{i=1}^4 X^i_{\{jk_i/N\}} = \prod_{i=1}^4 \sigma_{\{jk_i/N\}} e^{i\{jk_i/N\}(H_i - \bar{H}_i)}$

Ground or first excited states in each twisted sector.

Convenient notation to label twisted states in various rings: note twist operators in $e.g.$ $(c_{X_1}, c_{X_2}, c_{X_3}, a_{X_4})$-ring can be rewritten $X_j^{ccca} = \prod_{i=1}^3 X^i_{\{\frac{jk_i}{N}\}} (X^4_{1-\{\frac{jk_4}{N}\}})^* = \prod_{i=1}^3 X^i_{\{\frac{jk_i}{N}\}} (X^4_{\{\frac{-jk_4}{N}\}})^*$, resembling twist operators in $cccc$-ring of $\mathbb{C}^4/\mathbb{Z}_N(k_1, k_2, k_3, -k_4)$ with $X^4 \to (X^4)^*$.

$\mathbb{C}^4/\mathbb{Z}_N$ CFT, twisted sector states

Label $\mathbb{C}^4/\mathbb{Z}_N(1,p,q,r)$, with $p,q,r>0$: this is $cccc$-ring. Then the various rings are labelled as

$$cccc \equiv (1,p,q,r),\ \ ccca \equiv (1,p,q,-r),\ \ ccac \equiv (1,p,-q,r),$$
$$cacc \equiv (1,-p,q,r),\ \ ccaa \equiv (1,p,-q,-r),\ \ caca \equiv (1,-p,q,-r),$$
$$caac \equiv (1,-p,-q,r),\ \ caaa \equiv (1,-p,-q,-r).$$

Since all rings included, sufficient to restrict $0<p,q,r<N$. $e.g.$ $\mathbb{Z}_N(1,p,q,r) \equiv \mathbb{Z}_N(1,p,q,r-2N)$, and $cccc$-ring here equivalent to $ccca$-ring of $\mathbb{Z}_N(1,p,q,-(2N-r))$.

$X_j = \prod_i X^i_{\{jk_i/N\}}:\ R_j \equiv (\{\frac{jk_1}{N}\},\{\frac{jk_2}{N}\},\{\frac{jk_3}{N}\},\{\frac{jk_4}{N}\}) = \sum_i \{\frac{jk_i}{N}\}$.

GSO-allowed state: $X_j \rightarrow (-1)^{E_j} X_j = -X_j,\ \ E_j = \sum_i [\frac{jk_i}{N}] = odd.$

R_j R-charge (Minus sign from ghost contribution to E_j (in the $(-1,-1)$-picture), $i.e.$ total worldsheet $(-1)^F = even$.)

Spacetime masses (mass-shell condition): $m_j^2 = \frac{2}{\alpha'}(R_j - 1)$.

Terminal singularities

Spacetime masses: $m_j^2 = \frac{2}{\alpha'}(R_j - 1)$, R-charge: $R_j = \sum_i \{\frac{jk_i}{N}\}$.
$R_j < 1 \;\Rightarrow\; m_j^2 < 0$: tachyon. $R_j = 1 \;\Rightarrow\; m_j^2 = 0$: modulus.
$R_j > 1 \;\Rightarrow\; m_j^2 > 0$: irrelevant (massive).

Singularity terminal if no twisted sector tachyon or modulus,
$i.e.$ $R_j > 1$ for all chiral/antichiral ring states.
This means no geometric blowup modes, $i.e.$ singularity not physically resolved by (relevant or marginal) worldsheet string modes.
Type II GSO: only some states from each ring preserved.

Note: in mathematics literature, terminal singularities refer to no Kahler blowup modes (chiral ring alone). Milder. These usually admit nonKahler blowups (from some antichiral ring).

Physical question: any tachyons/moduli from any ring?
If not, then no unstable directions for decay.

Terminal $\mathbb{C}^4/\mathbb{Z}_N$ singularities

Susy terminal singularities: $R_j > 1$ if $E_j = odd$.

$\mathbb{C}^4/\mathbb{Z}_N(1,1,1,1)$: $cccc$: $E_j = 4[\frac{j}{N}] = 0 \;\Rightarrow$ no GSO-preserved state.

$ccca$: $R_j = 3\{\frac{j}{N}\} + \{\frac{j(-1)}{N}\} = 1 + 2\{\frac{j}{N}\} > 1, (E_j = odd)$, etc.

