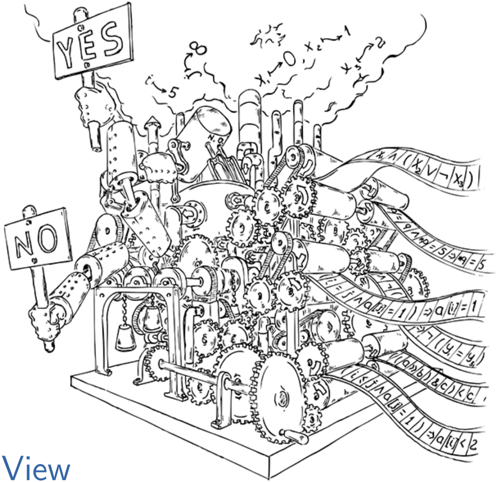
Propositional Encodings

Chapter 11



Decision Procedures

An Algorithmic Point of View

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Revision 1.0

Outline

1. Overview
2. Notation
3. A Basic Encoding Algorithm
4. Integration into DPLL
5. Theory Propagation and the DPLL(T) Framework
6. Theory Propagation and the DPLL(T) Framework
7. Optimizations and Implementation Issues

- Let T be a first-order Σ-theory such that:
  - T is quantifier-free.
  - There exists a decision procedure, denoted DP_T, for the conjunctive fragment of T.

- Example 1:
  - T is equality logic.
  - DP_T is the congruence closure algorithm.

- Example 2:
  - T is disjunctive linear arithmetic.
  - DP_T is the Simplex algorithm.

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- a SAT solver,

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in various ways, in order to construct a decision procedure for T.

This method is

- modular,
- efficient,
- competitive (all state-of-the-art SMT solvers work this way).

The two main engines in this framework work in tight collaboration:

- The SAT solver chooses those literals that need to be satisfied in order to satisfy the Boolean structure of the formula, and
- The theory solver DP_T checks whether this choice is consistent in T.

Let l be a Σ-literal.

- Denote by $e(l)$ the Boolean encoder of this literal.

Let t be a Σ-formula,

- Denote by $e(t)$ the Boolean formula resulting from substituting each Σ-literal in t with its Boolean encoder.

Notation

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- Denote by $e(t)$ the Boolean formula resulting from substituting each Σ-literal in t with its Boolean encoder.

For a Σ-formula t, the resulting Boolean formula $e(t)$ is called the propositional skeleton of t.

- Example I: Let $l := x = y$ be a Σ-literal. Then $e(x = y)$, a Boolean variable, is its encoder.

Boolean encoders – examples

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- Example II: Let

$$t := \; x = y \vee x = z$$

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$$e(t) := \; e(x = y) \vee e(x = z)$$

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Note that since we are encoding *literals* and not *atoms*, $e(\varphi)$ has no negations and hence is trivially satisfiable.

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Assume that the SAT solver returns the satisfying assignment

$$\alpha := \{e(x = y) \mapsto \text{TRUE},\ e(y = z) \mapsto \text{TRUE},\ e(x \neq z) \mapsto \text{TRUE},\\ e(x = z) \mapsto \text{FALSE}\} .$$

- Denote by $\hat{Th}(\alpha)$ the conjunction of the literals corresponding to this assignment.

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$\hat{Th}(\alpha)$ is not satisfiable, which means that the negation of this formula is a tautology.

Thus $\mathcal{B}$ is conjoined with $e(\neg \hat{Th}(\alpha))$, the Boolean encoding of this tautology:

$$e(\neg \hat{Th}(\alpha)) := \ (\neg e(x = y) \vee \neg e(y = z) \vee \neg e(x \neq z) \vee e(x = z)) \ .$$

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- This clause contradicts the current assignment, and hence blocks it from being repeated.
- Such clauses are called **blocking clauses**.

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- Such clauses are called **blocking clauses**.
- We denote by t the formula – also called the **lemma** – returned by DP_T (in this example $t := \neg \hat{Th}(\alpha)$).

After the blocking clause has been added, the SAT solver is invoked again and suggests another assignment, for example

$$\begin{aligned}\alpha' := \ & \{e(x = y) \mapsto \text{TRUE},\ e(y = z) \mapsto \text{TRUE},\ e(x = z) \mapsto \text{TRUE}, \\ & e(x \neq z) \mapsto \text{FALSE}\}\ .\end{aligned}$$

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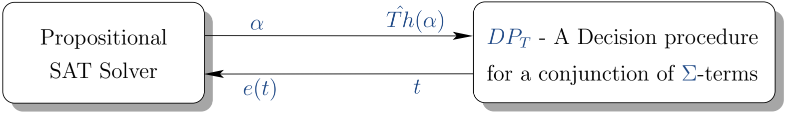
The corresponding Σ-formula

$$\hat{Th}(\alpha') := \ x = y \wedge y = z \wedge x = z \wedge \neg(x \neq z) \tag{3}$$

is satisfiable, which proves that φ, the original formula, is satisfiable.

