Functional Programming in Haskell

Part I : Basics

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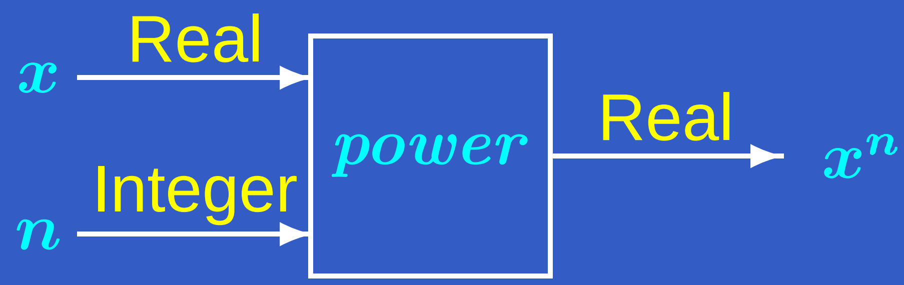
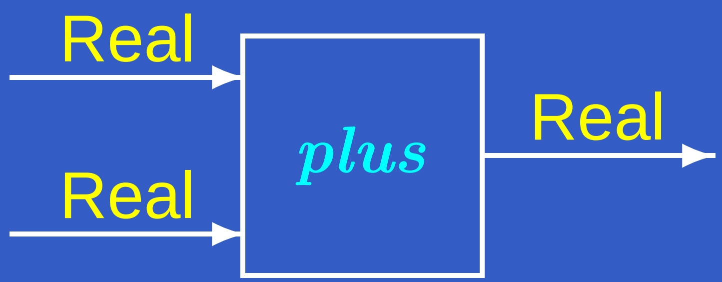
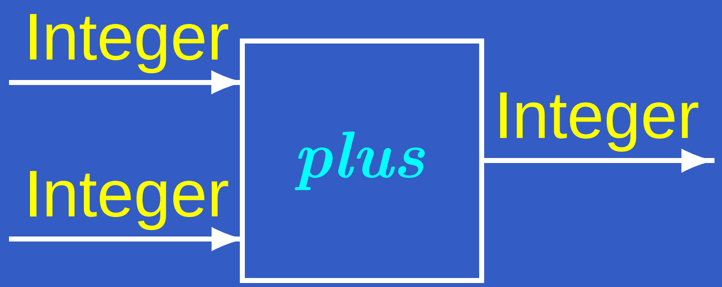
Functions

- Transform inputs to output
- Operate on specific types



- These functions are different





Functional programming

- Program $\Leftrightarrow$ set of function definitions
- Function definition $\Leftrightarrow$ how to “calculate” the value
- Declarative programming
  - Provably correct programs

    Functional program closely follows mathematical definition
  - Rapid prototyping

    Easy to go from specification (what we require) to implementation (working program)

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  with basic operations `+`, `-`, `*`, `/`, ||, &&, `...`

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- Need not be a “total” function

  What is `power 2.0 -1`?

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power ::  Float -> Int -> Float
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```

- Implicit “pattern matching” of arguments

```
xor ::  Bool -> Bool -> Bool
xor True True     =  False
xor False False   =  False
xor x y           =  True
```

Ways of defining functions . . .

- Multiple options with conditional guards

```
max ::  Int -> Int -> Int

max i j | (i >= j) = i
        | (i < j)  = j
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- Default conditional value — `otherwise`

```
max3 :: Int -> Int -> Int -> Int

max3 i j k | (i >= j) && (i >= k) = i
           | (j >= k)             = j
           | otherwise            = k
```

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max3 :: Int -> Int -> Int -> Int

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- Note: Conditional guards are evaluated top to bottom!

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  (Float,Float,Float) -> Float
```

- Functions can return n-tuples

```
maxAndMinOf3 ::
  Int -> Int -> Int -> (Int,Int)
```

Local definitions using `where`

Example: Compute distance between two points in 2D

```
distance ::
  (Float,Float)->(Float,Float)->Float

distance (x1,y1) (x2,y2) =
  sqrt((sqr xdistance) + (sqr ydistance))
  where

    xdistance = x2 - x1
    ydistance = y2 - y1


    sqr ::  Float -> Float
    sqr z      = z*z
```

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~> 9.0 * 1.0
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Functions that manipulate functions

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twice ::  (Int -> Int) -> Int -> Int
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  ```
- `twice sqr 7` $\leadsto$ `sqr (sqr 7)` $\leadsto$ `sqr (7*7)` $\cdots$ $\leadsto$ `49*49` $\leadsto$ `2401`

