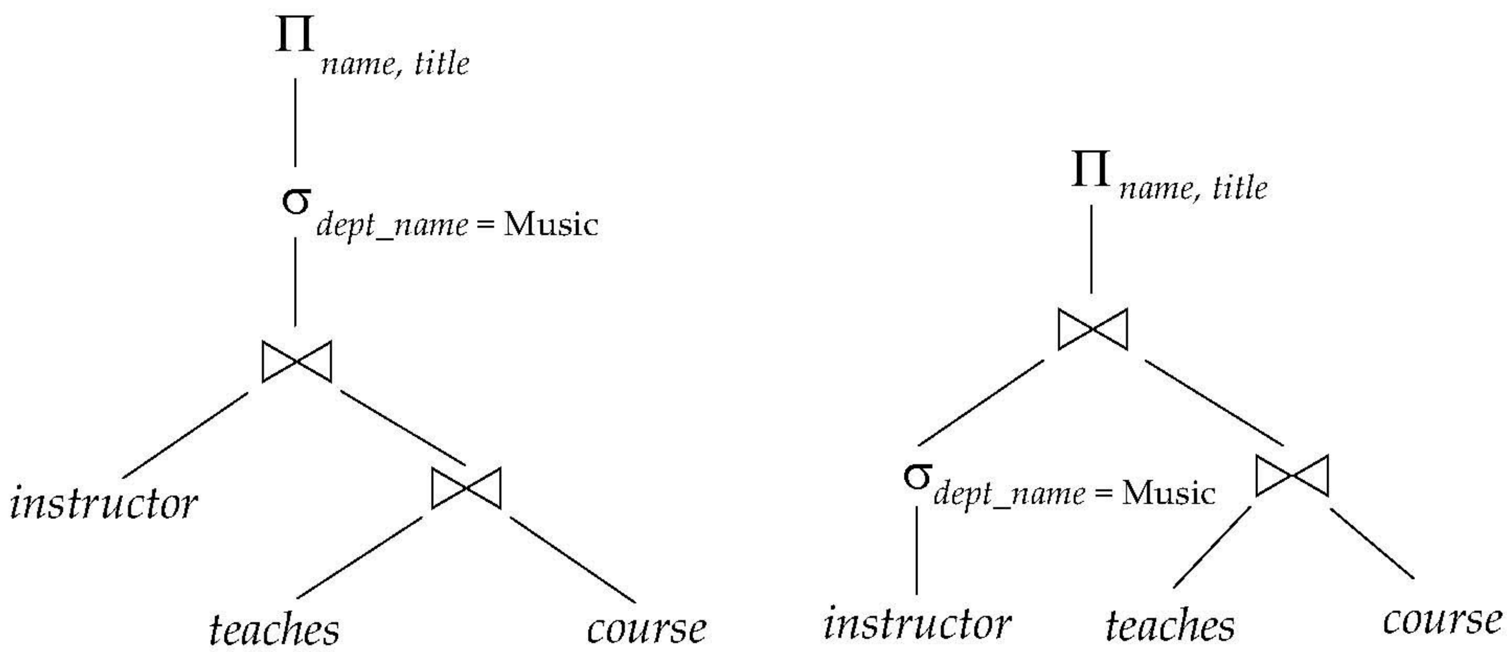
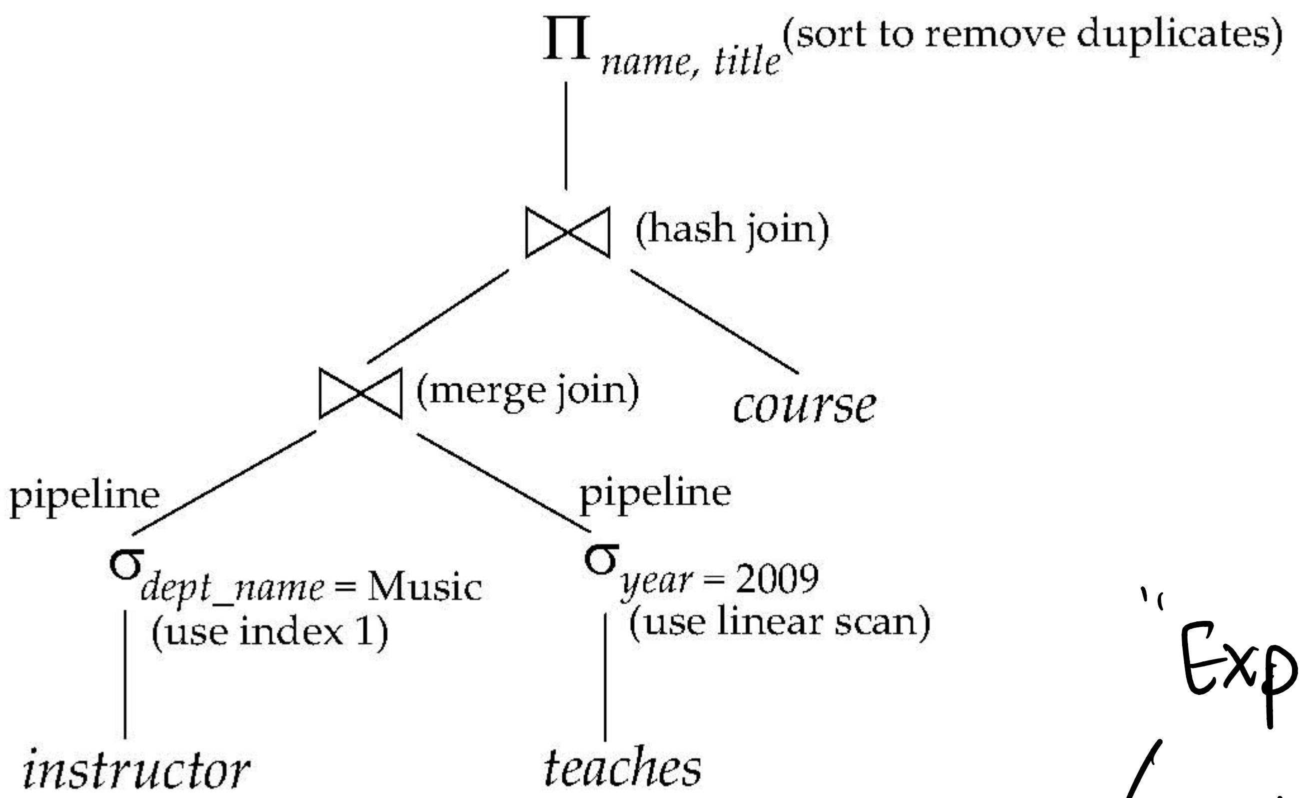
Introduction

