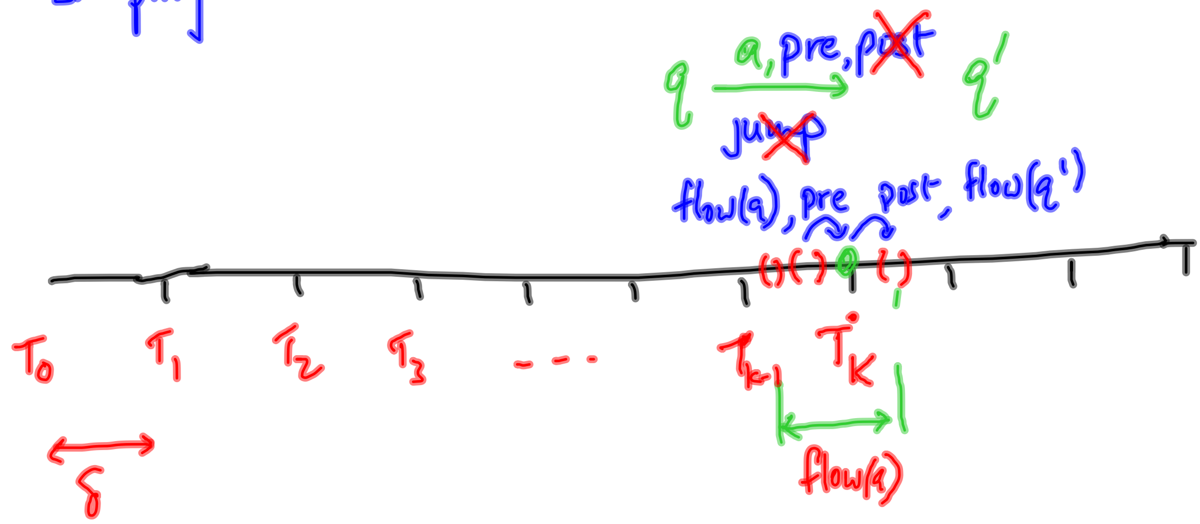
Lazy Hybrid Automata

- Reading values & effecting changes take time
- Sampling values at discrete time intervals.



n vars : $\{x_1, \ldots, x_n\} = X$

$\mathcal{A} = (Q, Act, q_{in}, V_{in}, D, \{\rho_q\}_{q \in Q}, B, \rightarrow)$

