NPTEL MOOC

PROGRAMMING, DATA STRUCTURES AND ALGORITHMS IN PYTHON

Week 4, Lecture 1

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$O(n^2)$ sorting algorithms

- Selection sort and insertion sort are both $O(n^2)$
- $O(n^2)$ sorting is infeasible for n over 5000

A different strategy?

- Divide array in two equal parts
- Separately sort left and right half
- Combine the two sorted halves to get the full array sorted

Combining sorted lists

- Given two sorted lists `A` and `B`, combine into a sorted list `C`
  - Compare first element of `A` and `B`
  - Move it into `C`
  - Repeat until all elements in `A` and `B` are over
- Merging `A` and `B`

Merging two sorted lists

32 74 89

21 55 64

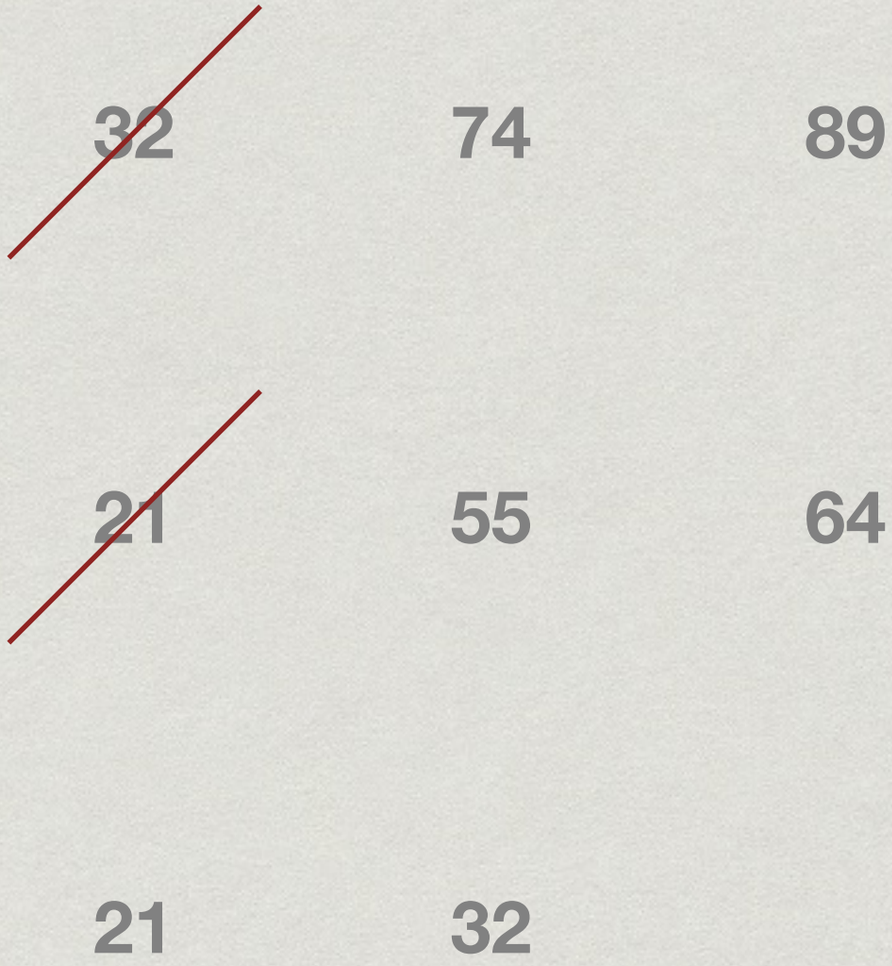
Merging two sorted lists

32 74 89

21 55 64

21

Merging two sorted lists



Merging two sorted lists

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21 32 55 64 74 89

Merge Sort

- Sort `A[0:n//2]`
- Sort `A[n//2:n]`
- Merge sorted halves into `B[0:n]`
- How do we sort the halves?
  - Recursively, using the same strategy!