- Alternative ways of evaluating a given query
  - Equivalent expressions
  - Different algorithms for each operation



Introduction (Cont.)

- An **evaluation plan** defines exactly what algorithm is used for each operation, and how the execution of the operations is coordinated.



"Explain"
✓ in MySQL

- Find out how to view query execution plans on your favorite database

Introduction (Cont.)

- Cost difference between evaluation plans for a query can be enormous
  - E.g. seconds vs. days in some cases
- Steps in **cost-based query optimization**
  1. Generate logically equivalent expressions using **equivalence rules**
  2. Annotate resultant expressions to get alternative query plans
  3. Choose the cheapest plan based on **estimated cost**
- Estimation of plan cost based on:
  - Statistical information about relations. Examples:
    - number of tuples, number of distinct values for an attribute
  - Statistics estimation for intermediate results
    - to compute cost of complex expressions
  - Cost formulae for algorithms, computed using statistics

Measures of Query Cost

- Cost is generally measured as total elapsed time for answering query
  - Many factors contribute to time cost
    - *disk accesses, CPU*, or even network *communication*
- Typically disk access is the predominant cost, and is also relatively easy to estimate. Measured by taking into account
  - Number of seeks * average-seek-cost
  - Number of blocks read * average-block-read-cost
  - Number of blocks written * average-block-write-cost
    - Cost to write a block is greater than cost to read a block
      - data is read back after being written to ensure that the write was successful

Measures of Query Cost (Cont.)

- For simplicity we just use the **number of block transfers** *from disk and the* **number of seeks** as the cost measures
  - t_T – time to transfer one block
  - t_S – time for one seek
  - Cost for b block transfers plus S seeks
    $b * t_T + S * t_S$
- We ignore CPU costs for simplicity
  - Real systems do take CPU cost into account
- We do not include cost to writing output to disk in our cost formulae

Join Operation

- Several different algorithms to implement joins
  - Nested-loop join
  - Block nested-loop join
  - Indexed nested-loop join
  - Merge-join
  - Hash-join
- Choice based on cost estimate
- Examples use the following information
  - Number of records of *student*: 5,000 *takes*: 10,000
  - Number of blocks of *student*: 100 *takes*: 400

50 rows per block

25 rows per block

Nested-Loop Join

- To compute the theta join $r \bowtie_{\theta} s$

  for each tuple t_r **in** r **do begin**

      for each tuple t_s **in** s **do begin**

          test pair (t_r, t_s) to see if they satisfy the join condition θ

          if they do, add $t_r \cdot t_s$ to the result.] Not counted in calculation

      end

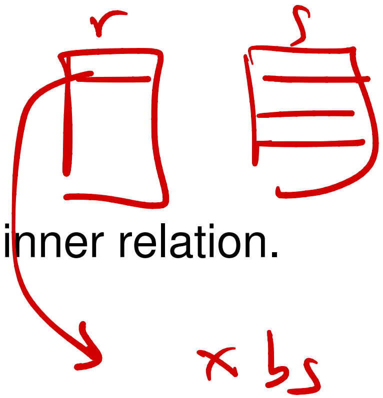
  end
- r is called the **outer relation** and s the **inner relation** of the join.
- Requires no indices and can be used with any kind of join condition.
- Expensive since it examines every pair of tuples in the two relations.

