

Junior Batch

This Math Circle session was divided into two distinct themes, separated by the break. During the first half, students worked on ruler-and-compass constructions.

Ruler-and-compass constructions

Ruler-and-compass constructions are classical geometric problems, asking what can be constructed purely with a ruler without markings (also called a straight-edge) and compass. The students started with understanding the ‘rules of the game’:

1. Given two points, we can draw a straight line through them. (Uses the ruler.)
2. Given a point and a distance, we can draw a circle with the given point as the centre and the given distance as the radius. (Uses the compass.)
3. Using the lines and the circles already drawn, we can mark their intersection points.
4. We should only transfer distances using the compass; the markings on the ruler should not be used for this.

Using these rules, students tried to make exact copies of line segments and of circles. From there, they proceeded to construct perpendicular bisectors of line segments.

One important goal of these constructions is to determine which numbers can be constructed, when starting with a given distance as the unit distance. The students constructed the number line, marked a few positive and negative integers, and multiples of $\frac{1}{2}$.

Multiplication Principle

After the break, we revisited the multiplication principle from the previous session. We recalled that this principle is useful when a task consists of several steps that must all be completed. For example, if Step 1 can be performed in m ways and Step 2 can be performed in n ways, then completing both steps can be done in $m \times n$ ways.

We then introduced a variation of the multiplication principle involving “shrinking choices.” In the examples from the previous session, making a choice for one position did not affect the choices available for the next position. This time, we considered situations where items are selected from the same group without repetition.

For example, suppose a class of 15 students is conducting an election for President and Vice-President. There are 15 choices for President. Once the President is selected, that student cannot also be Vice-President, leaving only 14 choices for the second position. Thus, the total number of possible outcomes is

$$15 \times 14 = 210.$$

Through several examples, students recognized that whenever an item is used and cannot be selected again, the number of available choices decreases.

Next, we considered the question: “*What if we want to arrange all the items in a group?*” For instance, if 5 friends need to sit in 5 chairs in a cinema hall, the shrinking-choices principle gives

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

possible arrangements.

We then introduced factorial notation, which provides a convenient way to represent such products without writing out every factor. The factorial of a positive integer n , written as $n!$, is the product of all positive integers from n down to 1. Thus,

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

We calculated several factorials together on the board to illustrate how quickly these numbers grow:

$$3! = 6, \quad 4! = 24, \quad 5! = 120, \quad 6! = 720, \quad 7! = 5040.$$

We also observed that

$$6! = 6 \times 5!,$$

which provides a quick way to compute successive factorials.

Finally, we considered situations where we do not need to arrange all the items. If there are n total items but we only want to fill k positions, we write this as $P(n, k)$, read as “the permutation of n items taken k at a time.”

For example, if a class has 8 students and there are three only prizes and we only need to fill three positions. Applying the shrinking-choices principle gives

$$P(8, 3) = \underbrace{8 \times 7 \times 6}_{3 \text{ slots}} = 336$$

possible outcomes.

