NPTEL MOOC,JAN-FEB 2015
Week 8, Module 5

DESIGN AND ANALYSIS OF ALGORITHMS

Reductions

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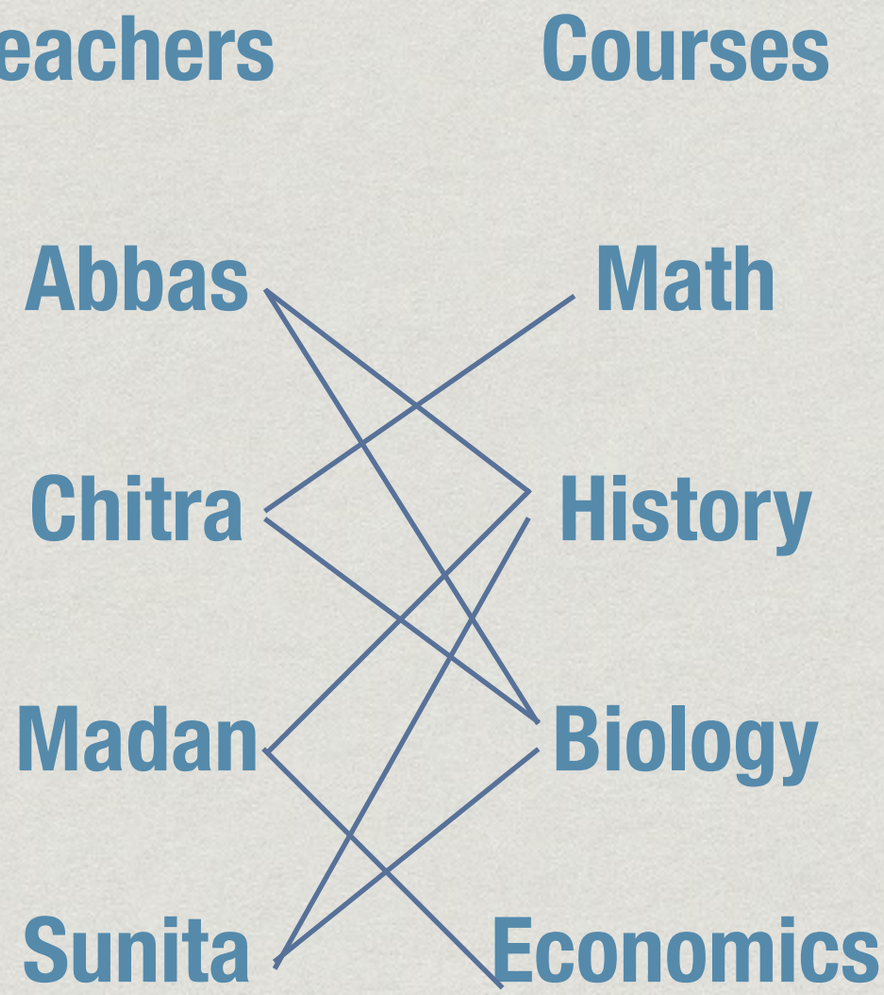
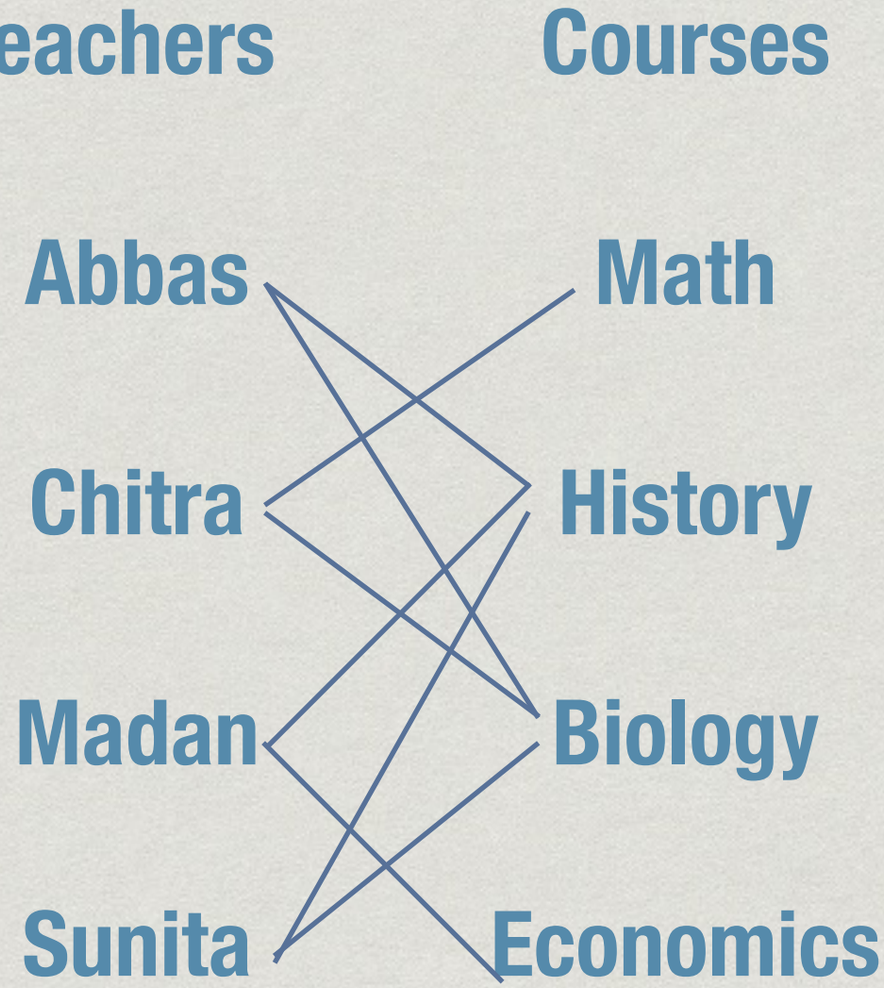
Bipartite Matching