- D: delays
- $\{\rho_q\}_{q \in Q}$: flows at q
- B: bounds on value

$\rightarrow \subseteq Q \times Act \times I^n \times Q$

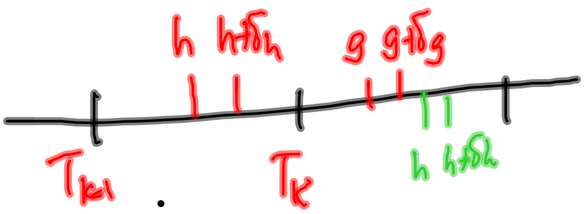
I^n: product of n intervals within B

$\rho_q : X \rightarrow \mathbb{Q}$

$B = [B_{min}, B_{max}]$

$D = \{g, \delta_g, h, \delta_h\}$

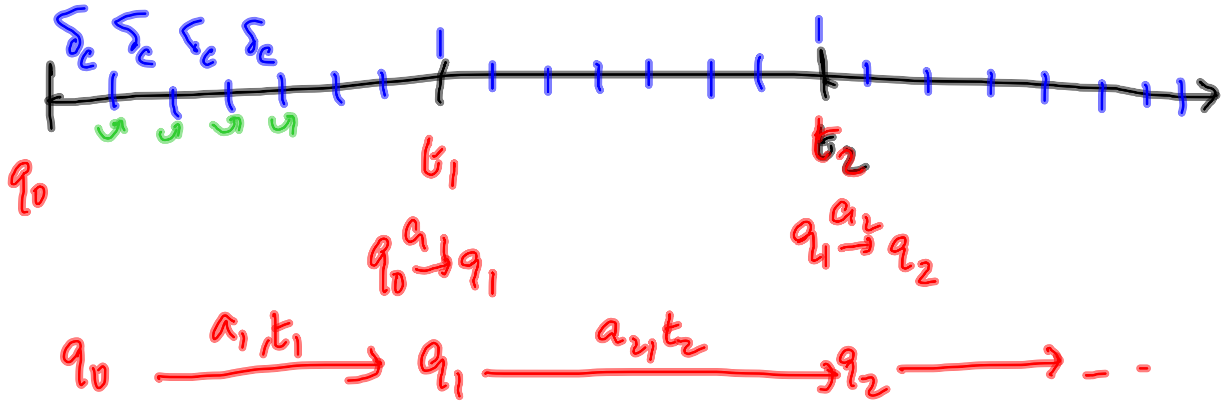
$g < g+\delta_g < h < h+\delta_h$



Simplifying assumptions: $q \xrightarrow{a,I} q'$, $q \neq q'$

Add a silent move $q \xrightarrow{\tau} q'$ where $q = q'$ to denote time elapsing

Delay of one unit δ_c



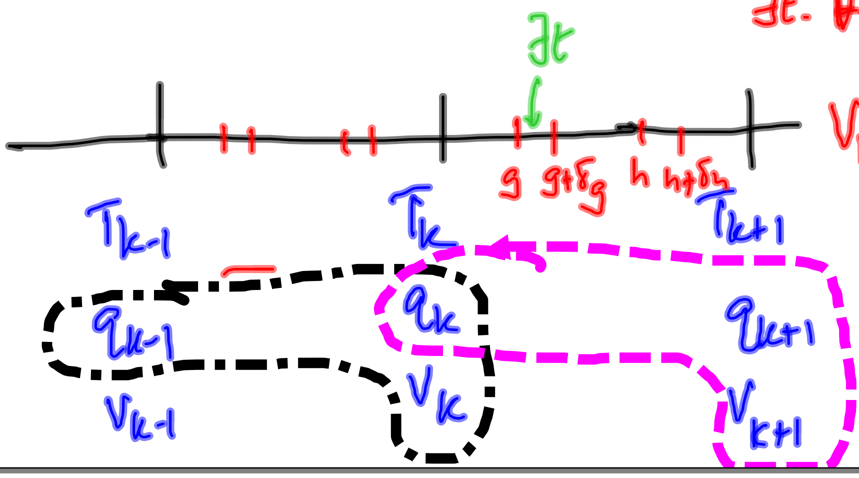
Record values precisely at T_k points

At T_k state : q_k

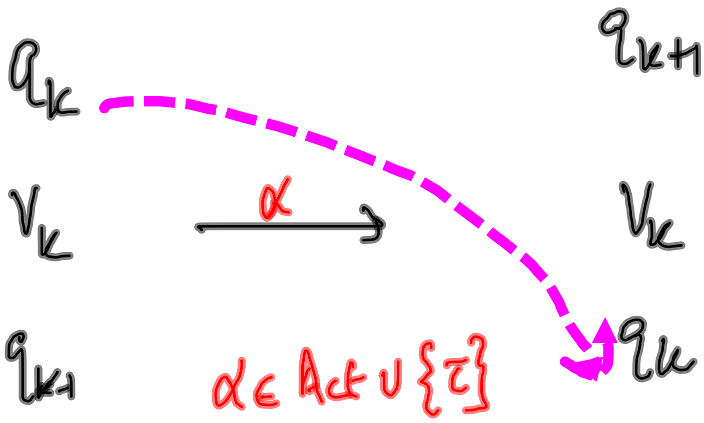
valuation: $V_k \in B^n$

$\in [g, g+\delta_g]$

$\exists t.\ \forall x_i$:



$V_{k+1} = (\delta_{q_{k-1}})(x_i) \cdot t + (\delta_{q_k})(x_i)(1-t)$



$\alpha = \tau$: V_{k+1} follows from V_k as given before

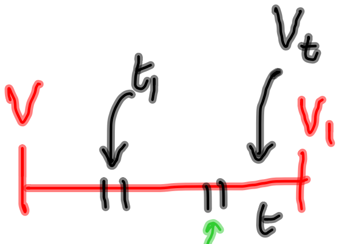
$q_{k+1} = q_k$

$$(q, V, q') \xrightarrow{\alpha \in Act} (q_1, V_1, q_1')$$

current prev → current prev

- $q_1' = q$
- $\exists \; q \xrightarrow{\alpha, I} q_1$ s.t.