Nested-Loop Join (Cont.)

- In the worst case, if there is enough memory only to hold one block of each relation, the estimated cost is seeks in r

  seeks in s $n_r * b_s + b_r$ block transfers, plus

  $n_r + b_r$ seeks $n_r * b_s + b_r$



- If the smaller relation fits entirely in memory, use that as the inner relation.
  - Reduces cost to $b_r + b_s$ block transfers and 2 seeks
- Assuming worst case memory availability cost estimate is
  - with *student* as outer relation:
    - 5000 ∗ 400 + 100 = 2,000,100 block transfers,
    - 5000 + 100 = 5100 seeks
  - with *takes* as the outer relation
    - 10000 ∗ 100 + 400 = 1,000,400 block transfers and 10,400 seeks
- If smaller relation (*student*) fits entirely in memory, the cost estimate will be 500 block transfers.
- Block nested-loops algorithm (next slide) is preferable.

Block Nested-Loop Join

- Variant of nested-loop join in which every block of inner relation is paired with every block of outer relation.

for each block B_r **of** r **do begin**
    for each block B_s **of** s **do begin**
        for each tuple t_r **in** B_r **do begin**
            for each tuple t_s **in** B_s **do begin**
                Check if (t_r, t_s) satisfy the join condition
                if they do, add $t_r \cdot t_s$ to the result.
            end
        end
    end
end

$b_r * b_s + b_r$

vs

$n_r * b_s + b_r$

Block Nested-Loop Join (Cont.)

- Worst case estimate: $b_r \ast b_s + b_r$ block transfers + 2 * b_r seeks
  - Each block in the inner relation *s* is read once for each *block* in the outer relation
- Best case: $b_r + b_s$ block transfers + 2 seeks.
- Improvements to nested loop and block nested loop algorithms:
  - In block nested-loop, use M — 2 disk blocks as blocking unit for outer relations, where M = memory size in blocks; use remaining two blocks to buffer inner relation and output
    - Cost = $\lceil b_r / (M\text{-}2) \rceil \ast b_s + b_r$ block transfers + $2 \lceil b_r / (M\text{-}2) \rceil$ seeks
  - If equi-join attribute forms a key or inner relation, stop inner loop on first match
  - Scan inner loop forward and backward alternately, to make use of the blocks remaining in buffer (with LRU replacement)
  - Use index on inner relation if available (next slide)

Indexed Nested-Loop Join

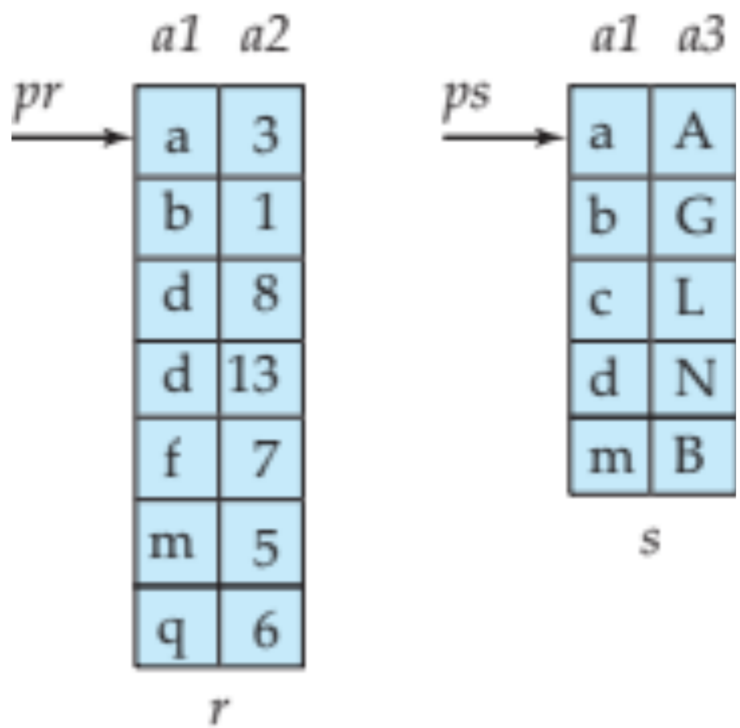
- Index lookups can replace file scans if
  - join is an equi-join or natural join and
  - an index is available on the inner relation's join attribute
    - Can construct an index just to compute a join.
- For each tuple t_r in the outer relation *r,* use the index to look up tuples in *s* that satisfy the join condition with tuple t_r.
- Worst case: buffer has space for only one page of *r*, and, for each tuple in *r*, we perform an index lookup on *s.*
- Cost of the join: $b_r (t_T + t_S) + n_r * c$
  - Where *c* is the cost of traversing index and fetching all matching *s* tuples for one tuple or *r*
  - *c* can be estimated as cost of a single selection on *s* using the join condition.
- If indices are available on join attributes of both *r* and *s,* use the relation with fewer tuples as the outer relation.

