CMI Math Circle

Synopsis for August 23, 2026

1 Geometry

The session began with a discussion of congruence of triangles. While some students agreed on the definition of congruent triangles, some students had a difficulty in making the notion precise. In particular, the idea that congruent triangles 'overlap' was being considered as a rigorous definition by some. However, even if this is a good aid for visualisation and understanding, it does not constitute a rigorous argument, and is not helpful in proving results or solving exercises.

So we began by rigorously defining congruence of triangles and proceeded to the discussion of what properties of triangles are preserved by congruency. Some answers that came up were: corresponding sides, corresponding angles, ratios of side lengths (incorrect in general) and areas of triangles.

There was some confusion about whether congruence does or does not imply equality of areas, so we proceeded to prove it. Students had some difficulty grasping the flow of logic, so this took some time. Students came up with two ways of proving this, one with Heron's formula for area, and another with the formula $\frac{1}{2} \times$ base $\times$ height , which requires one to prove that the corresponding heights of congruent triangles are equal.

Then we had a short discussion on why congruency tests work. In particular, why is it sufficient to prove the equality of certain sets of 3 corresponding entities, when congruency actually requires the equality of 6 corresponding entities. This questioning of the congruency tests was totally new and unexpected for some students, so we asked them to think about it over the week. However, we could list the tests that work, and the ones that don't (SSA ans AAA). Students could also come up with counterexamples for the tests that don't work.

With the promise to return to this discussion next week, we moved on to solving two simple problems involving congruency of triangles.

2 Magic squares

We had an exploration worksheet on 3×3 magic squares which explored the following ideas:

The students are asked to arrange

$$1, 2, \ldots, 9$$

in a 3×3 square so that every row, column and main diagonal has the same sum.

A standard solution is

8	1	6
3	5	7
4	9	2

with magic sum

$$15.$$

There are, of course, many arrangements obtained by rotations and reflections.

We explored the following questions:

1. What must the magic sum be?
2. What must be in the centre?
3. What should the opposite pairs of numbers be?
4. How much freedom is there in filling up the numbers?