EXERCISES

Problem Set, due 2020-02-04. Preliminary version; final on 2020-02-01

(1) Show that the matrix for the dual representation is given by the transpose of the inverse.

(2) Let V and W be $\Bbbk$-linear representations of G. For $g \in G$, describe the matrix of g for its action on $V \otimes_{\Bbbk} W$ in terms of those for its action on V and on W.

(3) Define the *exterior* (or the *wedge*) product $\bigwedge^\bullet(V)$ of V to be the quotient of $T^\bullet(V)$ by the two-sided ideal generated by $\{v \otimes w + w \otimes v \mid v, w \in V\}$ and $\{v \otimes v \mid v, w \in V\}$. Show that $\bigwedge^\bullet(V)$ is a finite-dimentional vector-space over $\Bbbk$ and that it inherits the graded G-action from $T^\bullet(V)$. Describe this action in terms of matrices.

(4) Let the cyclic group $C_2 = \langle \sigma \rangle$ of two elements act on $R = \mathbb{F}_2[x_1, y_1, x_2, y_2, x_3, y_3]$ by $\sigma(y_i) = y_i$ and $\sigma(x_i) = x_i + y_i$ for $i = 1, 2, 3$. Show that $\left(R^{C_2}\right)_2$ is generated by $y_i, i = 1, 2, 3$ and $x_i y_j + x_j y_i, 1 \leq i < j \leq 3$.

(5) Determine the symmetric polynomial of degree k in n variables, with the smallest leading term in degree-lexicographic order.

(6) Let V be a finite-dimentional representation of G. Let H be a normal subgroup of G. Then $V^G = (V^H)^{G/H}$.

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