Example of Nested-Loop Join Costs

- Compute *student* ⋈ *takes,* with *student* as the outer relation.
- Let *takes* have a primary B^+-tree index on the attribute *ID,* which contains 20 entries in each index node.
- Since *takes* has 10,000 tuples, the height of the tree is 4, and one more access is needed to find the actual data
- *student* has 5000 tuples
- Cost of block nested loops join
  - 400*100 + 100 = 40,100 block transfers + 2 * 100 = 200 seeks
    - assuming worst case memory
    - may be significantly less with more memory
- Cost of indexed nested loops join
  - 100 + 5000 * 5 = 25,100 block transfers and seeks.
  - CPU cost likely to be less than that for block nested loops join

Merge-Join

1. Sort both relations on their join attribute (if not already sorted on the join attributes).
2. Merge the sorted relations to join them
   1. Join step is similar to the merge stage of the sort-merge algorithm.
   2. Main difference is handling of duplicate values in join attribute — every pair with same value on join attribute must be matched
   3. Detailed algorithm in book



Merge-Join (Cont.)

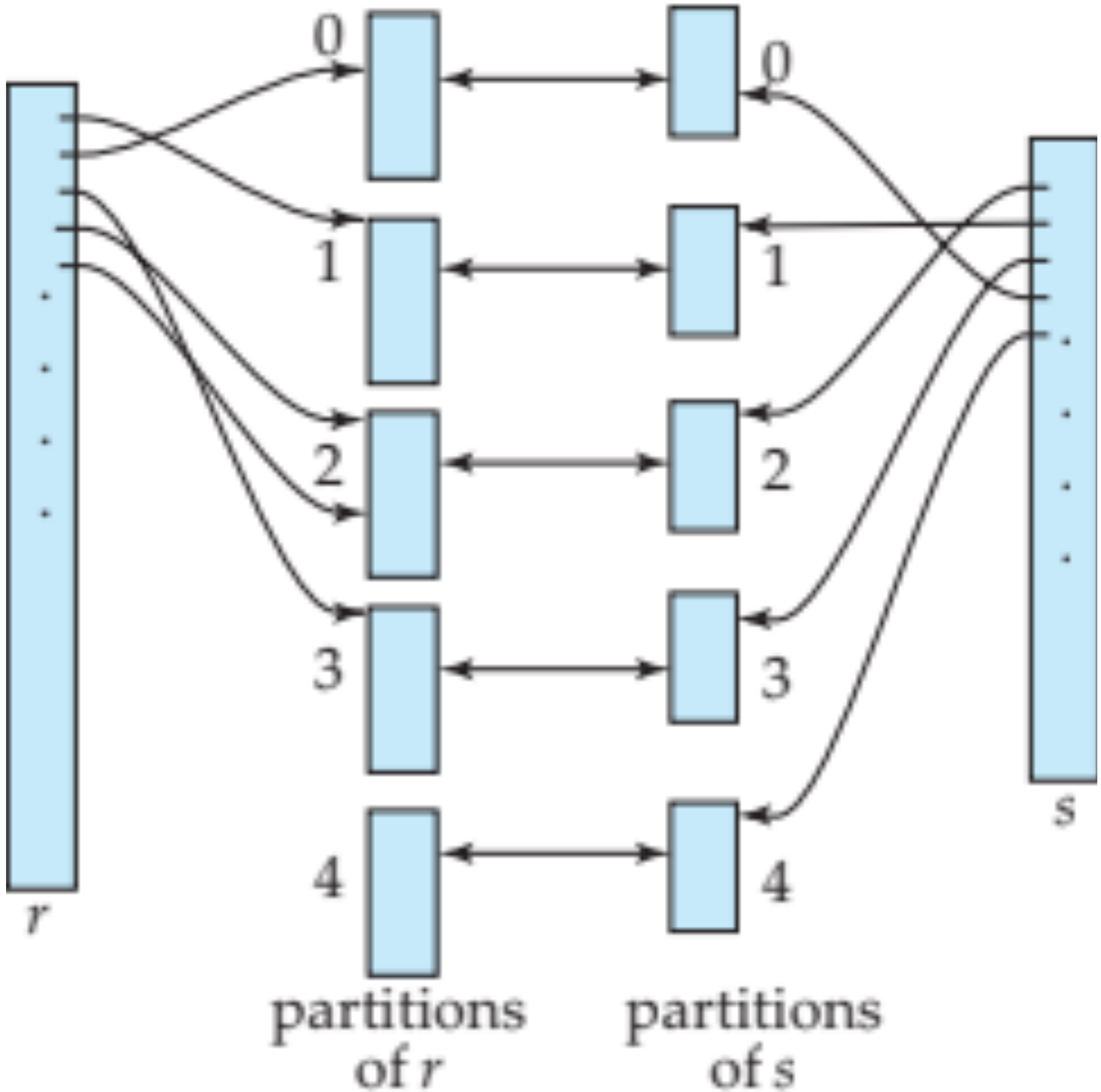
- Can be used only for equi-joins and natural joins
- Each block needs to be read only once (assuming all tuples for any given value of the join attributes fit in memory
- Thus the cost of merge join is:
  $b_r + b_s$ block transfers $+ \lceil b_r / b_b \rceil + \lceil b_s / b_b \rceil$ seeks
  - + the cost of sorting if relations are unsorted.
- **hybrid merge-join:** If one relation is sorted, and the other has a secondary B^+-tree index on the join attribute
  - Merge the sorted relation with the leaf entries of the B^+-tree .
  - Sort the result on the addresses of the unsorted relation's tuples
  - Scan the unsorted relation in physical address order and merge with previous result, to replace addresses by the actual tuples
    - Sequential scan more efficient than random lookup