Merge Sort

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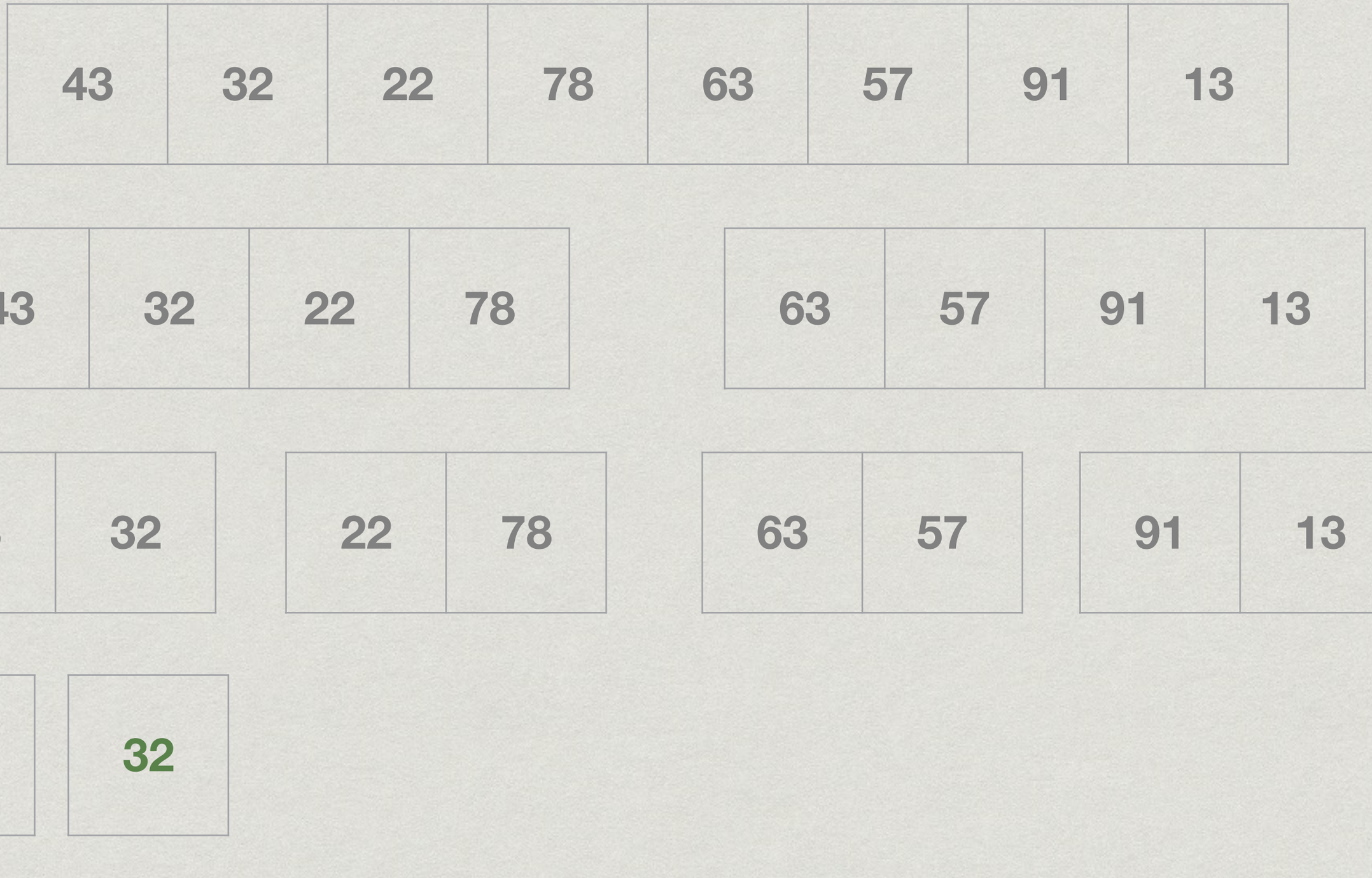
Merge Sort