Similarly, $\mathbb{C}^4/\mathbb{Z}_N(1,1,p,p)$ terminal if p coprime w.r.t. N.

Nonsusy $\mathbb{C}^4/\mathbb{Z}_N(1,p,q,r)$: combinatorics much richer than $\mathbb{C}^3/\mathbb{Z}_N$.

Proving Type II non-terminality difficult.

Constraints from (i) no susy ($\pm 1 \pm p \pm q \pm r \neq 0 (mod\, 2N)$),

(ii) Type II ($\sum_i k_i = 1 + p + q + r = even$),

(iii) $j = 1$ sector all-ring terminality ($R_{j=1} > 1$), give

$$q - p + 1 < r < p + q - 1, \qquad r - p + 1 < q < p + r - 1\,,$$
$$r - q + 1 < p < q + r - 1$$

as strongest inequalities on $p, q, r > 0$ for potential terminality of $\mathbb{C}^4/\mathbb{Z}_N(1,p,q,r)$.

Terminal $\mathbb{C}^4/\mathbb{Z}_N$ singularities

- $(1,p,q,r) = \ldots, (1,3,5,7), (1,5,7,13), \ldots$: supersymmetric. $e.g.$ $caac$-ring of $(1,3,5,7)$ is $cccc$-ring of $(1,-3,-5,7)$ (susy).
- $\ldots, (1,1,5,9), (1,3,7,13), \ldots$: do not satisfy inequalities above, $i.e.$ tachyon in some $j=1$ sector.
- $(1,5,7,9), (1,7,11,13), \ldots$: satisfy inequalities above, potentially terminal. Maple program check, $N \leq 400$, shows no nonsusy Type II terminal singularity.
- Miscellaneous Maple checks, $N \leq 30$, assorted weights.
- Type 0 terminal singularities exist: $e.g.$ $\mathbb{Z}_3(1,1,1,2), \mathbb{Z}_4(1,1,1,2), \mathbb{Z}_4(1,1,2,3), \mathbb{Z}_5(1,1,2,3)$.

Maple output shows number of tachyons/moduli increasing as order N increases. Terminal singularities less likely as N increases.
This "experimental" search not equivalent to proof. But strongly suggestive of absence of nonsusy Type II $\mathbb{C}^4/\mathbb{Z}_N$ singularities.

$\mathbb{C}^4/\mathbb{Z}_N$: CFT, geometry

Toric cone real 4-dim. Hard to visualise geometry of interior lattice points. Use $\mathbb{Z}_N(1,p,q,r)$ CFT and GLSM as surrogate for geometry.

Cone: $e_1=(N,-p,-q,-r), e_2=(0,1,0,0), e_3=(0,0,1,0), e_4=(0,0,0,1)$. “Marginality hyperplane” Δ (tetrahedral cone) thro e_i.

Beautiful correspondence: twisted sector states $\leftrightarrow$ blowup modes.

Generic lattice point $(x_j,y_j,z_j,w_j)=r_1e_1+r_2e_2+r_3e_3+r_4e_4$,

$i.e.\ P_j=(j,-[\frac{jp}{N}],-[\frac{jq}{N}],-[\frac{jr}{N}])\ \equiv\ R_j=(\frac{j}{N},\{\frac{jp}{N}\},\{\frac{jq}{N}\},\{\frac{jr}{N}\})$.

Interior points *on* Δ have $R_j=1$: moduli.

Points “below” (in 4D) hyperplane Δ have $R_j<1$: tachyons.

Condensation of tachyon/modulus: blowup which is subdivision of cone by corresponding lattice point. Type II GSO preserved.

Typically multiple tachyons: tachyon masses get renormalized (become more irrelevant) under condensation of some tachyon.

$\mathbb{C}^4/\mathbb{Z}_N$ tachyon condensation

No canonical resolution. Different tachyons are different decay channels. Physically, expect most relevant tachyon ($i.e.$ smallest R_j, $i.e.$ most negative m^2) is most dominant decay channel.
Study its condensation and endpoints. Typically these are also unstable. Further decay $\ldots$
Absence of Type II terminal singularities means final endpoints smooth or susy singularities (possibly terminal).

As in 3-dim, flips: blowdown of one cycle and blowup of topologically distinct cycle, if more dominant tachyon condenses during condensation of some tachyon. Typically $\mathbb{P}^1$ blows down, $w\mathbb{P}^2$ blows up (or vice versa).
Inherent time dependence here: geometry dynamically evolves towards less singular resolution.

