CMI Math Circle

Synopsis for August 16 2026

Junior Batch

We started the session by briefly recalling the definition of a graph (nodes and edges) and concepts from our graph colouring session last week. We moved to a hands-on activity: **The "Do Not Lift Your Pen" Challenge**.

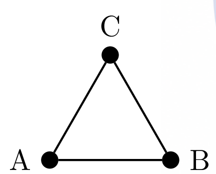
Students were asked to trace various geometric shapes using three simple rules:

- They could start at any dot.
- They could not lift their pen from the paper.
- They could not draw over the same line more than once.

Based on where their pen stopped, students categorized the traceable shapes into two mathematical types:

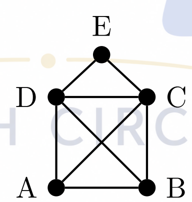
- **Circuit:** A path that traces every line exactly once and finishes **exactly at the dot where it started**.
- **Euler Path:** A path that traces every line exactly once but finishes at a **different** dot from where it started.

Circuit



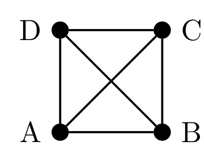
(Starts and ends at the same dot. Starting point does not matter.)

Euler Path



(Starts and ends at different dots. Starting point matters: must start at C or E.)

Impossible



(Cannot be drawn in one stroke. Starting point does not matter.)

For larger graphs, guessing with a pencil quickly became frustrating.

We introduced the concept of a **Degree**—the total number of lines connected to a single dot. Dots were classified as either **Even** (2, 4, 6 lines) or **Odd** (1, 3, 5 lines).

By counting the number of odd-degree vertices on the shapes they had just traced, the students independently discovered **Euler's Theorem**:

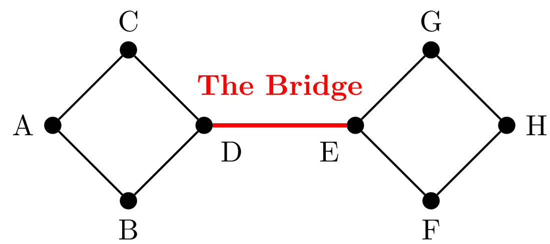
- **0 vertices of odd degree:** The graph contains an **Euler Circuit**.
- **Exactly 2 vertices of odd degree:** The graph contains an **Euler Path**.
- **More than 2 vertices of odd degree:** It is **mathematically impossible** to trace the graph in one continuous stroke.

The logic behind this is simple: every time a pen passes through a dot, it needs one line to enter and one line to exit (an even number). The only dots allowed to have an odd degree are the starting point (needs a line to leave) and the finishing point (needs a line to enter).

Fleury's Algorithm

Knowing a Euler path exists is different from actually finding it. For massive graphs (like computer networks or city grids), the students learned a systematic method called **Fleury's Algorithm**.

The motto of this algorithm is a famous proverb: **"Don't burn your bridges behind you!"** In Graph Theory, a **bridge** is an edge that, if deleted, breaks the graph into two disconnected pieces.



When tracing a graph, the algorithm tells us to always choose a normal line and *only* cross a bridge if there is absolutely no other choice. Crossing a bridge too early traps you on one side of the graph, making it impossible to finish tracing the rest.

We concluded the session with few room maze puzzles: "Can you trace a path through this house plan going through every door once and only once?", which was borrowed from Maths Week Ireland.