  I is true at some t_2 in



$$V_{t_2} = V + \delta_{q'}(x) \cdot t_1 + \delta_q(x)(t_2 - t_1) \text{ satisfies } I$$

OLD CURR

$$V_1 = V + \delta_{q'} t_1 + \delta_q (1 - t_1)$$

Start at (q_{in}, V_{in}) $\longrightarrow$ Initial config (q_{in}, V_{in}, q_{in})

Assume $\delta_{q_{in}}$ holds from $t=0$

$(q_{in}, V_{in}, q_{in}) \xrightarrow{\alpha_1} (q_1, V_1, q_{in}) \xrightarrow{\alpha_2} (q_2, V_2, q_1) \ldots$

$L_{st}(A)$: state language of A — Regular over Q

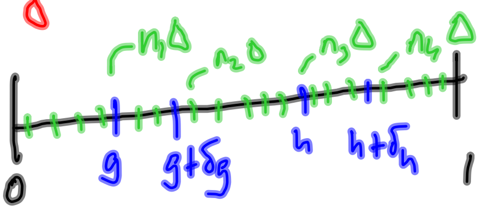
$L_{act}(A)$: action language of A — Regular over $Act \cup \{\tau\}$

Quotient space of configurations (i.e. space of valuations) into a finite number of equivalence classes

Find Γ s.t. all interesting values in evolution of δ are integral multiples of $\Gamma \in \mathbb{Q}$

Start with $D = \{g, h, \delta_g, \delta_h\}$

Δ is largest rational that divides all of these



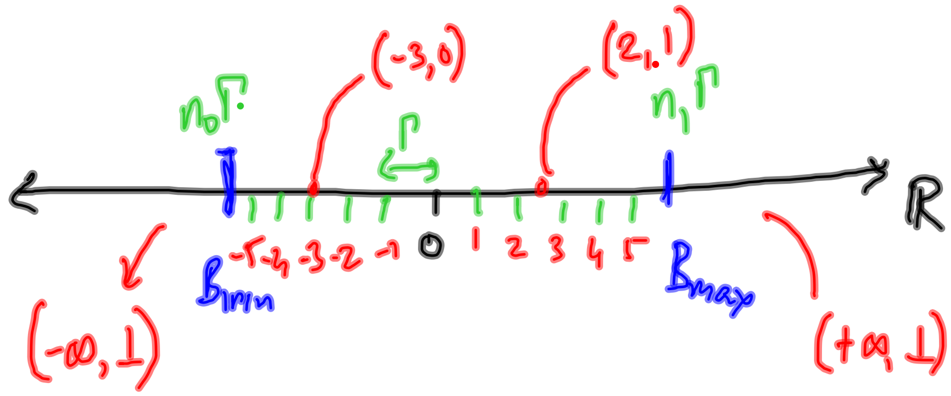
Set Γ to gcd $\{\delta_q \cdot \Delta \mid q \in Q, x \in X\}$

$\cup \{B_{min}, B_{max}\}$

$\cup \{l, r \mid (q, a, [l, r], q') \in \rightarrow\}$.

