DESIGN AND ANALYSIS OF ALGORITHMS

Quicksort: Analysis

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Quicksort

* Choose a pivot element

  * Typically the first value in the array

* Partition A into lower and upper parts with respect to pivot

* Move pivot between lower and upper partition

* Recursively sort the two partitions

Analysis of Quicksort

* Partitioning with respect to pivot takes O(n)

* If pivot is median

    * Each partition is of size n/2

    * $t(n) = 2t(n/2) + n = O(n \log n)$

* Worst case?

Analysis of Quicksort

Worst case

* Pivot is maximum or minimum

  * One partition is empty

  * Other is size n-1

  * $t(n) = t(n-1) + n = t(n-2) + (n-1) + n$
    $= \ldots = 1 + 2 + \ldots + n = O(n^2)$

* Already sorted array is worst case input!

Analysis of Quicksort

But …

* Average case is O(n log n)

  * Sorting is a rare example where average case can be computed

* What does average case mean?

Quicksort: Average case

* Assume input is a permutation of {1,2,…,n}

  * Actual values not important

  * Only relative order matters

  * Each input is equally likely (uniform probability)

* Calculate running time across all inputs

* Expected running time can be shown O(n log n)

Quicksort: randomization

✳ Worst case arises because of fixed choice of pivot

   ✳ We chose the first element

   ✳ For any fixed strategy (last element, midpoint), can work backwards to construct $O(n^2)$ worst case

✳ Instead, choose pivot randomly

   ✳ Pick any index in [0..n-1] with uniform probability

✳ Expected running time is again O(n log n)

Iterative Quicksort

* Recursive calls work on disjoint segments of array

    * No recombination of results required

* Can use an explicit stack to simulate recursion

    * Stack only needs to store left and right endpoints of interval to be sorted

Quicksort in practice

* In practice, Quicksort is very fast

  * Typically the default algorithm for in-built sort functions

    * Spreadsheets

  * Built in sort function in programming languages