- Each instructor is willing to teach a set of courses
- Find an allocation so that
  - Each course is taught by a single instructor
  - Each instructor teaches only one course, which he/she is willing to teach



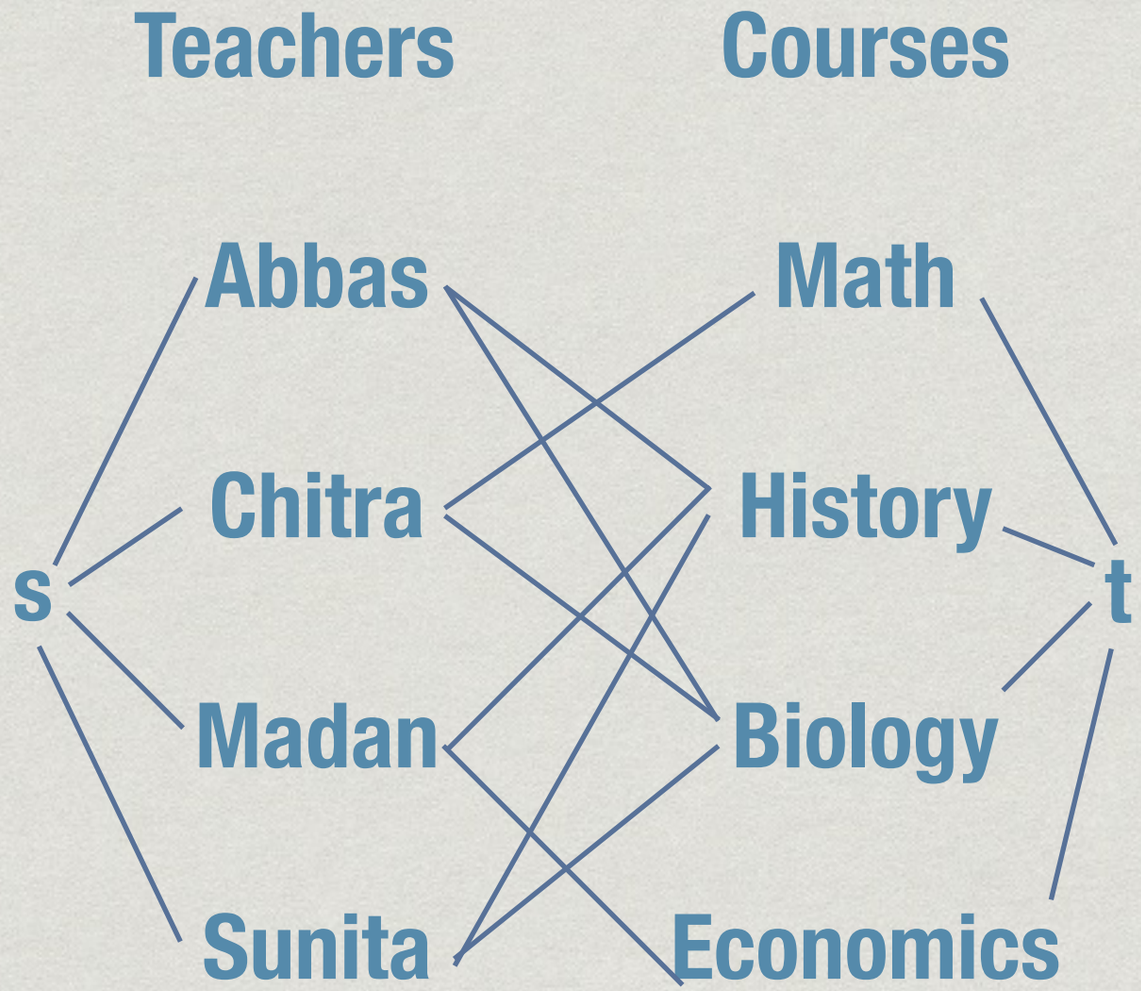
Bipartite Matching

- V partitioned into V_0,V_1
- All edges from V_0 to V_1
- Matching: subset of edges so that no two of them share an endpoint
- Find largest matching
  - If possible, a perfect matching, all nodes covered