Analysis for $X = \{x_1\}$

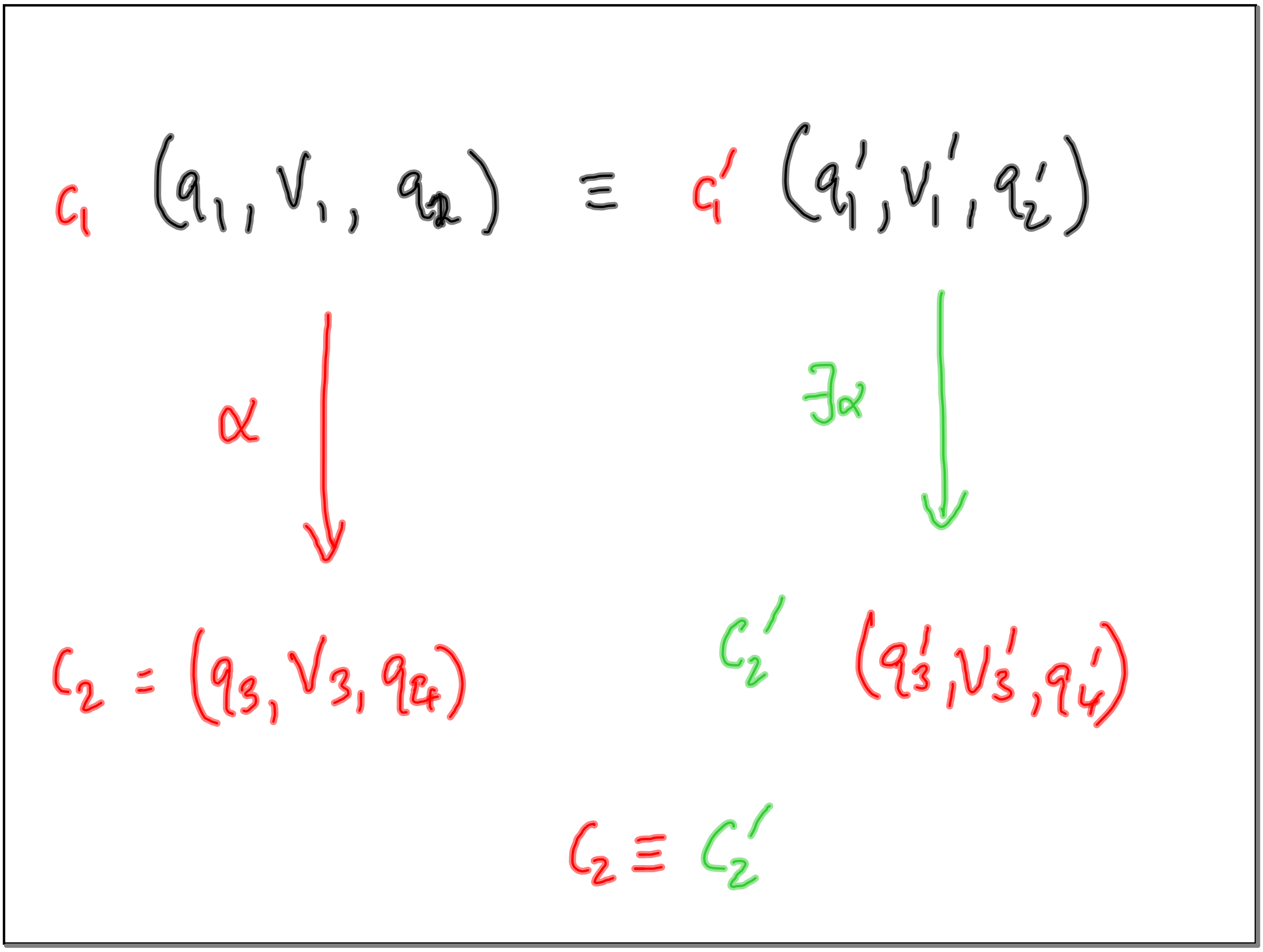
Given

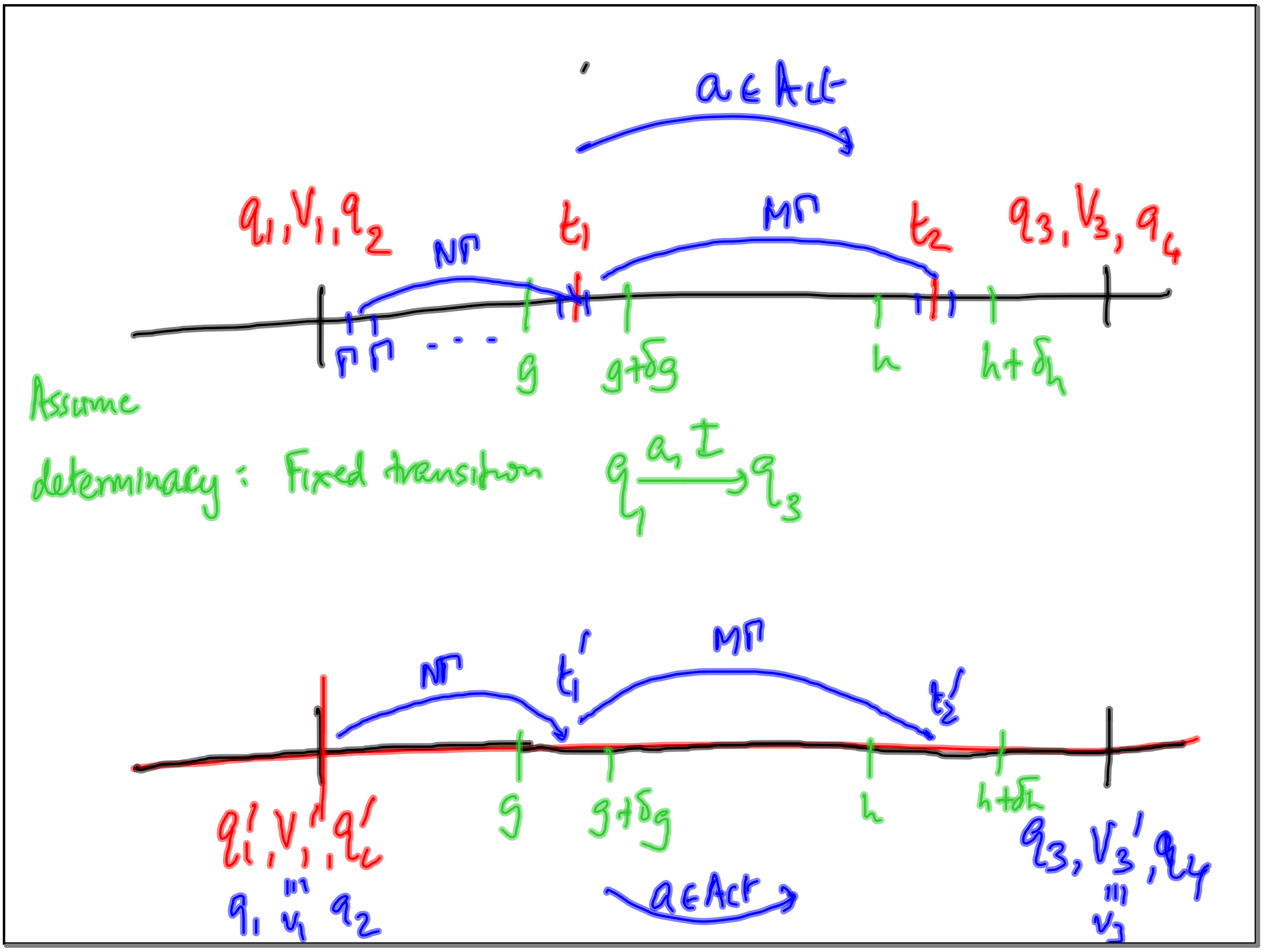


$V_1 \equiv V_2$ if they agree on this abstraction of the real value of x

$\|V_1\|$ — equivalence class of V_1

$(q_1, V_1, q_2) \equiv (q_1', V_1', q_2')$ if
$q_1 = q_1'$
$q_2 = q_2'$
$V_1 \equiv V_1'$


c_1 (q_1, v_1, q_2) $\equiv$ c_1' (q_1', v_1', q_2')
α
$\exists\alpha$
$c_2 = (q_3, v_3, q_4)$
c_2' (q_3', v_3', q_4')
$c_2 \equiv c_2'$


a ∈ Act
q1, V1, q2
NΓ
t1
MΓ
t2
q3, V3, q4
g
g+δg
h
h+δh
Assume
determinacy : Fixed transition
q1 —a, I→ q3
NΓ
t1'
MΓ
t2'
q1', V1', q2'
g
g+δg
h
h+δh
q3, V3', q4
q1 V1 q2
V3
a ∈ Act

<u>Claim:</u> Finite state abstraction captures $L_{st}(A)$, $L_{Act}(A)$

States are $\|(q, V, q')\|$

$$\|(q,V,q')\| \xrightarrow{\alpha} \|(q_1, V_1, q_1')\|$$

Compute this transition relation explicitly for each pair of configurations

Set up linear inequalities to solve for t_1, t_2

For n variables

Set up separate automata for each x_i

Synchronize them

$$(q, \{v_1, v_2, \ldots, v_n\}, q') \xrightarrow{\alpha} (q_1, \{v_1', \ldots, v_n'\}, q_1')$$

if $(q, v_i, q') \xrightarrow{\alpha} (q_1, v_i', q_1')$

for each i

Laziness allows analysis of a larger class

Linear hybrid automata