Theorem Proving, January–April 2014

Assignment 1, 21 March 2014
Due Monday, 7 April 2014

Instructions for submitting solutions

Please submit your solutions electronically by email to madhavan@cmi.ac.in *to facilitate evaluation.*

Preferrably, send an electronic document in PDF (you can generate it in LaTeX or OpenOffice or Word or whatever, but send only PDF). If you can't do this, send scanned copies of handwritten pages.

The numbers quoted with each question refer to the book *Term Rewriting and All That* by Franz Baader and Tobias Nipkow. Cross check if you think there are typos!

1. **Exercise 2.6**

   A relation $\rightarrow$ is called **bounded** iff for each element the length of all paths starting from it is bounded: $\forall x.\ \exists n.\ \nexists y.\ x \xrightarrow{n} y$.

   (a) Is every terminating relation bounded?

   (b) Show that a finitely branching relation terminates iff it is bounded.

2. **Exercise 2.14**

   Show that $>^*_{lex}$ is linear if $>$ is.

3. **Exercise 2.15**

   Why do the following two programs terminate, provided all variables range over positive natural numbers?

   while $m \neq n$ **do**
     if $m > n$ **then** $m := m - n$ **else** $n := n - m$

   while $m \neq n$ **do**
     if $m > n$ **then** $m := m - n$
     else begin $h := m$; $m := n$; $n := h$ **end**

   What if the variables range over positive rational numbers?

4. **Exercise 2.31**

   Does strong confluence imply the following property?

   $$y_1 \leftarrow z \rightarrow y_2 \Rightarrow \exists z.\ y_1 \xrightarrow{=} z \xleftarrow{=} y_2.$$

   Give a proof or counterexample.

5. **Exercise 5.13**

   Let $>$ be a rewrite orer. Show that the subterm property folllows from the following simpler property:

   $$f(\ldots, x, \ldots) > x \text{ for all } f \in \Sigma \text{ and all } x \in V.$$

6. **Exercise 5.17**

   Show that the TRS $R := \{f(f(x)) \to f(g(f(x)))\}$ is terminating.

7. **Exercise 5.22**

   Show that it is not possible to prove termination of the term rewriting system $R := \{f(f(x)) \to g(x), g(g(x)) \to f(x)\}$ with the help of a lexicographic path order.

8. **Exercise 5.25**

   Show that "$s >_{lpo} t$" can be decided in time $O(|s| \cdot |t|)$.

9. **Exercise 6.3**

   Find r_1 and r_2 such that $\{f(g(x)) \to r_1, g(h(x)) \to r_2\}$ is confluent.

10. **Exercise 6.6**

    Show that the following system is convergent:

    $$\begin{array}{ll} f(f(x)) \to f(x), & f(g(x)) \to g(x), \\ g(g(x)) \to f(x), & g(f(x)) \to g(x). \end{array}$$

    Can you determine the normal form of a term as a function of the numbers of fs and gs in it?

11. **Exercise 7.1**

    Consider the following sets of identities:

    $$E_1 := \{f(g(f(x))) \approx x\} \text{ and } E_2 := \{f(g(f(x))) \approx f(g(x))\}.$$

    Choose an appropriate reduction order $>$ and apply the basic completion procedure to the input $(E_i, >), (i = 1, 2)$¿

12. **Exercise 7.2**

    Show that the TRS

    $$\{(x * y) * (y * x) \to y, x * ((x * y) * z) \to x * y, (x * (y * z)) * z) \to y * z\}$$

    is confluent.
