NPTEL MOOC,JAN-FEB 2015
Week 7, Module 5

DESIGN AND ANALYSIS OF ALGORITHMS

Edit Distance

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Document similarity

- “The students were able to appreciate the concept optimal substructure property and its use in designing algorithms”

- “The lecture taught the students to appreciate how the concept of optimal substructures can be used in designing algorithms”

- “The lecture taught the students ~~were able~~ to appreciate how the concept of optimal substructures ~~property~~ can~~d~~ ~~it~~b~~s~~e used in designing algorithms”

- 28 characters inserted, 18 ~~deleted~~, 2 ~~subst~~ituted

Edit distance

- Minimum number of editing operations needed to transform one document to the other
  - Insert a character
  - Delete a character
  - Substitute a character by another one
- In our example,
  28 characters inserted, 18 ~~deleted~~, 2 ~~subst~~ituted
- Edit distance is at most 48

Edit distance

- Also called Levenshtein distance
  - First proposed by Vladimir Levenshtein
- Applications
  - Suggest spelling corrections in word processor, search engine queries
  - Another way of comparing genetic similarity across species

Edit distance and LCS

- Longest common subsequence of u and v
    - What remains after minimum number of deletes to make them equal
- Deleting a letter in u equivalent to inserting it in v
    - “secret”, “bisect” — LCS is “sect”
        - delete “r”, “e” in “secret”, “b”, “i” in “bisect”
        - delete “r”, “e” then insert “b”, “i” in “secret”
- LCS is equivalent to edit distance without substitution

Inductive structure for edit distance

u	a_i	a_{i+1}	a_{i+2}	…	…	a_{m-1}	a_m
v	b_j	b_{j+1}	b_{j+2}	…	b_{n-1}	b_n	

- Recall LCS
  - If $a_i = b_j$, LCS(i,j) = 1 + LCS(i+1,j+1)
  - If $a_i \neq b_j$, LCS(i,j) = max(LCS(i+1,j), LCS(i,j+1))
  - Boundary condition when one of the words is empty

Edit distance…

u	a_i	a_{i+1}	a_{i+2}	…	…	a_{m-1}	a_m
v	b_j	b_{j+1}	b_{j+2}	…	b_{n-1}	b_n	

- Aim is to transform u into v
  - If $a_i = b_j$, ED(i,j) = ED(i+1,j+1) — nothing to be done at (a_i,b_j)
  - If $a_i \neq b_j$, can do one of three things
    - Substitute a_i by b_j : 1 + ED(i+1,j+1)
    - Delete a_i : 1 + ED(i+1,j)
    - Insert b_j before a_i: 1 + ED(i,j+1)
    - Take the minimum of these

Inductive structure

u	a_i	a_{i+1}	a_{i+2}	…	…	a_{m-1}	a_m
v	b_j	b_{j+1}	b_{j+2}	…	b_{n-1}	b_n	

- ED(i,j) stands for ED($a_i a_{i+1} \ldots a_m$, $b_j b_{j+1} \ldots b_n$)
- If $a_i = b_j$, ED(i,j) = ED(i+1,j+1)
- If $a_i \neq b_j$, LCS(i,j) =