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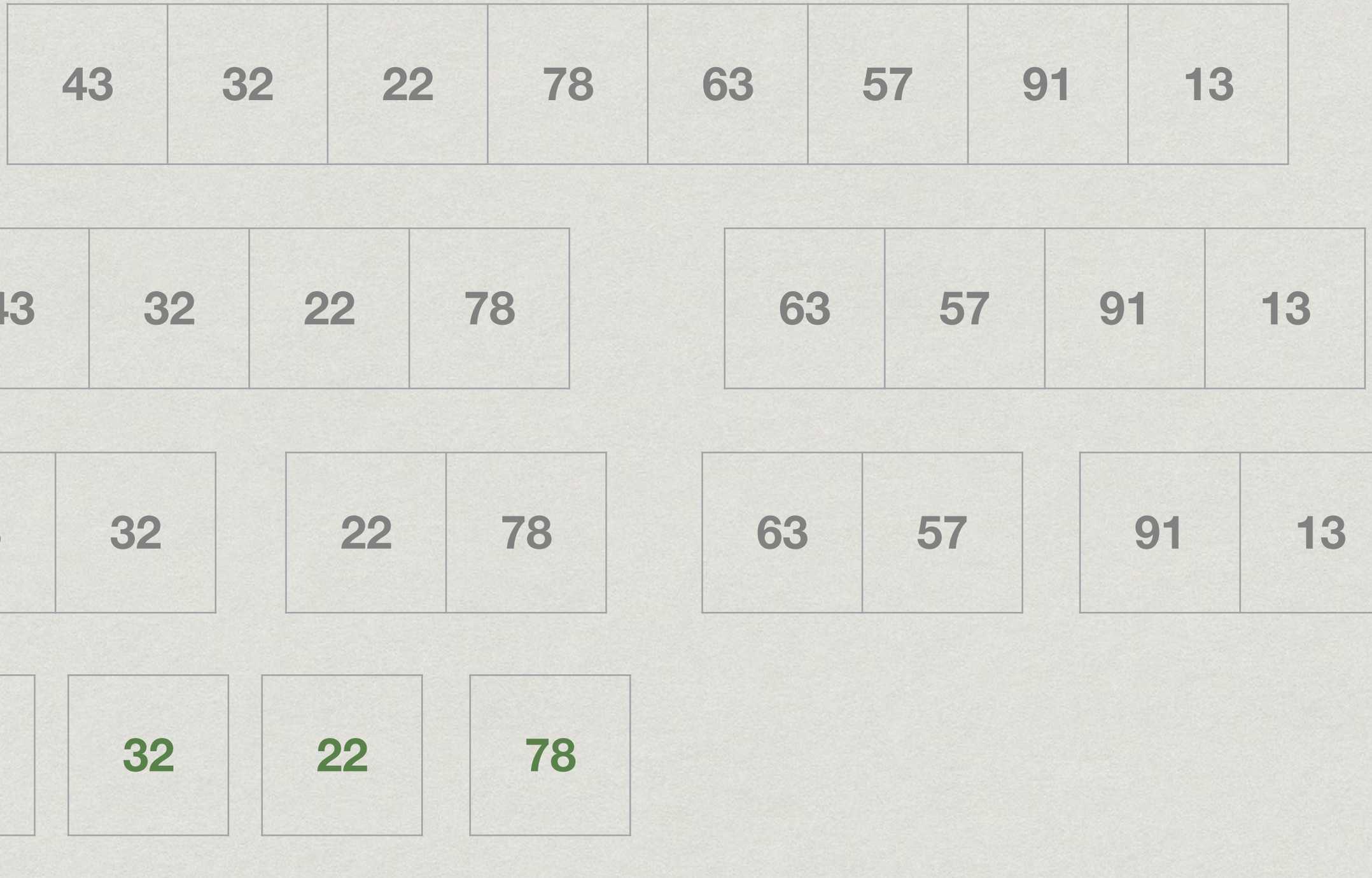
Merge Sort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Merge Sort