Indeed, any assignment that satisfies $\hat{Th}(\alpha')$ also satisfies φ.



The information flow between the two components of the decision procedure.

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There are many improvements to this basic procedure.

One such improvement is:
"Invoke the decision procedure DP_T after partial assignments, rather than waiting for a full assignment."

- A contradicting partial assignment leads to a more powerful lemma t, as it blocks all assignments that extend it.
- Theory propagation: When the partial assignment is not contradictory, it can be used to derive implications that are propagated back to the SAT solver.

Continuing the example above, consider the partial assignment

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DP_T concludes that $x = z$ is implied, and hence inform the SAT solver that $e(x = z) \mapsto \text{TRUE}$ and $e(x \neq z) \mapsto \text{FALSE}$ are implied by the current partial assignment α.

Plan

We will now formalize three versions of the algorithm:

1. Simple
2. Incremental
3. DPLL(T)

Notation

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- $lit_i(\varphi)$ – the i-th distinct literal in φ
(assuming some predefined order on the literals).

- α – For a given encoding $e(\varphi)$, denotes an assignment (either full or partial), to the encoders in $e(\varphi)$.

Notation

- $Th(lit_i, \alpha)$ – For an encoder $e(lit_i)$ that is assigned a truth value by α, denotes the corresponding literal:

$$Th(lit_i, \alpha) \doteq \begin{cases} lit_i & \alpha(lit_i) = \text{TRUE} \\ \neg lit_i & \alpha(lit_i) = \text{FALSE}\,. \end{cases} \tag{6}$$

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- $Th(\alpha) \doteq \{Th(lit_i, \alpha) \mid e(lit_i) \text{ is assigned by } \alpha\}$
- $\hat{T}h(\alpha)$ – a conjunction over the elements in $Th(\alpha)$.

Example Use of the Notation

Let

$$lit_1 = (x = y),\ lit_2 = (y = z),\ lit_3 = (z = w)\ , \tag{7}$$

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Conjoining these terms gives us

$$\hat{Th}(\alpha) := \ \neg(x = y) \wedge (y = z)\ .$$

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- Let DEDUCTION be a procedure based on DP_T, which receives a conjunction of Σ-literals as input, and
  - decides whether it is satisfiable, and,
  - if the answer is negative, returns constraints over these literals.

function LAZY-BASIC(φ)
    $\mathcal{B} := e(\varphi)$;
    while (TRUE) **do**
        $\langle \alpha, res \rangle :=$ SAT-SOLVER($\mathcal{B}$);
        if $res =$ "Unsatisfiable" **then return** "Unsatisfiable";
        else
            $\langle t, res \rangle :=$ DEDUCTION($\hat{Th}(\alpha)$);
            if $res =$ "Satisfiable" **then return** "Satisfiable";
            $\mathcal{B} := \mathcal{B} \wedge e(t)$;

Consider the following three requirements from the clause t that is returned by DEDUCTION:

1. The formula t is T-valid, i.e., t is a tautology in T. For example, if T is the theory of equality, then $x = y \wedge y = z \longrightarrow x = z$ is T-valid.

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The second and third requirements are sufficient for guaranteeing termination.

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- Requirement 1: the clause t can be any formula that is implied by φ, and not just a T-valid formula.
- Requirement 2: the clause t may refer to atoms that do not appear in φ, as long as the number of such new atoms is finite.
  - For example, in equality logic, we may allow t to refer to all atoms of the form $x_i = x_j$ where x_i, x_j are variables in $var(\varphi)$, even if only some of these equality predicates appear in φ.

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- As a result, any conflict clause that is learned while solving $\mathcal{B}^i$ can be reused when solving $\mathcal{B}^j$ for $i < j$.
- This is a special case of incremental satisfiability.

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- This algorithm uses a procedure ADDCLAUSES, which adds new clauses to the current set of clauses at run time.