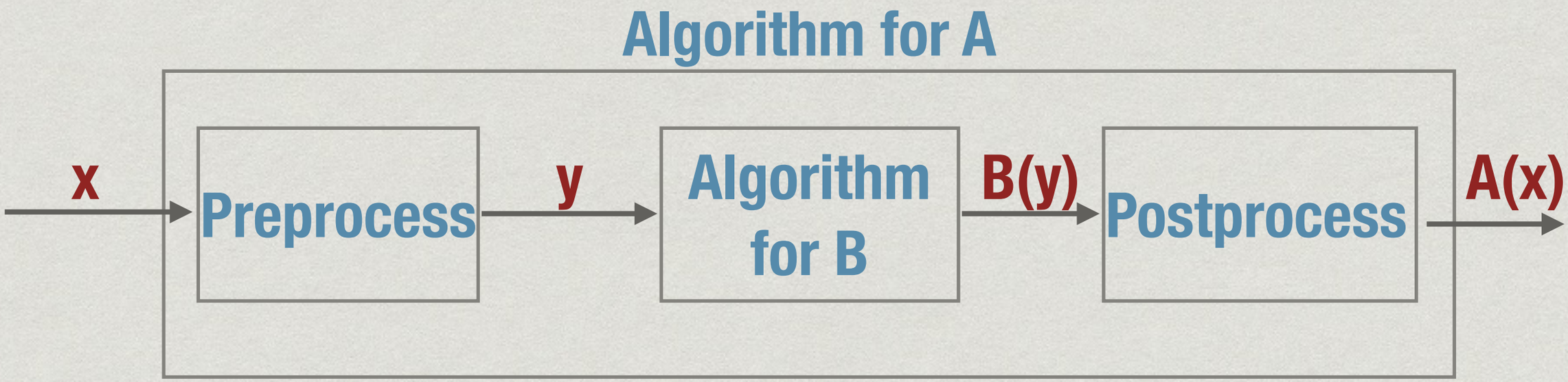
Bipartite Matching

- Add a source and sink
- All edge capacities are 1
- Find a maximum flow from s to t!



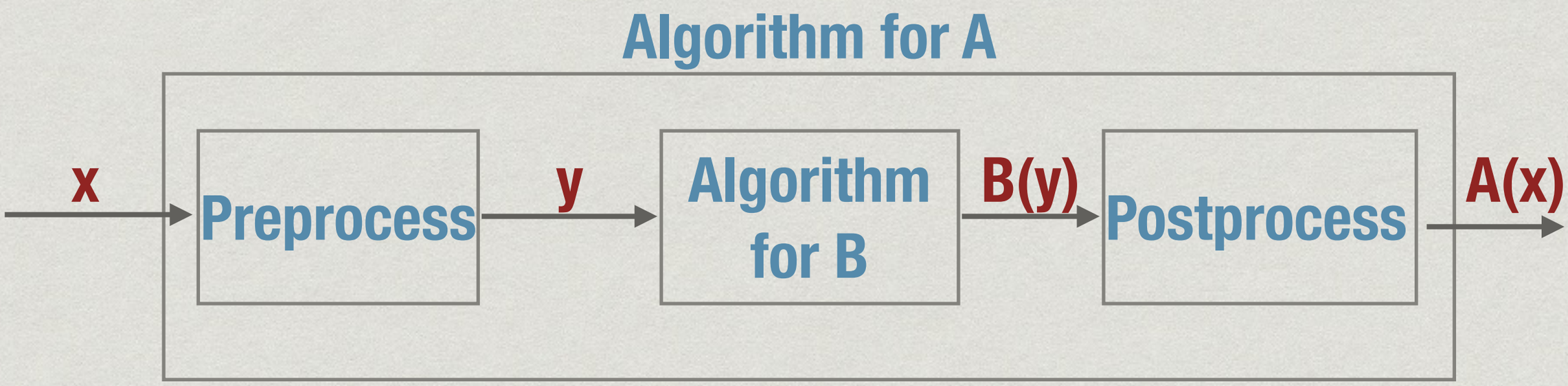
Reductions

- We want to solve problem A
- We know how to solve problem B
- Convert input for A into input for B
- Interpret output of B as output of A