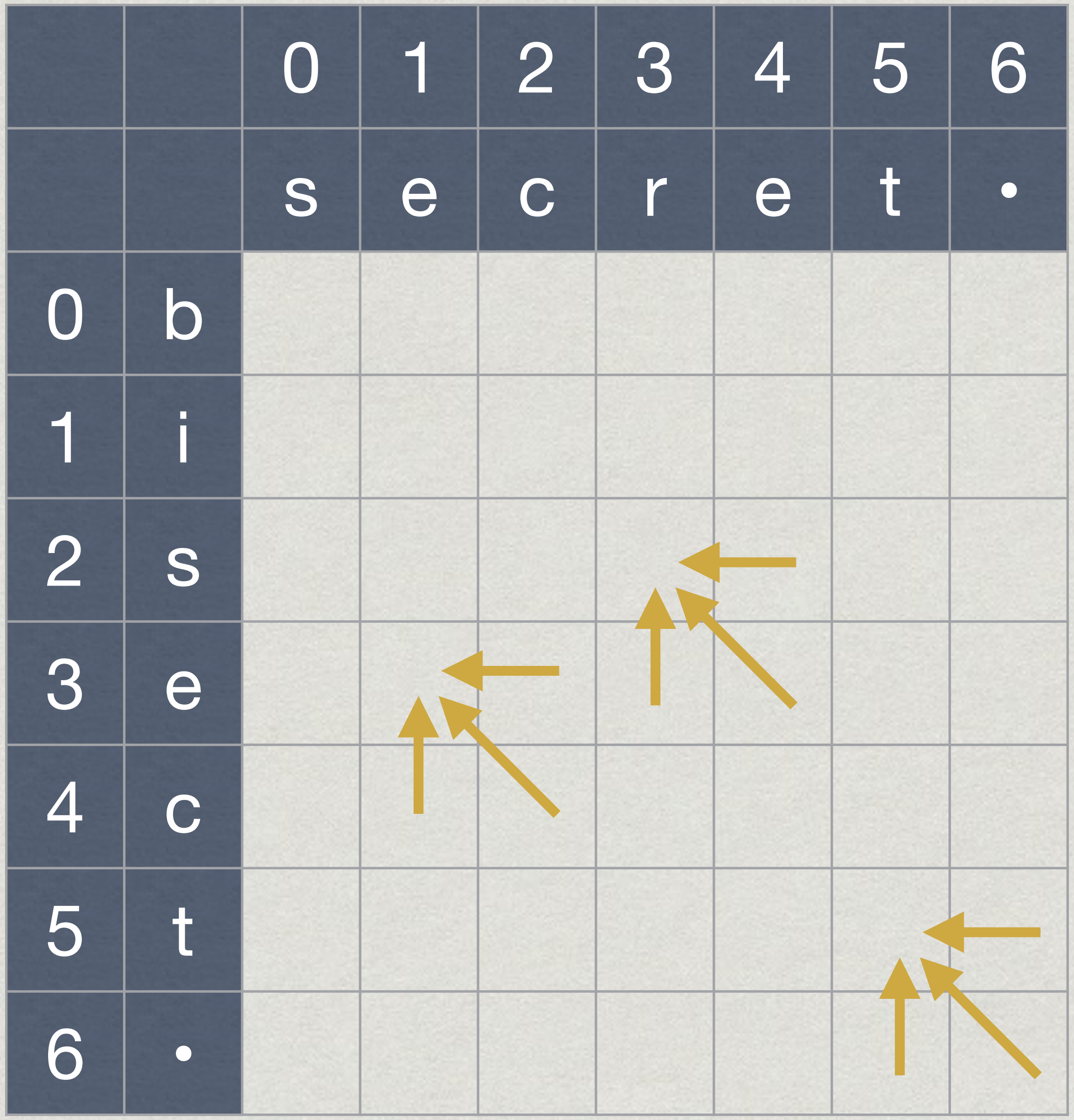
  1 + min(ED(i+1,j+1),ED(i+1,j), ED(i,j+1))
- As with LCS/LCW, extend positions to m+1, n+1
  - ED(m+1,j) = n-j+1 for all j # Insert $b_j b_{j+1} \ldots b_n$ in u
  - ED(i,n+1) = m-i+1 for all i, # Insert $a_i a_{i+1} \ldots a_m$in v

Subproblem dependency

- Like LCS, ED(i,j) depends on ED(i+1,j+1), ED(i+1,j) and ED(i,j+1)

- Dependencies for ED(m,n) are known

- Start at ED(m,n) and fill by row, column or diagonal



		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b							
1	i							
2	s							
3	e							
4	c							
5	t							
6	•							

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		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b							**6**
1	i							**5**
2	s							**4**
3	e							**3**
4	c							**2**
5	t							**1**
6	•	**6**	**5**	**4**	**3**	**2**	**1**	**0**

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		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b						**5**	**6**
1	i						**4**	**5**
2	s						**3**	**4**
3	e						**2**	**3**
4	c						**1**	**2**
5	t						**0**	**1**
6	•	**6**	**5**	**4**	**3**	**2**	**1**	**0**

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		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b					**5**	**5**	**6**
1	i					**4**	**4**	**5**
2	s					**3**	**3**	**4**
3	e					**2**	**2**	**3**
4	c					**1**	**1**	**2**
5	t					**1**	**0**	**1**
6	•	**6**	**5**	**4**	**3**	**2**	**1**	**0**

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		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b				5	5	5	6
1	i				4	4	4	5
2	s				3	3	3	4
3	e				2	2	2	3
4	c				2	1	1	2
5	t				2	1	0	1
6	•	6	5	4	3	2	1	0

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		s	e	c	r	e	t	•
0	b			5	5	5	5	6
1	i			4	4	4	4	5
2	s			3	3	3	3	4
3	e			3	2	2	2	3
4	c			2	2	1	1	2
5	t			3	2	1	0	1
6	•	6	5	4	3	2	1	0

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0	b		5	5	5	5	5	6
1	i		4	4	4	4	4	5
2	s		3	3	3	3	3	4
3	e		2	3	2	2	2	3
4	c		3	2	2	1	1	2
5	t		4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

