Functional Programming in Haskell

Part 2 : Abstract dataypes and “infinite” structures

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Haskell review

- Program $\equiv$ Collection of function definitions

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- Computation $\equiv$ Rewriting using definitions

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isDigit '0' = True
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isDigit '9' = True
isDigit c = False
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```

- `isDigit c`

```
      | (c >= '0' && c <= '9')  =  True
      | otherwise               =  False
```

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sum []     =  0
sum (x:l)  =  x + (sum l)
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- (Conditional) polymorphism

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- Example Adding up a list of integers

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sum []      =  0
sum (x:l)  =  x + (sum l)
```

- (Conditional) polymorphism

  Most general type of `sum` is

  ```
  sum ::  (Num a) => [a] -> a
  ```

  where `Num a` is true for any type `a` that supports basic arithmetic operations `+`, `-`, . . .

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- Using "infinite" data structures

User defined datatypes

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- What if we write instead

```
stopwhen2 ::  Signal -> Bool
stopwhen2 c |  (c == Red)  =  True
            |  otherwise   =  False
```

User defined types and type classes

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- How about

```
nextlight ::  Signal -> Signal
nextlight Green   =  Yellow
nextlight Yellow  =  Red
nextlight Red     =  Green
```

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- Displaying result of `nextlight` requires `Show Signal`
- `Show a` is true of type `a` if there is a function

```
show ::  a -> String
```

  that allows values of `a` to be displayed

Adding user defined types to type classes

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data Signal = Red | Yellow | Green
  deriving (Eq, Show, Ord)
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  - `Red` $<$ `Yellow` $<$ `Green`

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instance Ord Signal where
  Green <= Yellow  =  True
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  x <= y           =  False
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Note: `<=` need not even be a consistent ordering!

Recursive datatypes

- A binary tree to store integers at each node

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              Nil |
              Node Int Btreeint Btreeint
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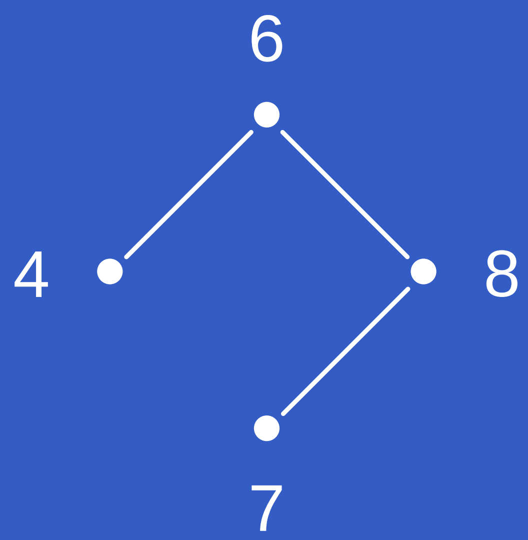
  A constant, like `Red`, `Green`, . . .
- The constructor `Node` has three arguments
  - First component `Int`: value stored at the node
  - Other two components `Btreeint`: left and right subtrees

Example

The tree



would be written as

```
Node 6 (Node 4 Nil Nil)
       (Node 8 (Node 7 Nil Nil) Nil)
```

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```
listout ::  Btreeint -> [Int]
listout Nil             = []
listout (Node n t1 t2)  =
  [n] ++ listout t1 ++ listout t2
```

Polymorphic recursive datatypes

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data Btree a =
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- Conditional polymorphism!

```
(Ord a) => data Btree a =
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              Node a (Btree a) (Btree a)
```

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- Built in list type is a polymorphic recursive datatype

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                Append a (Mylist a)
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- Since lists are built in, they can use special symbols `[]` and `:` for constructors `Emptylist` and `Append`

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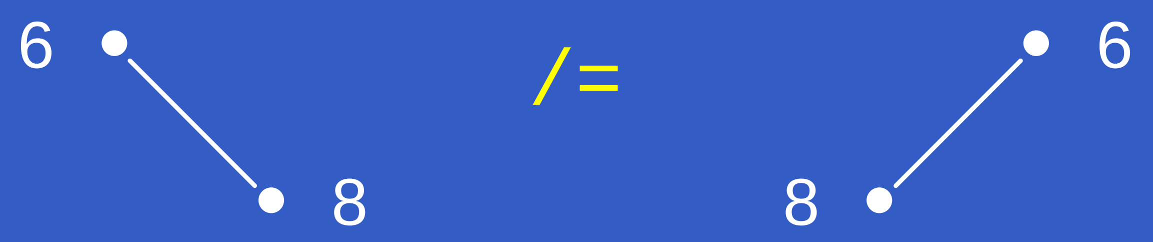

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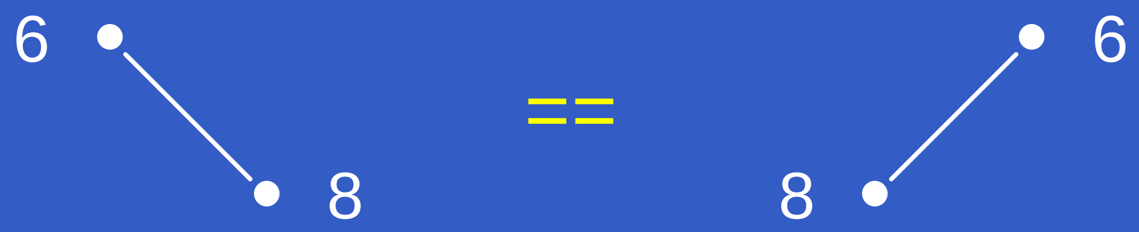

