

Aims of the session

- Introduce the students to **divisibility**, in particular, to prepare them for modular arithmetic in the following sessions.
- Familiarize them with **proof writing**: impress upon the students the importance of writing down the proof systematically, wrestling with the components of a proof and assimilating it slowly, till they develop a *feel* for it. In the end, the proof should mean something to them, rather than merely being a sequence of formal steps applied without introspection.

Mathematical concepts introduced

- When a, b are integers, $a \mid b$ is *shorthand* for ‘ a divides b ’, i.e. $b = ak$, for some integer k . This is not to be confused with the *value* of the fraction a/b , and that $k = b/a$, not a/b .
- Basic properties of divisibility:
 - Reflexive: $a \mid a$
 - Not symmetric: $a \mid b \not\Rightarrow b \mid a$
 - Anti-symmetric up to sign: $a \mid b, b \mid a \Rightarrow a = \pm b$
 - Transitive: $a \mid b, b \mid c \Rightarrow a \mid c$

$$a \mid b \Rightarrow a \mid bn \text{ for every integer } n \quad (1)$$

$$a \mid b, a \mid c \Rightarrow a \mid bc \quad (2)$$

- Converse of an implication. To illustrate this, we used

$$\text{It rained yesterday} \Rightarrow \text{It was cloudy yesterday}$$

whose converse (cloudy $\Rightarrow$ rained) is false. Here, the chronological order: first cloud forms then rain appears, is opposite to the logical order of implication. This demonstrated the necessity of careful verification when writing down implications, and their converses.

Principles of proof writing covered

- Starting by carefully noting down ‘What is given?’ and ‘To prove’.
- Verifying the proof and conclusions, using concrete examples.
- Noticing *redundant hypotheses*. For example, the hypothesis $a \mid c$ was observed to be not necessary for the conclusion of (2), as already evidenced by (1).
- Making sure the conclusion is as strong as possible. In the proof of (2), by making our observations sharper, we were able to upgrade our conclusion to

$$a \mid b, a \mid c \Rightarrow a^2 \mid bc \quad (3)$$

- Generalizing the result. Some students upon seeing (3), remarked that the result can be generalized to:

$$a \mid b_1, a \mid b_2, \dots, a \mid b_n \Rightarrow a^n \mid b_1 b_2 \dots b_n$$