Hash-Join

- Applicable for equi-joins and natural joins.
- A hash function *h* is used to partition tuples of both relations
- *h* maps *JoinAttrs* values to {0, 1, ..., *n*}, where *JoinAttrs* denotes the common attributes of *r* and *s* used in the natural join.
  - r_0, r_1, . . ., r_n denote partitions of *r* tuples
    - Each tuple $t_r \in r$ is put in partition r_i where $i = h(t_r[JoinAttrs])$.
  - r_0,, r_1. . ., r_n denotes partitions of *s* tuples
    - Each tuple $t_s \in s$ is put in partition s_i, where $i = h(t_s[JoinAttrs])$.

- *Note:* In book, r_i is denoted as H_{ri}, s_i is denoted as H_{si} and *n* is denoted as n_h.

Hash-Join (Cont.)



$x \neq y$
$h(x) = h(y)$ ✓

$h(x) \neq h(y)$
$\Rightarrow x \neq y$
✓

Hash-Join (Cont.)

- r tuples in r_i need only to be compared with s tuples in s_i

  Need not be compared with s tuples in any other partition, since:
  - an r tuple and an s tuple that satisfy the join condition will have the same value for the join attributes.
  - If that value is hashed to some value i, the r tuple has to be in r_i and the s tuple in s_i.

Transformation of Relational Expressions

- Two relational algebra expressions are said to be **equivalent** if the two expressions generate the same set of tuples on every *legal* database instance
  - Note: order of tuples is irrelevant
  - we don't care if they generate different results on databases that violate integrity constraints
- In SQL, inputs and outputs are multisets of tuples
  - Two expressions in the multiset version of the relational algebra are said to be equivalent if the two expressions generate the same multiset of tuples on every legal database instance.
- An **equivalence rule** says that expressions of two forms are equivalent
  - Can replace expression of first form by second, or vice versa

Equivalence Rules

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections.

$$\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

2. Selection operations are commutative.

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

3. Only the last in a sequence of projection operations is needed, the others can be omitted.

$$\Pi_{L_1}(\Pi_{L_2}(\ldots(\Pi_{Ln}(E))\ldots)) = \Pi_{L_1}(E)$$

4. Selections can be combined with Cartesian products and theta joins.

   α. $\sigma_{\theta}(E_1 \times E_2) = E_1 \bowtie_{\theta} E_2$

   β. $\sigma_{\theta 1}(E_1 \bowtie_{\theta 2} E_2) = E_1 \bowtie_{\theta 1 \wedge \theta 2} E_2$

Equivalence Rules (Cont.)

