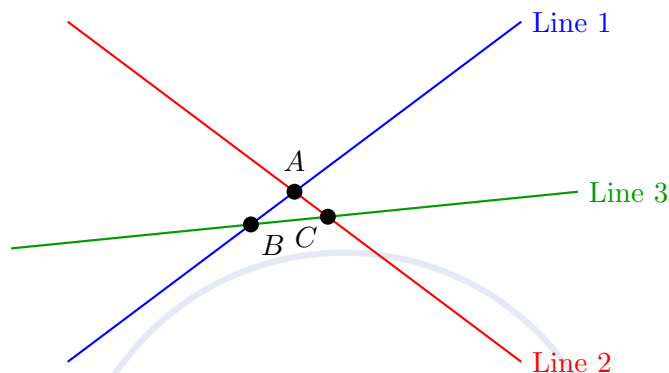


1 Concurrency

In this session we started with a discussion of why concurrency of lines is interesting. While it is always true that any two non-parallel lines in a plane will intersect in a point, the same is not necessarily true of 3 (mutually non-parallel) lines in a plane. For example, the 3 lines could meet in 3 distinct points as shown:



So when we talk about 3 or more lines meeting at one point in the plane, that's a special occurrence.

So it is indeed fascinating that in any given triangle,

- the medians are concurrent!
- the angle bisectors are concurrent!!
- the altitudes are concurrent!!!
- the perpendicular bisectors are concurrent!!!

We began discussing ideas to prove that medians are concurrent. We will pick up this topic again in a later math circle session.

2 Straight-edge and compass constructions - part 1

Ruler and compass constructions are classical, and explore what can be done 'purely geometrically'. We construct straight lines and circles using only the following:

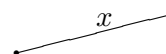
1. You may draw a straight line through two known points.
2. You may draw a circle with a known point as centre and a known distance as radius.
3. You may use intersection points of lines and circles to create new points.
4. No measuring with the ruler to transfer a length directly; the compass does that.

E.g., if a line segment AB is given, then by drawing circles with centres at A and at B and same radius AB , we can get a perpendicular bisector of AB .

3 Numbers using geometric constructions

Below is a number line, with 0 and 1 marked (so the unit length is marked). We say that a number a is **constructible**, if we can mark that distance from 0 on the number line.

E.g., a distance x is also marked separately. Using this, we can construct



Thus we see that if x constructible, so are $1 + x$, $2x$ and $\frac{1+x}{2}$, $\frac{x}{2}$ are constructible.

Similarly, if x and y are constructible, then $x + y$ and $x - y$ are constructible. If x and $y \neq 0$ are constructible, an angle θ with $\tan \theta = \frac{x}{y}$ can be constructed. Using similarity of triangles, we can construct xy and $\frac{x}{y}$ (if $y \neq 0$) and $\sqrt{x}$.

Therefore the set of constructible numbers contains all the integers, and all the numbers obtainable from the integers using the arithmetic operations $+$, $-$, $\times$, $\div$ and taking square-roots.

