

1 Tic-Tac-Toe - Part 1

Today we worked on developing a strategy for tic-tac-toe, a very popular game among kids.

The goal was to let students experience the progression

play $\longrightarrow$ notice patterns $\longrightarrow$ formulate a strategy
 $\longrightarrow$ test the strategy $\longrightarrow$ ask whether it is guaranteed.

The important distinction at the end is between

“I have played many games and this seems to work”

and

“I can give an argument showing that this strategy cannot fail.”

The exploration proceeded as follows:

- Students formed pairs and played several games to find out if there is a preferred/recommended starting position (edge/corner/center).
- After playing many games, the students formulated a strategy:
 - If I can win at the next turn, I should play to win.
 - If my opponent can win next turn, I should block.
- The students observed that they need to make a distinction between *losing* and *not winning*.
- The students more-or-less agreed that if both sides play perfectly, then neither player can win or lose, and the game ends in a draw.
- There was a considerable argument among students about whether or not the above-mentioned strategy helps one win or even avoid losing. There were arguments on both sides, and a new strategy seemed to emerge - which will be taken up at the next discussion.
- The discussion ended by asking the students to think about whether the following two statements mean the same thing or not:
 - *“I have never lost when I use this strategy.”*
 - *“I can prove that this strategy means I will never lose.”*

2 Straight-edge and compass constructions - part 2

2.1 Constructing angles

In the last session, given lengths x and 1, you constructed an angle θ such that $\tan \theta = x$.

Which angles are constructible? In other words, given an angle θ , a line and a point A on that line, can you draw another line through A so that the angle between the lines is θ ?

The following angles are constructible: 180° , 90° , 45° , 60° , 30° . (This is not an exhaustive list.)

Using the identity: $\cos(36^\circ) = \frac{\sqrt{5} + 1}{4}$, we can see that $\sin^2(36^\circ)$, $\sin(36^\circ)$, 36° and 72° are constructible.

2.2 Regular polygons

Which regular polygons (i.e., polygons with all sides equal.) can be constructed entirely using a compass and a ruler (without using it to measure lengths)?

Equilateral triangles, squares, hexagon (not an exhaustive list).

Let us determine at what angle we should turn, if we need to construct a regular polygon with n sides. Draw a pentagon below in free hand. From the centre point, draw lines to each of the corners. Can you identify isosceles triangles? Can you determine the angles? Do this for a regular 7-sided polygon also and guess a formula.

To construct a pentagon, we need to construct 72° , which we have already done.

It takes much more work to show that $\frac{360^\circ}{7}$ is not constructible, so we *cannot* construct a regular 7-sided polygon.

But surprisingly, a regular 17-sided polygon can be constructed.