5. Theta-join operations (and natural joins) are commutative.

$$E_1 \bowtie_\theta E_2 = E_2 \bowtie_\theta E_1$$

6. (a) Natural join operations are associative:

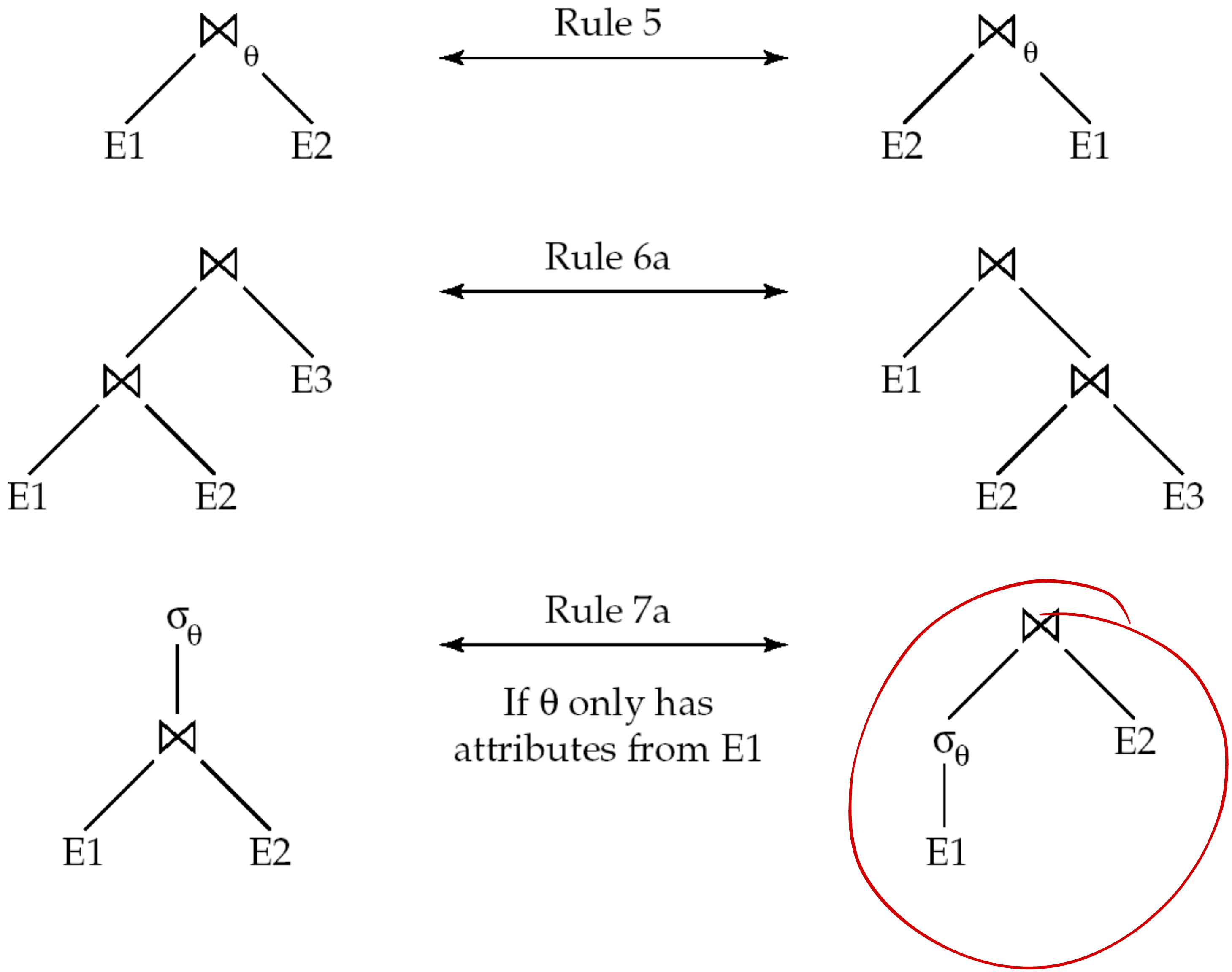
$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$$

(b) Theta joins are associative in the following manner:

$$(E_1 \bowtie_{\theta 1} E_2) \bowtie_{\theta 2 \wedge \theta 3} E_3 = E_1 \bowtie_{\theta 1 \wedge \theta 3} (E_2 \bowtie_{\theta 2} E_3)$$

where θ_2 involves attributes from only E_2 and E_3.

Pictorial Depiction of Equivalence Rules



Equivalence Rules (Cont.)

7. The selection operation distributes over the theta join operation under the following two conditions:
   (a) When all the attributes in θ_0 involve only the attributes of one of the expressions (E_1) being joined.

$$\sigma_{\theta 0}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta 0}(E_1)) \bowtie_{\theta} E_2$$

   (b) When θ_1 involves only the attributes of E_1 and θ_2 involves only the attributes of E_2.

$$\sigma_{\theta 1 \wedge \theta 2}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta 1}(E_1)) \bowtie_{\theta} (\sigma_{\theta 2}(E_2))$$

Equivalence Rules (Cont.)