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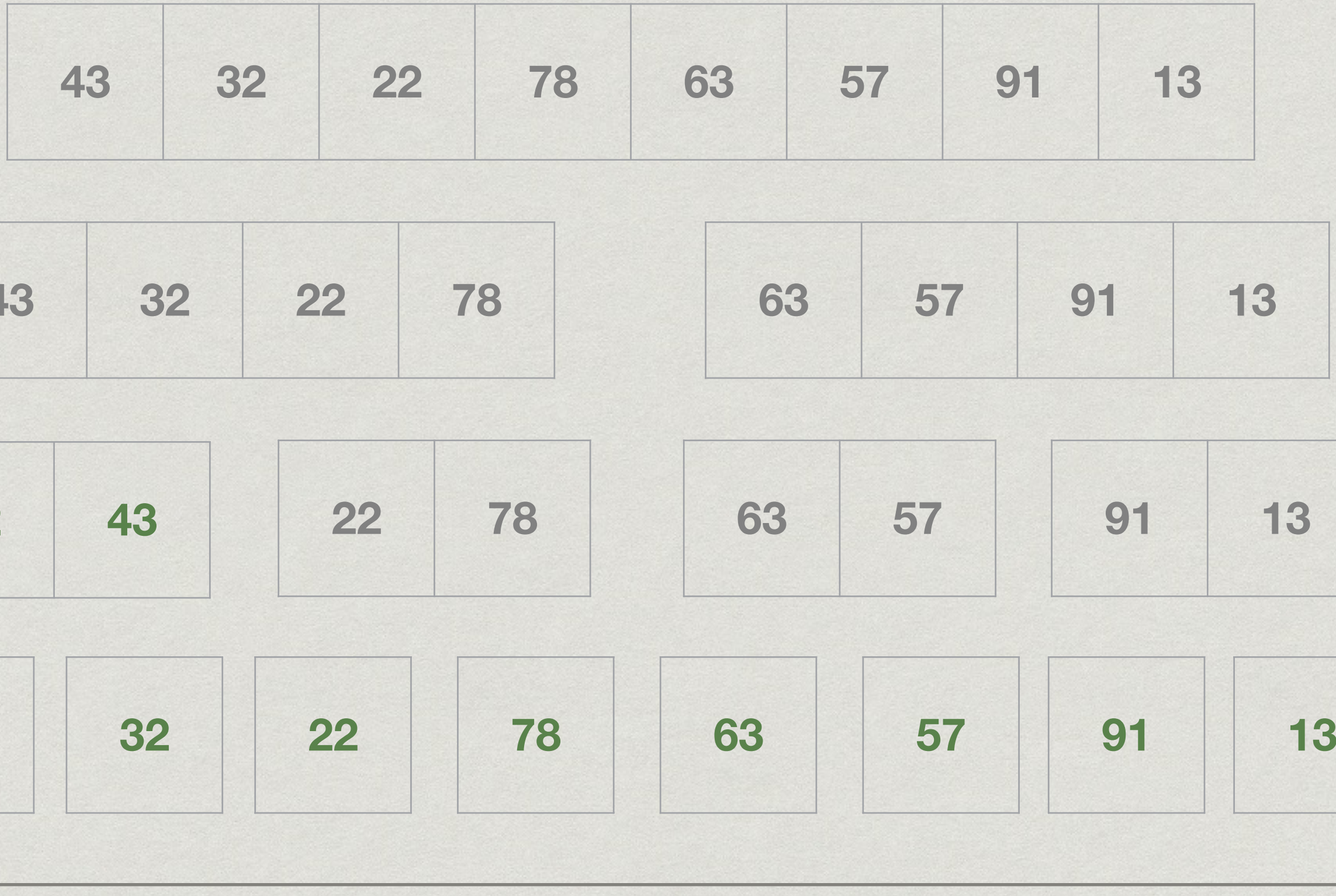
Merge Sort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