Gauged linear sigma models

Witten; Morrison,Plesser;...

Elegant way to realize the various phases of spacetime geometry.

2-dim $U(1)^r$ gauge theory with $(2,2)$ worldsheet susy and massless chiral superfields $\Psi_i \to e^{iQ_i^a\lambda}\Psi_i$, with charge matrix Q_i^a. No superpotential. There are couplings to $t_a = ir_a + \frac{\theta_a}{2\pi}$ (Fayet-Iliopoulos parameters, θ-angles). The potential energy is

$$U = \sum_a \frac{(D_a)^2}{2e_a^2} + 2\sum_{a,b}\bar{\sigma}_a\sigma_b\sum_i Q_i^a Q_i^b|\Psi_i|^2.$$

Vacuum structure ($U = 0$) exhibits several "phases" as r_a vary

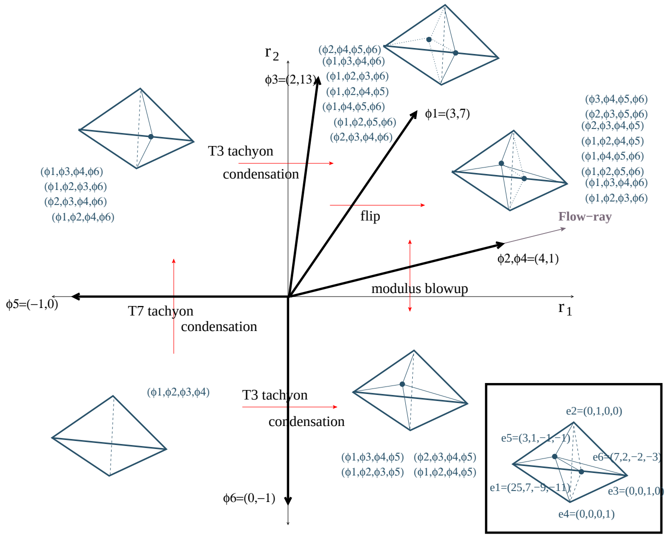
$$D_a \sim \sum_i Q_i^a|\Psi_i|^2 - r_a = 0.$$

Nonzero $r_a \Rightarrow$ some fields are Higgsed: light fields (moduli of GLSM) describe spacetime geometry of string propagation.

Quantum (1-loop) renormalization: $r_a(\mu) = (\sum_i Q_i^a)\log\frac{\mu}{\Lambda}$

Large r_a: worldsheet instanton corrections ignored.

Phases of $\mathbb{C}^4/\mathbb{Z}_{25}(1,-7,9,11)$



Most relevant tachyon (T_3) is most dominant decay channel.

T_3 lies in *cacc*-ring of $\mathbb{Z}_{25}(1,7,9,11)$. Also include tachyon T_7.

Analyse phase structure using $U(1)^2$ GLSM with fields

$\Psi_i = \phi_1, \phi_2, \phi_3, \phi_4, T_3, T_7$. Charge matrix $Q_i^a = \begin{pmatrix} 3 & 4 & 2 & 8 & -25 & 0 \\ 7 & 1 & 13 & 2 & 0 & -25 \end{pmatrix}$.

Most phase boundaries represent condensation of some tachyon.

ϕ_1 phase boundary: topology changing flip transition.

Conifold-like singularities

Toric singularities defined by $Q_i = (\ n_1 \quad n_2 \quad n_3 \quad -n_4 \quad -n_5\ )$, $n_i > 0$ and $\sum_i Q_i \neq 0$.

4-dim analogs of nonsusy conifold-like singularities. Exhibit flips.

Maximally susy subfamily: $\sum_i Q_i = 0$. Calabi-Yau 4-fold cones.

4-dim analogs of $L_{abc}s, Y_{pq}s$, in 3-dim.

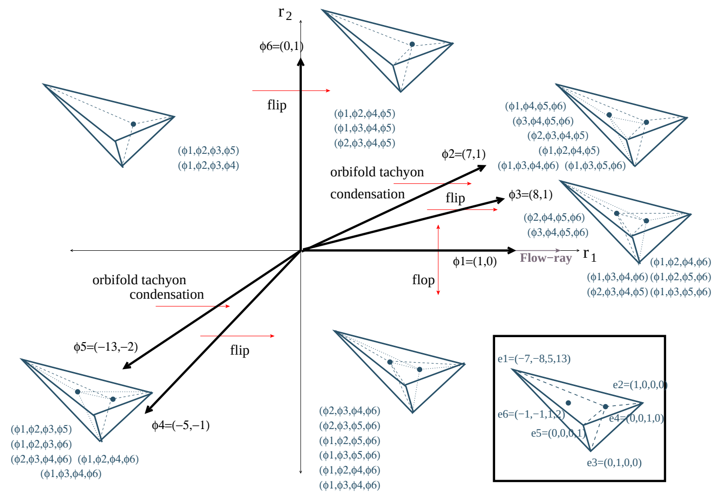
No obvious CFT description. Use GLSMs to glean decay structure.