8. The projection operation distributes over the theta join operation as follows:

   (a) if θ involves only attributes from $L_1 \cup L_2$:

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_{\theta} E_2) = (\Pi_{L_1}(E_1)) \bowtie_{\theta} (\Pi_{L_2}(E_2))$$

   (b) Consider a join $E_1 \bowtie_{\theta} E_2$.

   - Let L_1 and L_2 be sets of attributes from E_1 and E_2, respectively.
   - Let L_3 be attributes of E_1 that are involved in join condition θ, but are not in $L_1 \cup L_2$, and
   - let L_4 be attributes of E_2 that are involved in join condition θ, but are not in $L_1 \cup L_2$.

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_{\theta} E_2) = \Pi_{L_1 \cup L_2}((\Pi_{L_1 \cup L_3}(E_1)) \bowtie_{\theta} (\Pi_{L_2 \cup L_4}(E_2)))$$

Equivalence Rules (Cont.)

9. The set operations union and intersection are commutative

$$E_1 \cup E_2 = E_2 \cup E_1$$
$$E_1 \cap E_2 = E_2 \cap E_1$$

    9. (set difference is not commutative).

10. Set union and intersection are associative.

$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3)$$
$$(E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

9. The selection operation distributes over $\cup$, $\cap$ and $-$.

$$\sigma_\theta (E_1 - E_2) = \sigma_\theta (E_1) - \sigma_\theta(E_2)$$

and similarly for $\cup$ and $\cap$ in place of $-$

Also: $$\sigma_\theta (E_1 - E_2) = \sigma_\theta(E_1) - E_2$$

and similarly for $\cap$ in place of $-$, but not for $\cup$

12. The projection operation distributes over union

$$\Pi_L(E_1 \cup E_2) = (\Pi_L(E_1)) \cup (\Pi_L(E_2))$$

Transformation Example: Pushing Selections

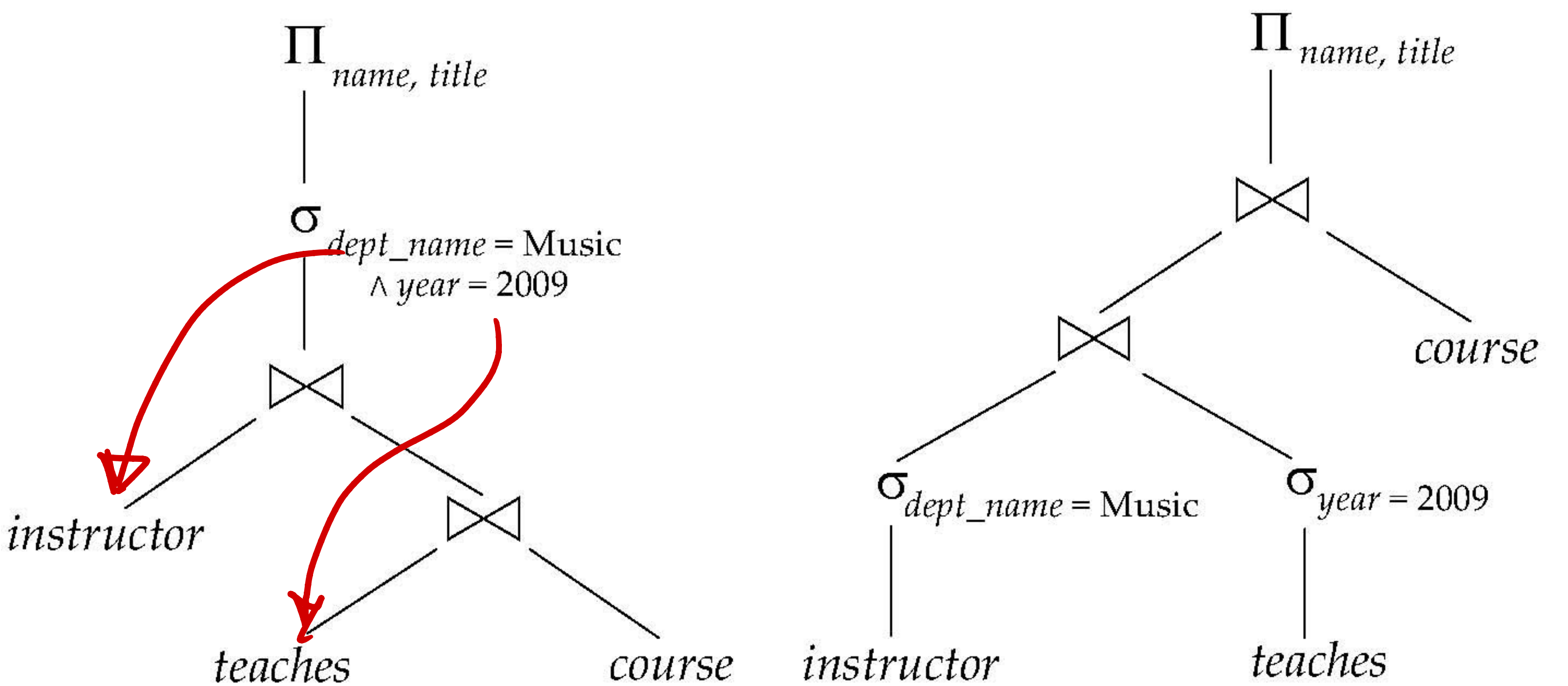
- Query: Find the names of all instructors in the Music department, along with the titles of the courses that they teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"}}(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course))))$
- Transformation using rule 7a.
  - $\Pi_{name,\ title}((\sigma_{dept\_name=\text{"Music"}}(instructor)) \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course)))$
- Performing the selection as early as possible reduces the size of the relation to be joined.