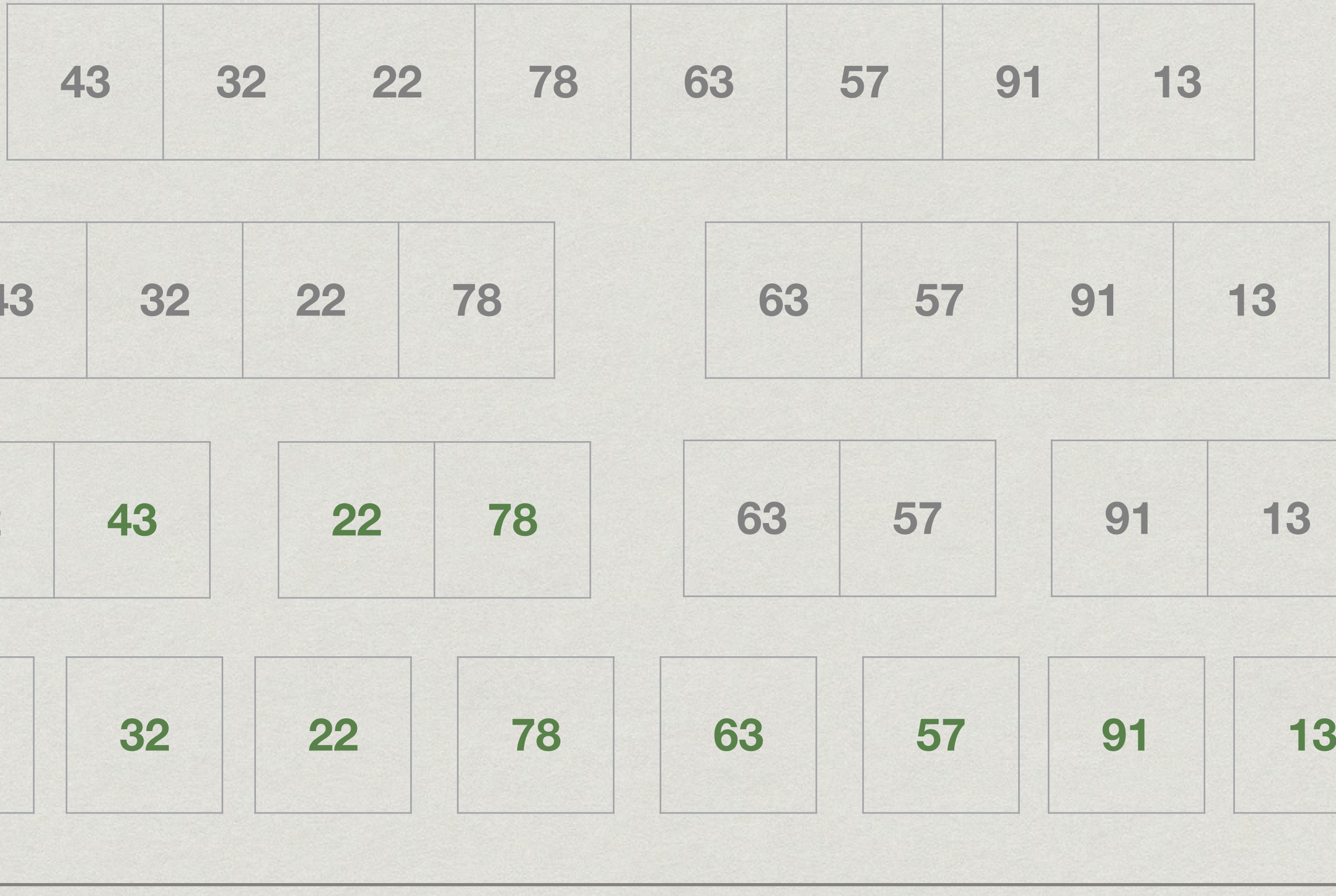
Merge Sort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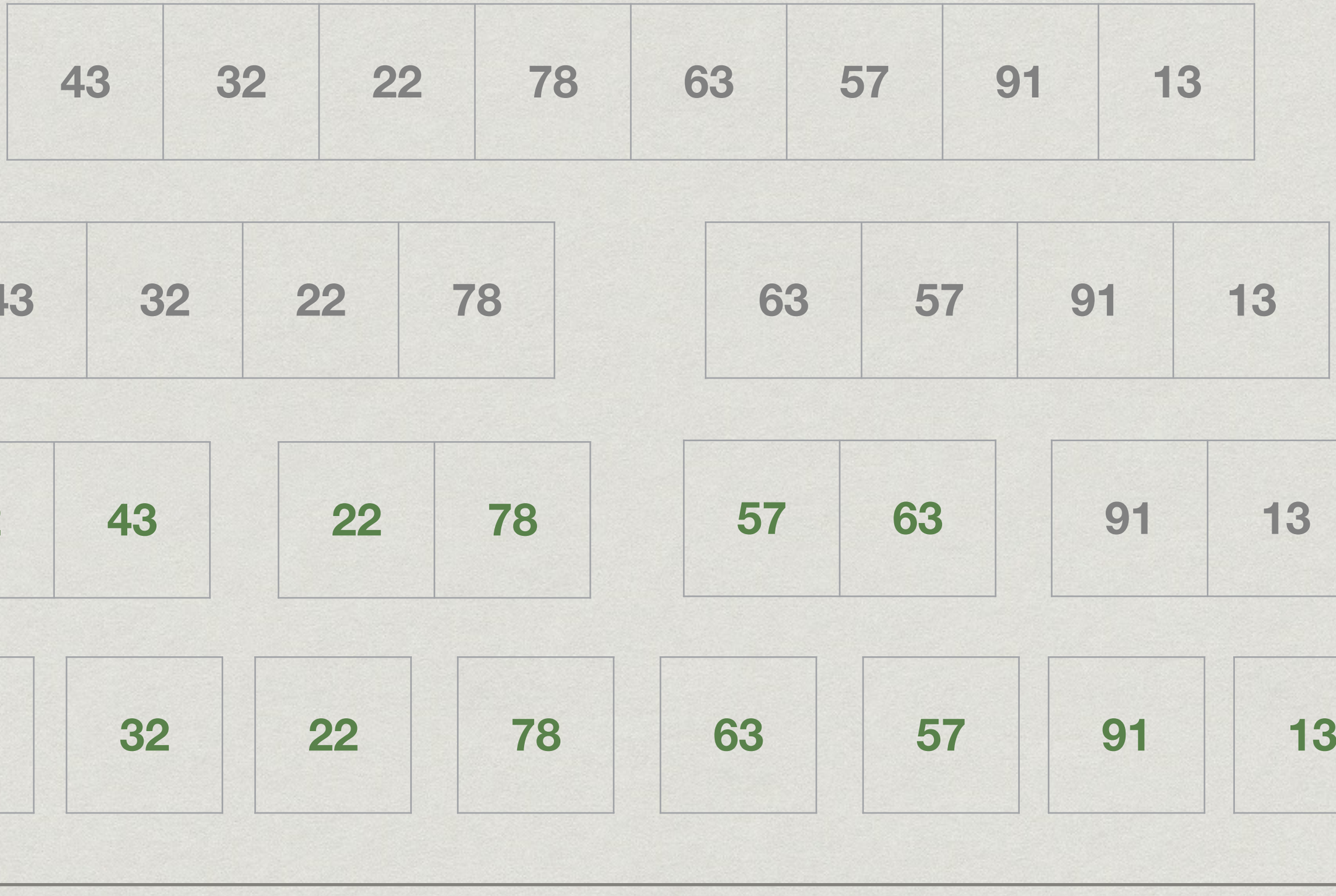