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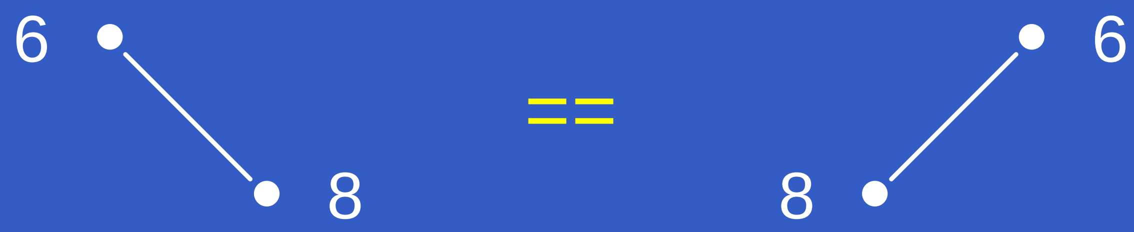

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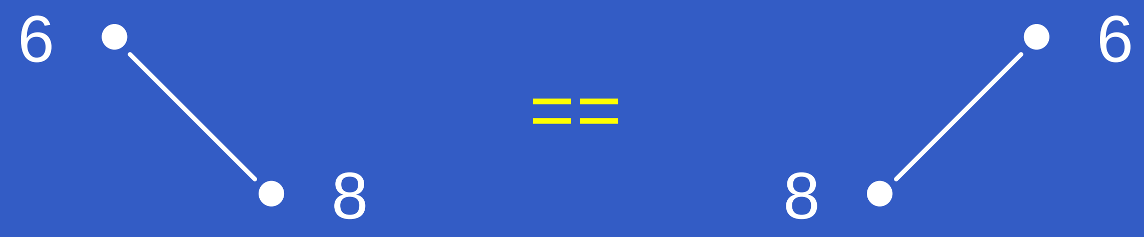

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- Rotate right — transformation used to balance trees

Declarative programming with abstract datatypes

- Rotate right — transformation used to balance trees

- `rotateright (Node x (Node y t1 t2) t3) =`
  `Node y t1 (Node x t2 t3)`

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- Stores sequence of values in FIFO fashion
- Append items at the tail of queue
- Want a datatype `Queue a` with functions

```
addq :: (Queue a) -> a -> (Queue a)
removeq :: (Queue a) -> (a,Queue a)
isemptyq :: (Queue a) -> Bool
emptyqueue :: (Queue a)
```

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  addq (Qu l) d = Qu (d:l)

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  removeq (Qu l)) = (last l,Qu (init l))

  isemptyq ::  (Queue a) -> Bool
  isemptyq (Qu []) = True
  isemptyq q = False

  emptyqueue ::  (Queue a)
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  ```

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- Use these definitions in another file
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- How do we prevent unauthorized access to a queue?

```
remsec ::  (Queue a) -> (a,Queue a)
remsec (Qu (x:y:l)) = (y, Qu (x:l))
```

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  Evaluate argument to a function only when needed.

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  Evaluate argument to a function only when needed.
- “Eager” rewriting — evaluate arguments before evaluating function

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```
power ::  Float -> Int -> Float

power x n = if (n == 0) then 1.0
                 else x * (power x (n-1))
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Lazy rewriting

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- `power (8.0/0.0) 0` $\leadsto$ `1.0`

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⇝ 2:(3:(from 4))
⇝ 2:(3:(4:(from 5)))
...
```

- Limit is the infinite list `[2,3,4,5,...]`
- Haskell can (and will) generate it incrementally, till you stop it, or it runs out of memory
- Can write `[2..]` to denote `[2,3,4,...]`

Why infinite lists?

- Can sometimes simplify a problem

Why infinite lists?

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- Consider the problem of computing the n^{th} prime number

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  - Repeat second step forever!

The Sieve of Eratosthenes

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  where sieve (x:l) =
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⇝2:3:sieve [z | z <- [y <- [4..],
                  mod y 2 > 0], mod z 3 > 0]
...
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...
⤳2:3:sieve [z | z <- [5,7,9...],
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⇝2:3:sieve [z | z <- [y <- [4..],
                 mod y 2 > 0], mod z 3 > 0]
...
⇝2:3:sieve [z | z <- [5,7,9...],
                           mod z 3 > 0]...
⇝2:3:sieve [5,7,11...] ⇝...
```

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- Once “enough” has been computed, the rest of `primes` is ignored!

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- Functional programming provides a framework for declarative programming
  - Provably correct programs
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- Haskell has a powerful typing mechanism
  - Conditional polymorphism
  - Modules with hiding and overriding for abstract datatypes
- Lazy evaluation permits infinite data structures

For more information

Software and other resources

- `http://www.haskell.org`

Quick tutorial

- *A Gentle Introduction to Haskell*
  by Paul Hudak et al

Textbooks

- *The Craft of Functional Programming*
  by Simon Thompson
- *Introduction to Functional Programming in Haskell*
  by Richard Bird