NPTEL MOOC,JAN-FEB 2015
Week 5, Module 3

# DESIGN AND ANALYSIS OF ALGORITHMS

## Priority queues

**MADHAVAN MUKUND, CHENNAI MATHEMATICAL INSTITUTE**
**http://www.cmi.ac.in/~madhavan**

# Job scheduler

- A job scheduler maintains a list of pending jobs with their priorities.
- When the processor is free, the scheduler picks out the job with maximum priority in the list and schedules it.
- New jobs may join the list at any time.
- How should the scheduler maintain the list of pending jobs and their priorities?

# Priority queue

- Need to maintain a list of jobs with priorities to optimise the following operations
  - delete_max( )
    - Identify and remove job with highest priority
    - Need not be unique
  - insert( )
    - Add a new job to the list

# Linear structures

- Unsorted list
  - insert( ) takes O(1) time
  - delete_max( ) takes O(n) time
- Sorted list
  - delete_max( ) takes O(1) time
  - insert( ) takes O(n) time
- Processing a sequence of n jobs requires O($n^2$) time

# Two dimensional structures

N = 25

## First attempt

- Assume N processes enter/leave the scheduler
- Keep an √N x √N array
- Each row is maintained in sorted order

| | | | | |
|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 |
| 8 | 19 | 22 | 33 | 37 |
| 10 | 13 | 14 | | |
| 13 | 20 | 25 | 43 | |
| 6 | 11 | | | |

# insert( )

## Insert 11

- Insert into first row that has free space
    - Maintain size of each row
- Takes time O(√N)

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | 5 |
| 8 | 19 | 22 | 33 | 37 | 5 |
| 10 | 13 | 14 |  |  | 3 |
| 13 | 20 | 25 | 43 |  | 4 |
| 6 | 11 |  |  |  | 2 |

# insert( )

## Insert 11

- Insert into first row that has free space
  - Maintain size of each row
- Takes time O(√N)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **11** | 12 | 17 | 29 | 31 | 40 | | 5 |
| | 8 | 19 | 22 | 33 | 37 | | 5 |
| | 10 | 13 | 14 | | | | 3 |
| | 13 | 20 | 25 | 43 | | | 4 |
| | 6 | 11 | | | | | 2 |

# insert( )

## Insert 11

- Insert into first row that has free space
    - Maintain size of each row
- Takes time O(√N)