Merge Sort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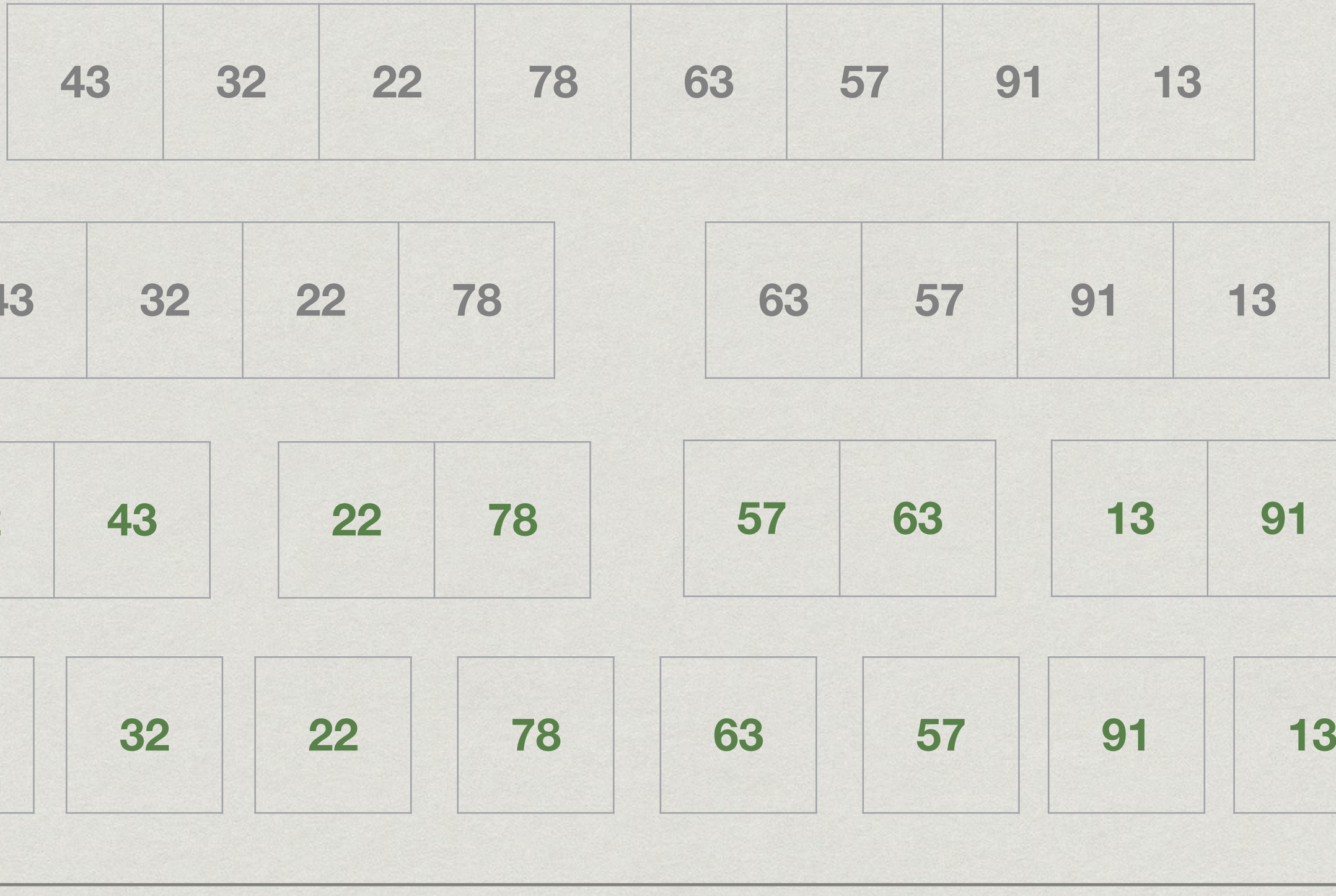
Merge Sort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Merge Sort