Admit spacetime fermions if Type II GSO projection $\sum_i Q_i = even$ holds (using Type II GSO for orbifolds): obtained by studying phase structure and assuming GSO not broken along RG flow of decay.

Decay phase structure cascade-like, containing lower-order conifold-like singularities, as well as resolutions of orbifold singularities contained within.

Conifold-like singularities: phases

Phases of the $(1\ \ 7\ \ 8\ -5\ -13)$ conifold-like singularity.



Fields $\phi_{1,\ldots,6}$, charge matrix $Q_i^a = \begin{pmatrix} 1 & 7 & 8 & -5 & -13 & 0 \\ 0 & 1 & 1 & -1 & -2 & 1 \end{pmatrix}$.

Occasionally, the geometry decays to lower-order conifold-like singularity, here $Q = (\ 1\ \ 1\ \ 1\ \ -1\ \ -2\ )$ $[\sum_i Q_i = 0 \Rightarrow susy]$.

M2-branes, nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$

"Purely geometric" (probe brane) limit: loads of tachyons.

Now consider stacking M M2-branes at isolated (pointlike) $\mathbb{C}^4/\mathbb{Z}_N$ singularity: full M-theory background $\mathbb{R}^{2,1} \times \mathbb{C}^4/\mathbb{Z}_N$.
Large M: near horizon limit $\to$ nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$.
Group $\mathbb{Z}_N$ acts freely on $S^7 \Rightarrow$ resulting $S^7/\mathbb{Z}_N$ space smooth.
No fixed points on S^7 where localized tachyons can arise.
Large flux $AdS_4 \times S^7/\mathbb{Z}_N$ limit apparently tachyon free, potentially stable nonsupersymmetric background ?

Similar question for D3-branes at $\mathbb{C}^3/\mathbb{Z}_N$ and potentially stable nonsusy $AdS_5 \times S^5/\mathbb{Z}_N$ addressed by Horowitz,Orgera,Polchinski.
Nonperturbative gravitational instability: Kaluza-Klein "bubble of nothing" obtained by regarding $S^5/\mathbb{Z}_N$ as KK compactification on S^1.
Conformal theory $\Rightarrow$ decay rate scale free. Decays rapidly.

M2-branes, nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$

Likely similar story here in $AdS_4 \times S^7/\mathbb{Z}_N$:
rate $\Gamma \sim e^{-B} \int^{r_c} dr r^2 \sim e^{-B} \Lambda^3, \quad B \sim \frac{r_0^9}{G_{11}} \sim \frac{M^{3/2}}{N^9}$.
B gravitational instanton action, $r_0 \sim \frac{R}{N}$ is radial coordinate value where instanton capped off.

Kachru,Simic,Trivedi: for $AdS_5 \times S^5/\mathbb{Z}_N$, embed singularity in compact Calabi-Yau, interpret Λ as physical cutoff (UV is compact CY), construct realistic stable nonsusy throats.

Story here likely similar: singularity embedded in compact space (say orbifold of CY 4-fold). For fixed orbifold order N and large number M of M2-branes, instanton action large, giving small decay rate.
Possibly stable nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$ throats in M-theory.

M2-branes, nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$

Question: expect susy case stable, $i.e.$ no instanton. Within supergravity, difficult to see why $e.g.$ $\mathbb{Z}_N(1,3,5,7)$ (susy) is stable, while $\mathbb{Z}_N(1,5,7,9)$ (nonsusy) unstable, and $\mathbb{Z}_N(1,5,7,11)$ (susy) stable. Not obvious (in sugra) if nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$ always unstable to decay via KK-instanton.

Stable nonsusy $AdS_4 \times S^7/\mathbb{Z}_N$ throats in compact CY-orbifold interesting.
Dual field theory? Expect nonsusy Chern-Simons ABJM-like theory.

More generally: gauge theory moduli space is geometry (susy).
Nonsusy singularities?