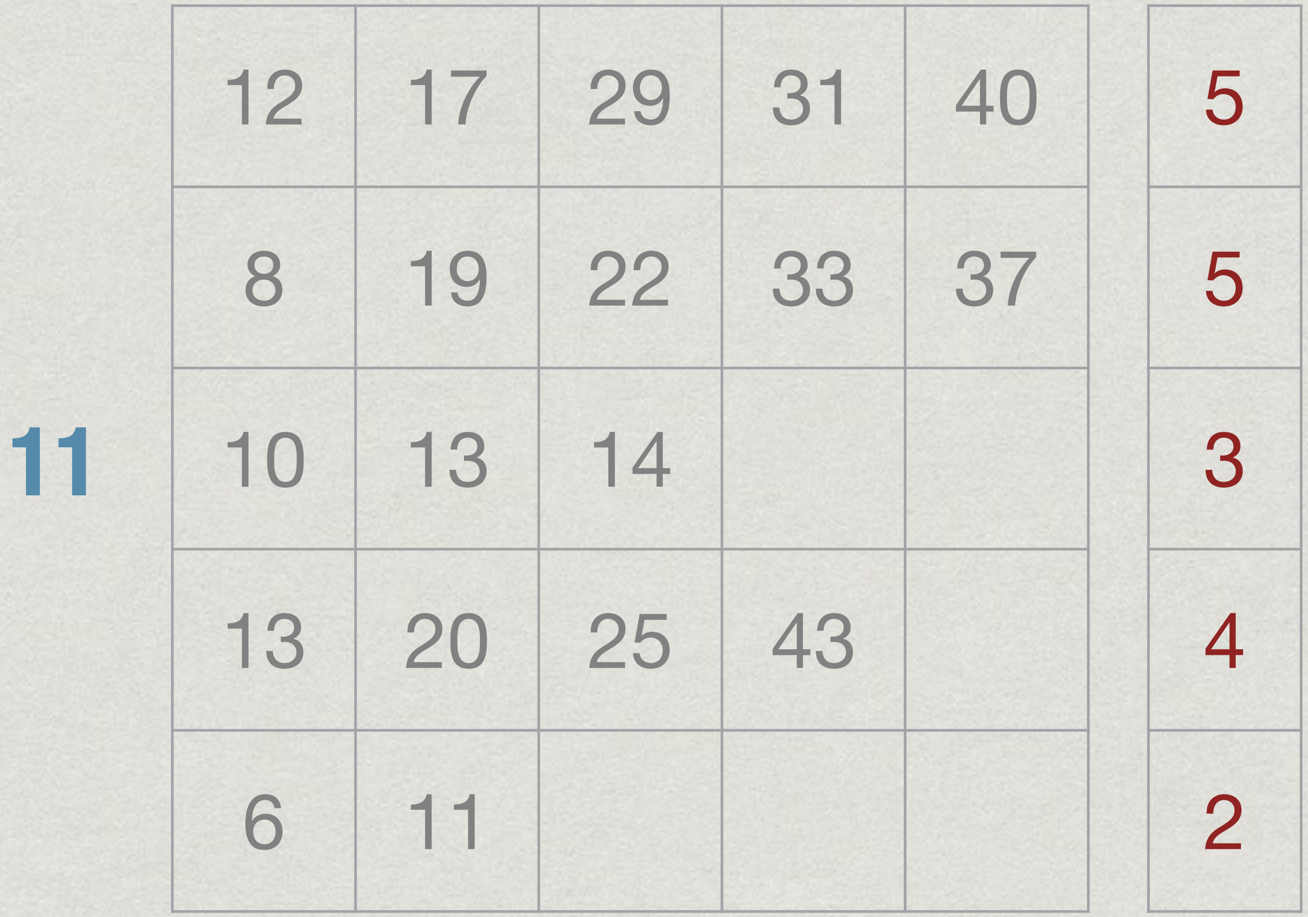
11

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | | 5 |
| 8 | 19 | 22 | 33 | 37 | | 5 |
| 10 | 13 | 14 | | | | 3 |
| 13 | 20 | 25 | 43 | | | 4 |
| 6 | 11 | | | | | 2 |

# insert( )

## Insert 11

- Insert into first row that has free space
    - Maintain size of each row
- Takes time O(√N)



|  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|
|  | 12 | 17 | 29 | 31 | 40 | 5 |
|  | 8 | 19 | 22 | 33 | 37 | 5 |
| 11 | 10 | 13 | 14 |  |  | 3 |
|  | 13 | 20 | 25 | 43 |  | 4 |
|  | 6 | 11 |  |  |  | 2 |

# insert( )

## Insert 11

- Insert into first row that has free space
    - Maintain size of each row
- Takes time O(√N)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | 5 |
| 8 | 19 | 22 | 33 | 37 | 5 |
| 10 | **11** | 13 | 14 | | 3 |
| 13 | 20 | 25 | 43 | | 4 |
| 6 | 11 | | | | 2 |

# insert( )

## Insert 11

- Insert into first row that has free space
  - Maintain size of each row
- Takes time O(√N)

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | | 5 |
| 8 | 19 | 22 | 33 | 37 | | 5 |
| 10 | **11** | 13 | 14 | | | 4 |
| 13 | 20 | 25 | 43 | | | 4 |
| 6 | 11 | | | | | 2 |

# delete_max( )

- Maximum in each row is the last element
- Maximum among these is to be deleted
- Again O(√N)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | 5 |
| 8 | 19 | 22 | 33 | 37 | 5 |
| 10 | 11 | 13 | 14 | | 4 |
| 13 | 20 | 25 | 43 | | 4 |
| 6 | 11 | | | | 2 |

# delete_max( )

- Maximum in each row is the last element
- Maximum among these is to be deleted
- Again O(√N)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | **40** | 5 |
| 8 | 19 | 22 | 33 | **37** | 5 |
| 10 | 11 | 13 | **14** | | 4 |
| 13 | 20 | 25 | **43** | | 4 |
| 6 | **11** | | | | 2 |

# delete_max( )

- Maximum in each row is the last element
- Maximum among these is to be deleted
- Again O(√N)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | **40** | 5 |
| 8 | 19 | 22 | 33 | **37** | 5 |
| 10 | 11 | 13 | **14** | | 4 |
| 13 | 20 | 25 | **43** | | 4 |
| 6 | **11** | | | | 2 |

# delete_max( )

- Maximum in each row is the last element
- Maximum among these is to be deleted
- Again O(√N)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | 5 |
| 8 | 19 | 22 | 33 | 37 | 5 |
| 10 | 11 | 13 | 14 | | 4 |
| 13 | 20 | 25 | | | 4 |
| 6 | 11 | | | | 2 |

# delete_max( )

- Maximum in each row is the last element
- Maximum among these is to be deleted
- Again O(√N)

| | | | | | |
|---|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 | 5 |
| 8 | 19 | 22 | 33 | 37 | 5 |
| 10 | 11 | 13 | 14 | | 4 |
| 13 | 20 | 25 | | | 3 |
| 6 | 11 | | | | 2 |

# Two dimensional structures

## Summary

- insert( ) takes $O(\sqrt{N})$
- delete_max( ) takes $O(\sqrt{N})$
- Processing N jobs takes $O(N\sqrt{N})$

Can we do better?

| | | | | |
|---|---|---|---|---|
| 12 | 17 | 29 | 31 | 40 |
| 8 | 19 | 22 | 33 | 37 |
| 10 | 13 | 14 | | |
| 13 | 20 | 25 | 43 | |
| 6 | 11 | | | |

# Trees

- Maintain a special kind of binary tree called a heap
  - Balanced: N node tree has height log N
- Both insert( ) and delete_max( ) take O(log N)
  - Processing N jobs takes time O(N log N)
- Truly flexible, need not fix upper bound for N in advance