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Merge Sort

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Divide and conquer

- Break up problem into disjoint parts
- Solve each part separately
- Combine the solutions efficiently

Merging sorted lists

Combine two sorted lists `A` and `B` into `C`

- If `A` is empty, copy `B` into `C`
- If `B` is empty, copy `A` into `C`
- Otherwise, compare first element of `A` and `B` and move the smaller of the two into `C`
- Repeat until all elements in `A` and `B` have been moved

Merging

```
def merge(A,B): # Merge A[0:m],B[0:n]
  (C,m,n) = ([],len(A),len(B))
  (i,j) = (0,0) # Current positions in A,B
  while i+j < m+n: # i+j is number of elements merged so far
    if i == m:  # Case 1: A is empty
      C.append(B[j])
      j = j+1
    elif j == n: # Case 2: B is empty
      C.append(A[i])
      i = i+1
    elif A[i] <= B[j]: # Case 3: Head of A is smaller
      C.append(A[i])
      i = i+1
    elif A[i] > B[j]: # Case 4: Head of B is smaller
      C.append(B[j])
      j = j+1
  return(C)
```

Merging, wrong

```
def mergewrong(A,B): # Merge A[0:m],B[0:n]
  (C,m,n) = ([],len(A),len(B))
  (i,j) = (0,0) # Current positions in A,B
  while i+j < m+n:
  # i+j is number of elements merged so far
    # Combine Case 1, Case 4
    if i == m or A[i] > B[j]:
      C.append(B[j])
      j = j+1
    # Combine Case 2, Case 3:
    elif j == n or A[i] <= B[j}:
      C.append(A[i])
      i = i+1
  return(C)
```

Merge Sort

To sort `A[0:n]` into `B[0:n]`

- If n is 1, nothing to be done
- Otherwise
  - Sort `A[0:n//2]` into `L` (left)
  - Sort `A[n//2:n]` into `R` (right)
  - Merge `L` and `R` into `B`

Merge Sort

```
def mergesort(A,left,right):
    # Sort the slice A[left:right]

    if right - left <= 1: # Base case
        return(A[left:right])

    if right - left > 1:  # Recursive call

        mid = (left+right)//2

        L = mergesort(A,left,mid)
        R = mergesort(A,mid,right)

        return(merge(L,R))
```