

TUTORIAL 1 (2026-OCT-05)

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1. BASIC M2 COMMANDS

```
R = QQ[x,y,z];
I = ideal(x^2-y*z, x*y-z^2);
```

```
gens I
class I
class gens I
R/I
res I
beti res I
```

What does the semi-colon at the end do? What do the output lines mean? Get help from M2 itself using ?. E.g.

```
? class
```

Now try

```
L = {x,y,z}
L#0
M = matrix{{x,y},{y,z}}
```

Loop the following command for use later:

```
? gb
? kernel
? eliminate
? res
? betti
```

2. GRÖBNER BASES AND ELIMINATION

Here is the example in class.

```
Rlex = QQ[x,y, MonomialOrder => Lex];
I = ideal(x^2-y, x*y-1);
G = gb I;
gens G
(y^3-1) % I
eliminate(x,I)
Rgrev = QQ[x,y];
J = ideal(x^2-y, x*y-1);
gens gb J
```

(1) What is %? Check using

```
? %
? symbol %
```

Compiled at 2026-10-06 09:39:08+05:30.

- (2) Compare the two Grobner bases for the two monomial orders: Does the Gröbner basis change? Which order makes elimination visible?
- (3) Repeat the same for $I = (x^2 + y^2 - 1, x - y^2) \subseteq R := \mathbb{Q}[x, y]$.

3. LEARN THE M2 LANGUAGE FROM THE DOCUMENTATION

Look up the M2 documentation to solve the following problems.

- (1) In $R = \mathbb{Q}[x, y, z]$, define the list $L = \{x, y, z, x + y, x + z, y + z\}$. Write a `for` loop to print f^2 for every $f \in L$.
- (2) Use `apply` to construct the list $\{f^2 : f \in L\}$. How does this differ from `for`?
- (3) Write a function of the form
`isInIdeal = (f,I) -> ...`
 which returns `true` or `false` according to whether $f \in I$. Run this on several examples.
- (4) Write a function that takes an ideal I and a variable and returns the corresponding elimination ideal. Do this for a list of variables.
- (5) Two useful command to explore the M2 language are `methods` and `code`. Learn how to use them with
`? methods`
`? code`
`methods gb`
`code oo#0`
 What does output of the last command say?

4. RATIONAL NORMAL CURVES

For $d \geq 2$, consider the rational normal curve $C_d \subset \mathbf{P}^d$ given by $[s : t] \mapsto [s^d : s^{d-1}t : \dots : st^{d-1} : t^d]$.

- (1) Do this explicitly for $2 \leq d \leq 4$. $R = \mathbb{Q}[x_0, x_1, x_2, x_3, x_d]$, $A = \mathbb{Q}[s, t]$, $\phi : R \rightarrow A$ by $x_i \mapsto s^{4-i}t^i$. Compute $\ker \phi$ and observe whatever patterns you can.
- (2) Now do this for an arbitrary d and produce a list (for several values of d) in which each entry is itself the list $\{d, m_d, D_d\}$ where m_d is the number of generators of the ideal of C_d and D_d is the degree of the generators. (Hint: `d=4; R=QQ[x_0..x_d]` works.)
- (3) Let $S = \mathbb{Q}[s, t, x_0, x_1, x_2, x_3, x_d]$. Let J be the S -ideal generated by $\{x_i - s^{4-i}t^i \mid 0 \leq i \leq d\}$. Eliminate s and t from J . What does this signify?
- (4) Compare the generators with the 2×2 minors of

$$\begin{pmatrix} x_0 & x_1 & \cdots & x_{d-1} \\ x_1 & x_2 & \cdots & x_d \end{pmatrix}.$$