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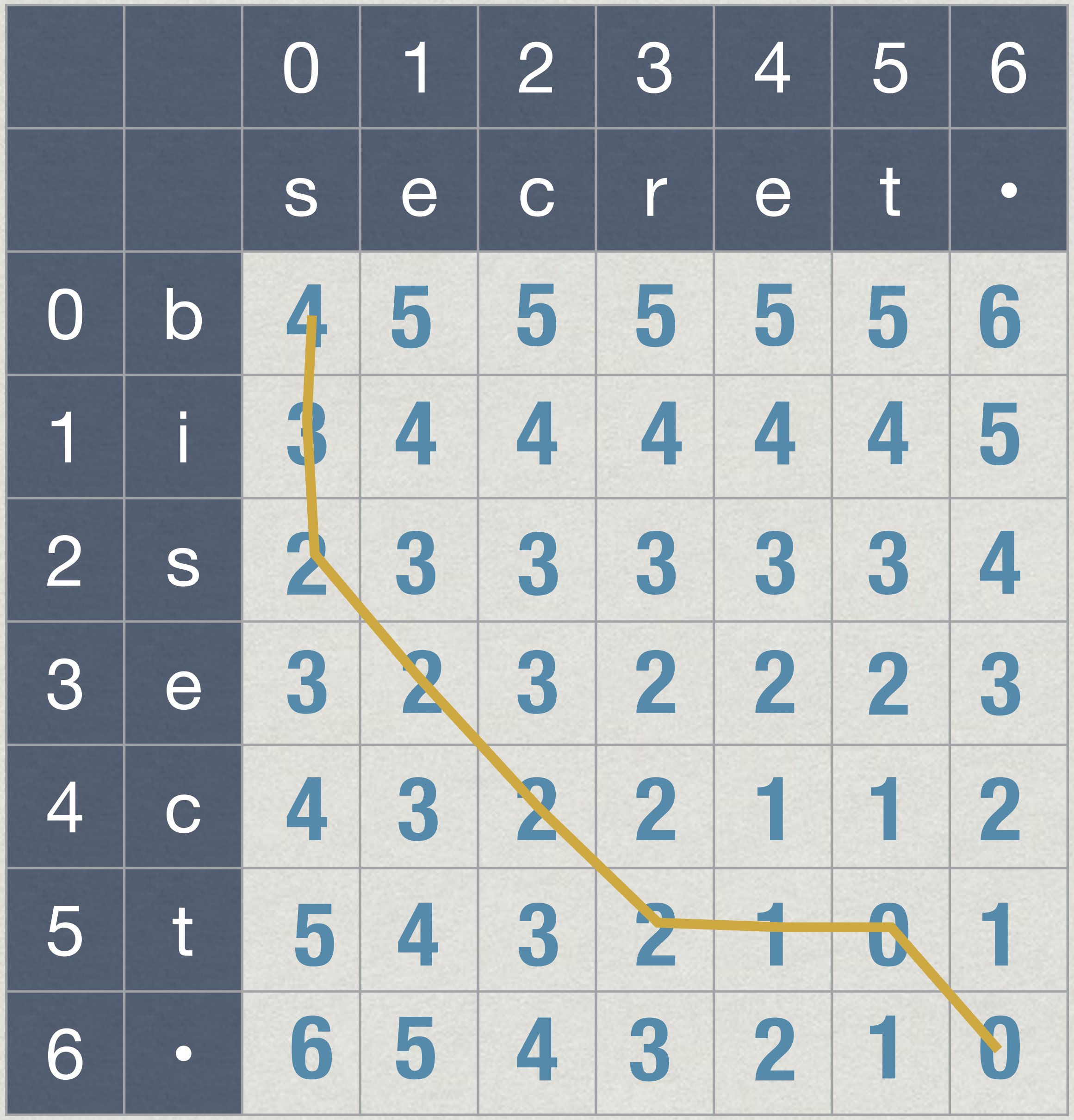
		s	e	c	r	e	t	•
0	b	4	5	5	5	5	5	6
1	i	3	4	4	4	4	4	5
2	s	2	3	3	3	3	3	4
3	e	3	2	3	2	2	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

Recovering the solution

- Trace back the path
- Transforming “secret” to “bisect”
  - Del “b” : (0,0)→(1,0)
  - Del “i” : (1,0)→(2,0)
  - Ins “r” : (5,3)→(5,4)
  - Ins “e”: (5,4)→(5,5)



		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	5	5	5	5	5	6
1	i	3	4	4	4	4	4	5
2	s	2	3	3	3	3	3	4
3	e	3	2	3	2	2	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

ED(u,v), DP

```
function ED(u,v) # u[0..m], v[0..n]

for r = 0,1,…,m+1  { ED[r][n+1] = m-r+1 }

for c = 0,1,…,m+1  { ED[m+1][c] = n-c+1 }

for c = n,n-1,…,0
  for r = m,m-1,…0
    if (u[r] == v[c])
      ED[r][c] = ED[r+1][c+1]
    else
      ED[r][c] = 1 + min(ED[r+1][c+1],
                         ED[r+1][c],
                         ED[r][c+1])

return(ED[0][0])
```

Complexity

- Again O(mn) using dynamic programming (or memoization)
  - Need to fill an O(mn) size table
  - Each table entry takes constant time to compute

Space complexity

- For LCW, LCS, ED
    - Need to fill an O(mn) size table
    - Do we need to store the entire table?
- Filling column by column, only need next column and current column
    - Or next row and current row
- Reduce space to O(n), assuming m ≥ n