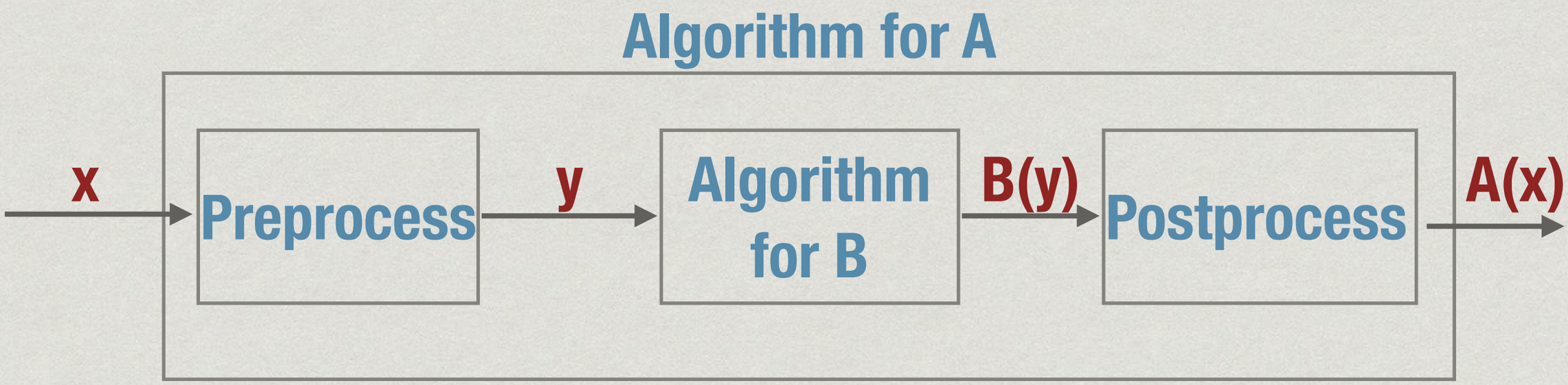
Reductions

- A reduces to B
- Can transfer efficient solution from B to A
  - But preprocessing and postprocessing must also be efficient!
- Typically, both should be polynomial time



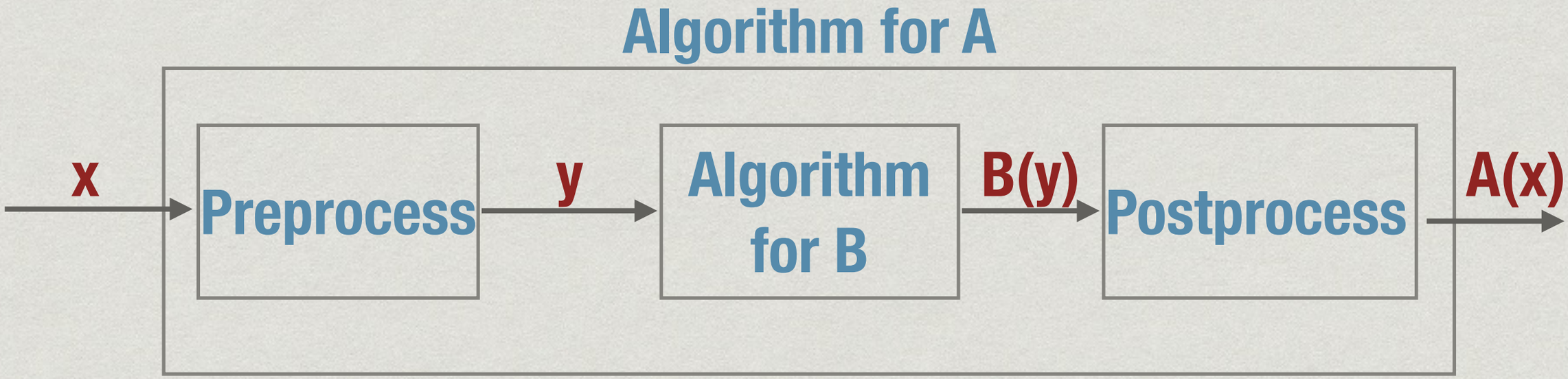
Reductions

- Bipartite matching reduces to max flow
- Max flow reduces to LP
  - Number of variables, constraints is linear in the size of the graph



Reductions

- Reverse interpretation is also useful
- If A is known to be intractable and A reduces to B, then B must also be intractable
- Otherwise, solution for B will yield solution for A



Big hammers

- LP and network flows are powerful tools
- Many algorithmic problems can be reduced to them
- Efficient, off-the-shelf implementations are available
- Useful to understand what can (and cannot) be modelled in terms of LP and flows