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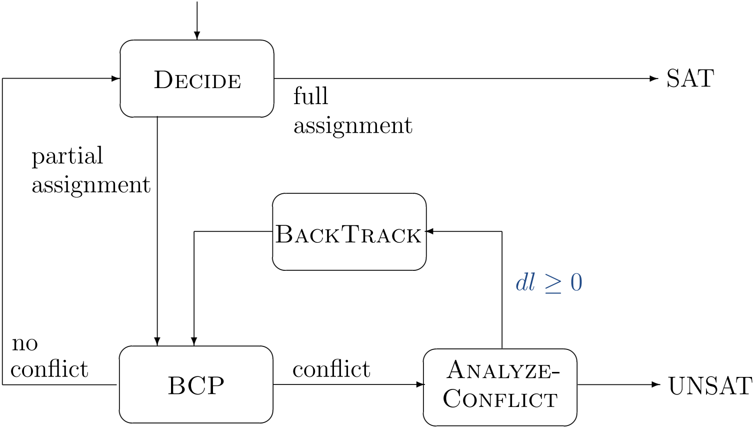
- A better option is to integrate DEDUCTION into the DPLL-SAT algorithm, as shown in the following algorithm.
- This algorithm uses a procedure ADDCLAUSES, which adds new clauses to the current set of clauses at run time.
- Before seeing this algorithm let us first recall DPLL...


Decide
SAT
full assignment
partial assignment
BackTrack
$dl \geq 0$
no conflict
BCP
conflict
Analyze-Conflict
UNSAT

2. Pseudo-code for DPLL

```
function DPLL
    if BCP() = "conflict" then return "Unsatisfiable";
    while (TRUE) do
        if ¬DECIDE() then return "Satisfiable";
        else
            while (BCP() = "conflict") do
                backtrack-level := ANALYZE-CONFLICT();
                if backtrack-level < 0 then return "Unsatisfiable";
                else BackTrack(backtrack-level);
```

function LAZY-DPLL
    ADDCLAUSES($cnf(e(\varphi))$);
    if BCP() = “conflict” **then return** “Unsatisfiable”;
    while (TRUE) **do**
        if $\neg$DECIDE() **then**   ▷ Full assignment
            $\langle t, res\rangle$:=DEDUCTION($\hat{Th}(\alpha)$);
            if res=“Satisfiable” **then return** “Satisfiable”;
            ADDCLAUSES($e(t)$);
            while (BCP() = “conflict”) **do**
                $backtrack\text{-}level$:= ANALYZE-CONFLICT();
                if $backtrack\text{-}level < 0$ **then return** “Unsatisfiable”;
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- Consider a formula φ that contains an integer variable x_1 and, among others, the literals $x_1 \geq 10$ and $x_1 < 0$.

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- Consider a formula φ that contains an integer variable x_1 and, among others, the literals $x_1 \geq 10$ and $x_1 < 0$.
- Assume that the DECIDE procedure assigns $e(x_1 \geq 10) \mapsto$ TRUE and $e(x_1 < 0) \mapsto$ TRUE.
- Inevitably, any call to DEDUCTION results in a contradiction between these two facts, independently of any other decisions that are made.

- However, the algorithms we saw so far do not call DEDUCTION until a full satisfying assignment is found.
  - Thus, the time taken to complete the assignment is wasted.

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  - Thus, the time taken to complete the assignment is wasted.

- Further, the refutation of this full assignment may be due to other reasons (i.e., a proof that a different subset of the assignment is contradictory).
  - Hence, additional assignments that include the same wrong assignment to $e(x_1 \geq 10)$ and $e(x_1 < 0)$ are not ruled out.

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1. Contradictory partial assignments are ruled out early.

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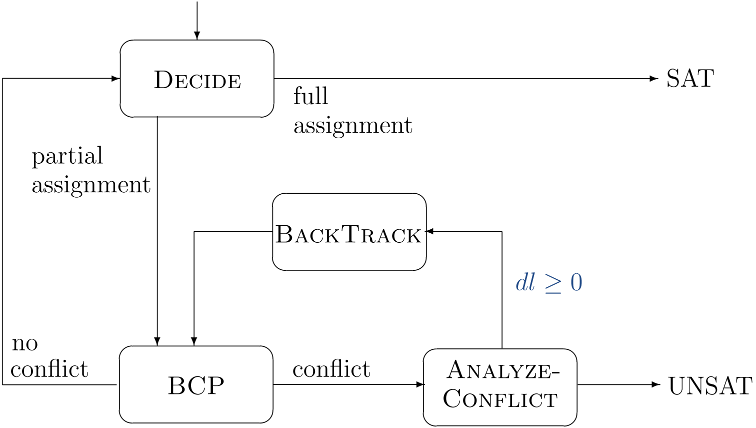
1. Contradictory partial assignments are ruled out early.
2. Allows theory propagation.
   - Continuing our example, once $e(x_1 \geq 10)$ has been assigned TRUE, we can infer that $e(x_1 < 0)$ must be FALSE and avoid the conflict altogether.