Running Haskell programs

- hugs — A Haskell interpreter
  Available for Linux, Windows, . . .
- ghc — the Glasgow Haskell Compiler
- Look at `http://www.haskell.org`

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- Lists are written as follows:
  `[2,3,1,7]`
  `[(3.0,7.5),(7.6,9.2),(3.3,7.868)]`
- Empty list is denoted `[]` (for all types)

Basic operations on lists

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head [1,3,5,7] = 1
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- Write `x:l` to denote the list with head `x`, tail `l`

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- ```
  reverse ::  [Int] -> [Int]
  reverse []      = []
  reverse (x:l)   = (reverse l) ++ [x]
  ```

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    Example `[3..7] = [3,4,5,6,7]`

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  - Arithmetic progressions

    `[1,3..8] = [1,3,5,7]`

    `[9,8..5] = [9,8,7,6,5]`

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square ::  Int -> Int
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even ::  Int -> Bool
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```
map square (filter even [1..10]) ⇝
                         [4,16,36,64,100]
```

List comprehension: New lists from old

- The set of squares of the even numbers between 1 and 10

$$\{x^2 \mid x \in \{1, \ldots, 10\}, even(x)\}$$

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```
[ square x | x <- [1..10], even x ]
     where even x = (mod x 2 == 0)
           square x = x*x
```

Using list comprehensions ...

- ```
  divisors ::  Int -> [Int]
  divisors n = [ m | m <- [1..n],
                     mod n m == 0 ]
  ```
  ```

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- ```
  divisors :: Int -> [Int]
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  ```

- ```
  prime :: Int -> Bool
  prime n = (divisors n == [1,n])
  ```

Example: Quicksort

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```
qsort [] = []
qsort l =
 qsort lower ++ [splitter] ++ qsort upper
 where
  splitter = head l
  lower = [i | i <- tail l, i <= splitter]
  upper = [i | i <- tail l, i > splitter]
```

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- Similarly, the most general type of `reverse` is `reverse :: [a] -> [a]`

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- What about subclass polymorphism in OO programming?

Polymorphism versus overloading in OO

```
class Shape {
}
class Circle extends Shape {
  double size {return pi*radius*radius}
}
class Square extends Shape {
  double size {return side*side}
}
Shape s1 = new Circle; print s1.size();
Shape s2 = new Square; print s2.size();
```

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- ```
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  Shape s1 = new Circle; print s1.size();
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  ```
- Implementation of `size` is different!!
  ```

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  Equality of functions cannot be checked effectively
- The underlying type should support equality

Type classes

- Haskell organizes types into classes. A type class is a subset of all types.
- The class `Eq` contains all types that support `==` on their elements.

  The "predicate" `Eq a` tells whether or not `a` belongs to `Eq`
- Haskell would type this as

```
member ::  Eq a => a -> [a] -> Bool
```

- Likewise `Ord a` is the set of types that support comparison, so

```
quickSort:  Ord a => [a] -> [a]
```

Examples of declarative programming

Compute all initial segments of a list

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Compute all initial segments of a list

- Initial segments of [ ] are empty

Compute all initial segments of a list

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- Initial segments of `x:l` — all initial segments of `l` with x in front, plus the empty segment

Examples of declarative programming

Compute all initial segments of a list

- Initial segments of `[]` are empty
- Initial segments of `x:l` — all initial segments of `l` with x in front, plus the empty segment
- ```
  initial ::  [a] -> [[a]]
  initial [] = [[]]
  initial (x:l) = [[]] ++
                      [ x:z | z <- initial l]
  ```
  ```

Examples of declarative programming . . .

- The empty list [ ] has no permutations

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- Permutations of `x:l`
  "Interleave" `x` through each permutation of `l`

Examples of declarative programming . . .

- The empty list `[]` has no permutations
- Permutations of `x:l`
  "Interleave" `x` through each permutation of `l`
- ```
  interleave ::  a -> [a] -> [[a]]
  interleave x [] = [[x]]
  interleave x (y:l) =
   [x:y:l] ++
   [y:l2 | l2 <- (interleave x l)]

  perms ::  [a] -> [[a]]
  perms [] = [[]]
  perms (x:l) =
    [z | y <- perms l, z <- interleave x y]
  ```
  ```

Second lecture

- User defined datatypes

  Stacks, queues, trees, . . .
- Hiding implementation details using modules
- “Infinite” data structures