Example with Multiple Transformations

- Query: Find the names of all instructors in the Music department who have taught a course in 2009, along with the titles of the courses that they taught
  - $\Pi_{name,\ title}(\sigma_{dept\_name= \text{“Music”} \wedge year = 2009}$ $(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}\ (course))))$
- Transformation using join associatively (Rule 6a):
  - $\Pi_{name,\ title}(\sigma_{dept\_name= \text{“Music”} \wedge year = 2009}$ $((instructor \bowtie teaches) \bowtie \Pi_{course\_id,\ title}\ (course)))$
- Second form provides an opportunity to apply the “perform selections early” rule, resulting in the subexpression

$$\sigma_{dept\_name\ =\ \text{“Music”}}\ (instructor) \bowtie \sigma_{\ year\ =\ 2009}\ (teaches)$$

Multiple Transformations (Cont.)



(a) Initial expression tree

(b) Tree after multiple transformations

Transformation Example: Pushing Projections

- Consider: $\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches) \bowtie \Pi_{course\_id,\ title}(course))))$
- When we compute

  $(\sigma_{dept\_name = \text{"Music"}}(instructor \bowtie teaches)$

  we obtain a relation whose schema is:
  (*ID, name, dept_name, salary, course_id, sec_id, semester, year)*
- Push projections using equivalence rules 8a and 8b; eliminate unneeded attributes from intermediate results to get:

  $$\Pi_{name,\ title}(\Pi_{name,\ course\_id}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches)) \bowtie \Pi_{course\_id,\ title}(course))))$$
- Performing the projection as early as possible reduces the size of the relation to be joined.

Join Ordering Example

- For all relations r_1, r_2 and r_3,

  $$(r_1 \bowtie r_2) \bowtie r_3 = r_1 \bowtie (r_2 \bowtie r_3)$$

  (Join Associativity)

- If $r_2 \bowtie r_3$ is quite large and $r_1 \bowtie r_2$ is small, we choose

  $$(r_1 \bowtie r_2) \bowtie r_3$$

  so that we compute and store a smaller temporary relation.

Join Ordering Example (Cont.)

- Consider the expression

  $\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Music”}}(instructor) \quad teaches) \bowtie$

  $\bowtie \ \Pi_{course\_id,\ title}(course))))$

- Could compute $teaches \bowtie \Pi_{course\_id,\ title}(course)$ first, and join result with

  $\sigma_{dept\_name=\text{“Music”}}(instructor)$

  but the result of the first join is likely to be a large relation.

- Only a small fraction of the university's instructors are likely to be from the Music department
  - it is better to compute

    $\sigma_{dept\_name=\text{“Music”}}(instructor) \quad teaches \bowtie$

    first.

Enumeration of Equivalent Expressions

- Query optimizers use equivalence rules to **systematically** generate expressions equivalent to the given expression
- Can generate all equivalent expressions as follows:
  - Repeat
    - apply all applicable equivalence rules on every subexpression of every equivalent expression found so far
    - add newly generated expressions to the set of equivalent expressions

    Until no new equivalent expressions are generated above
- The above approach is very expensive in space and time
  - Two approaches
    - Optimized plan generation based on transformation rules
    - Special case approach for queries with only selections, projections and joins

Choice of Evaluation Plans

- Must consider the interaction of evaluation techniques when choosing evaluation plans
  - choosing the cheapest algorithm for each operation independently may not yield best overall algorithm. E.g.
    - merge-join may be costlier than hash-join, but may provide a sorted output which reduces the cost for an outer level aggregation.
    - nested-loop join may provide opportunity for pipelining
- Practical query optimizers incorporate elements of the following two broad approaches:
  1. Search all the plans and choose the best plan in a cost-based fashion.
  2. Uses heuristics to choose a plan.

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