This brings us to the next version of the algorithm, called $\mathrm{DPLL}(T)$.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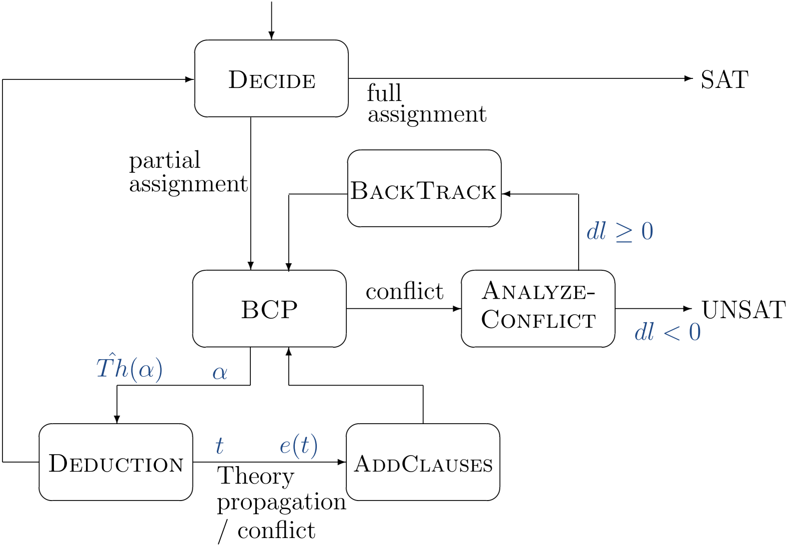

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$e(t)$
ADDCLAUSES
Theory propagation / conflict

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        until $t \equiv$ TRUE

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1. The clause $e(t)$ is an asserting clause under α. This implies that the addition of $e(t)$ to $\mathcal{B}$ and a call to BCP leads to an assignment to the encoder of some literal.
2. When $\mathrm{D{\small EDUCTION}}$ cannot find an asserting clause t as defined above, t and $e(t)$ are equivalent to TRUE.

The second case occurs, for example, when all the Boolean variables are already assigned, and thus the formula is found to be satisfiable.

3. Theory Propagation

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- Partial / Exhaustive theory propagation – propagate all that is implied by the current partial assignment.
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Exhaustive theory propagation after each assignment: what does this mean ?

That's right, no possible conflicts on the theory side.

How to check whether a predicate p is implied by $\hat{Th}(\alpha)$?

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- Theory-specific propagation. For example, in equality logic build the equality graph corresponding to $Th(\alpha)$. Infer equalities/disequalities from the graph.

How to check whether a predicate p is implied by $\hat{Th}(\alpha)$?

- Plunging – is $\hat{Th}(\alpha) \wedge \neg p$ satisfiable ?
- Theory-specific propagation. For example, in equality logic build the equality graph corresponding to $Th(\alpha)$. Infer equalities/disequalities from the graph.
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- How much propagation is cost-effective is a subject for research, and depends on T.

- Normally theory propagation is done by transferring clauses to the the DPLL solver.

- It turns out to be inefficient – few (less than 0.5%) are actually used.

- Instead – add implied literals directly to the implication stack.
  - This causes a problem in ANALYZE-CONFLICT() – can you see what problem ?

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- Theory propagation without clauses breaks this mechanism – there are implications without antecedents.
- Solution – DP_T should be able to explain an implication post-mortem, in the form of a clause.

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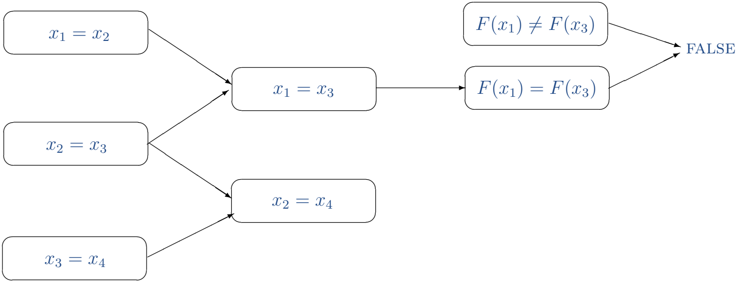
- Solution: analyze the reason for unsatisfiability.
  Build lemma accordingly.


$x_1 = x_2$
$x_2 = x_3$
$x_3 = x_4$
$x_1 = x_3$
$x_2 = x_4$
$F(x_1) \neq F(x_3)$
$F(x_1) = F(x_3)$
FALSE