Junior Batch

We started the session by counting the number of people in the class, the number of chairs and the number of even and odd numbers from 1 to 100.

We asked a fundamental question: **"What is counting?"** Students came up with various ideas, discussing exact numbers versus approximate counts. We discussed *how* we count (using our hands, numbering items) and how we can verify if our count is correct to avoid **over-counting** or **under-counting**. Eventually, the class agreed that to verify a count, we can make a list and compare it with someone else's list to ensure no items are missed.

Next, we asked the students:

- How many multiples of 3 are there from 30 to 300?
- How many multiples of 5 are there from 30 to 300?
- How many multiples of 7 are there from 30 to 300?

Students quickly realized that listing every single number is too difficult and takes too long.

We discussed whether counting and listing are the same thing. If we only need the final count, do we really need to list everything? No! We can count without listing by finding a pattern or inventing a formula.

For example, since even and odd numbers alternate, the number of even numbers from 1 to 100 is simply $100/2 = 50$. When solving the problem for multiples of 3, students tried different approaches:

- Some tried listing all the numbers manually.
- Some tried guessing a formula and gave approximate answers (which we wanted to avoid).
- A couple of students computed it logically: The number of multiples of 3 from 1 to 300 is 100. The number of multiples from 1 to 29 is 9. So, the total is $100 - 9 = 91$.
- Another group of students noticed that $30 = 3\times10$, $33 = 3\times11$, all the way up to $300 = 3\times100$.

This observation led to the concept of **Smart Listing**. Instead of writing huge numbers, we can write them as a multiple of a list of consecutive numbers!

$$\begin{aligned}
&\text{Multiples of 3 from 30 to 300:} \quad 3 \times \boxed{10, 11, 12, \ldots, 100} \\
&\text{Multiples of 5 from 30 to 300:} \quad 5 \times \boxed{6, 7, 8, \ldots, 60} \\
&\text{Multiples of 7 from 30 to 300:} \quad 7 \times \boxed{5, 6, 7, \ldots, 42}
\end{aligned}$$

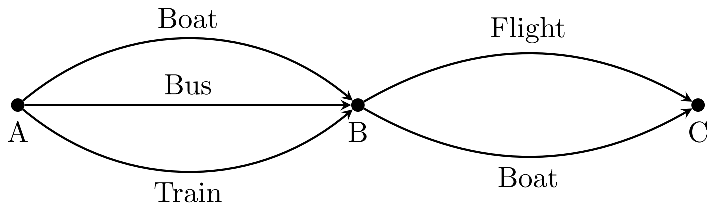
However, many students had trouble counting the exact number of items in a consecutive list like $\{a, a+1, a+2, \ldots, b\}$. Is it $b - a$? Is it $b - a + 1$? Or $b - a + 2$? After a few attempts testing it themselves, the students successfully figured out the exact formula:

$$\boxed{\text{Number of items in } \{a, a+1, a+2, \ldots, b\} = b - a + 1} \tag{1}$$

Students then applied Equation (1) to find the number of numbers that are multiples of *both* 3 and 5 (multiples of 15), multiples of 3 and 7 (multiples of 21), and multiples of 5 and 7 (multiples of 35) from 30 to 300. They created the smart lists and easily found the exact counts using the reference formula.

Arithmetic in Counting: AND vs OR

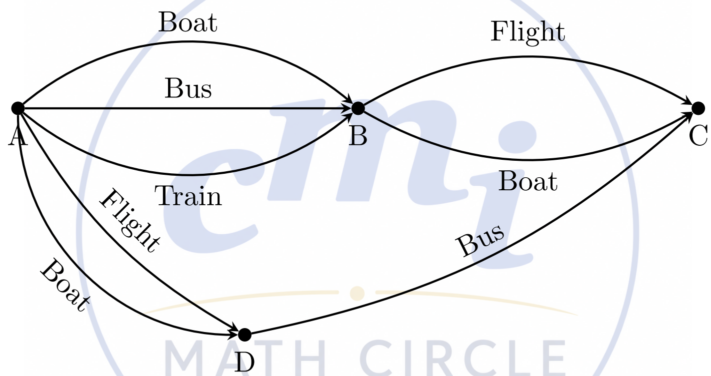
Next, we introduced combinatorics (arithmetic in counting). We looked at the following map: *There are three towns: Town A, Town B, and Town C. Three roads connect A and B, and two roads connect B and C.*



Question: How many different routes can one take from Town A to Town C? (Assume you can only travel forward, from left to right).

As students listed out the routes, they discovered that they had to make 2 independent choices: Choice 1 is from A to B, and Choice 2 is from B to C. In total, they have $3 \times 2 = 6$ routes.

Now, we added a new town, Town D:



Question: How many different routes can one take from Town A to Town C now?

Students quickly calculated the answer to be 8. They saw there were 6 routes via Town B, and 2 new routes via Town D. Because you can travel through Town B *OR* Town D, you add the results ($6 + 2 = 8$).

This led us to establish two golden rules of counting:

- **The Multiplication Principle (AND):** If you are making several independent choices in a row (e.g., picking a path from A to B **AND** then B to C), you find the total number of options by **multiplying** your choices.
- **The Addition Principle (OR):** If you have to choose exactly ONE outcome between entirely separate options (e.g., traveling via Town B **OR** via Town D), you find the total number of options by **adding** them together.

Next, we tested these rules using a relatable shopping scenario:
Scenario 1: Shop A sells only 1 colour of shirt and 1 colour of pants. Each item costs Rs. 500.

- Suppose you have exactly Rs. 500 and want to buy exactly one shirt **OR** one pair of pants. How many choices do you have? *(Answer: 2 choices)*.
- Suppose you have exactly Rs. 1000 and want to buy exactly one shirt **AND** one pair of pants. How many choices do you have? *(Answer: 1 choice)*.

Scenario 2: Shop A upgrades its inventory! It now sells 3 colours of shirts (Red, Blue, White) and 2 colours of pants (Black, White). Everything still costs Rs. 500.

- You have Rs. 1000 and want to buy one shirt **AND** one pair of pants. How many choices do you have? *(Answer:* $3 \times 2 = 6$ *choices).*
- You have Rs. 500 and want to buy one shirt **OR** one pair of pants. How many choices do you have? *(Answer:* $3 + 2 = 5$ *choices).*

Scenario 3: A new shop, Shop B, opens next door. Shop B sells 2 colours of shirts (Pink, Yellow) and 1 colour of pants (Blue) for Rs. 500 each.

- You have Rs. 1000 and want to buy a shirt and pant pair. If you can only buy your complete pair from **either Shop A OR Shop B**, how many choices do you have? *(Answer: 6 choices from A + 2 choices from B = 8 total choices).*
- You have Rs. 500 and want to buy one shirt **OR** one pair of pants from anywhere. How many choices do you have? *(Answer: 5 choices at A + 3 choices at B = 8 total choices).*

The students compared these shopping problems with the town/route problems and discussed the logical similarities between them. Both rely entirely on the exact same Addition and Multiplication principles!

Students were given more problems involving these principles to practice. Some were able to finish the work in class, while others took the remaining problems as a take-home challenge. *(Please refer to the worksheet for August 23, 2026, Junior batch, for the specific problems).*