Hybrid Automata

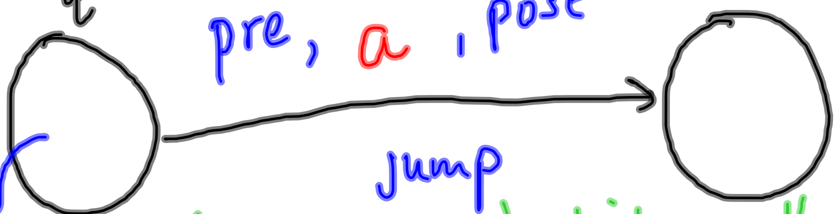
Clocks $\rightarrow$ arbitrary real valued variables

$\dot{x}$ depends on state

guard reset - can be nondet

pre, a, post

q q'



jump

which variables are reset $\in 2^X$

init(q) - initial values of each var if q is initial state

inv(q) - invariant conditions for values

flow(q) - specifies $\dot{x}$ for each x

What's decidable about hybrid automata?

Henzinger et al, JCSS

$$X = \{\underbrace{x_1, x_2, \ldots, x_n}_{\subseteq \mathbb{R}^n}\} \qquad V : X \to \mathbb{R}$$

Rectangular hybrid automata

$R \subseteq \mathbb{R}^n$ is a rectangle if it is a product of intervals

Rectangular hybrid automaton

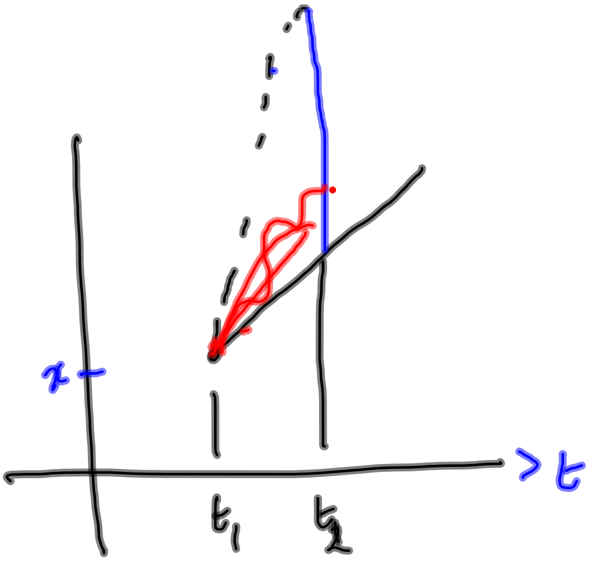
$$\left.\begin{array}{ll} flow(q) & pre(t) \\ inv(q) & post(t) \\ init(q) & \end{array}\right\}$$ a conjunction of one rectangular constraint per variable

$$post(t): \quad x = 3y + 7z + 8y^3$$

$$x \in [\ell, u)$$

$flow(x) \in [1, 5)$

effectively assume evolution is piecewise linear



Initialized automaton

$q \xrightarrow{a} q'$ if $flow(q)(x) \neq flow(q')(x)$

then $x \in jump(t)$

Whenever $\dot{x}$ changes, x is reset

Initialized + rectangular $\Leftrightarrow$ positive

Deterministic reset

Reachability is decidable for initialized rectangular automata

reachability of (q, Z)

where Z is a "zone" or rectangle

Initialized Rectangular

$\downarrow$

Initialized Singular – each $\dot{x}$ is a constant

deterministic resets.

$\downarrow$

Initialized Stopwatch $\dot{x} \in \{0,1\}$

deterministic resets

$\downarrow$

Timed Automata with constant resets

Initialized Stopwatch

$\downarrow$

Timed Automata

$(q, x_1, x_2, \ldots, x_n,$ some $\dot{x}_i = 0)$

if $\dot{x}_i = 0$, x_i is post(t) for last transition when $\dot{x}_i$ went from 1 to 0

$(q, x_1, \ldots, x_n, f : X \to \mathbb{R} \cup \{\perp\})$

If $\dot{x}_i = 1$, same value as stopwatch and $f(x) = \perp$

If $\dot{x}_i = 0$, $f(x) =$ stopped value of x

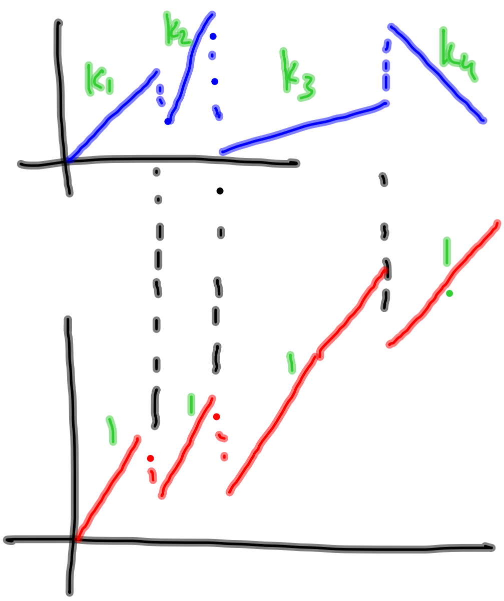
Initialized singular

$\downarrow$

Initialized stopwatch

$\forall q\ \exists k\ \ \dot{x} \in [k,k]$ in q.

det. jumps



Simple scaling relates configurations

Initialized Rectangular

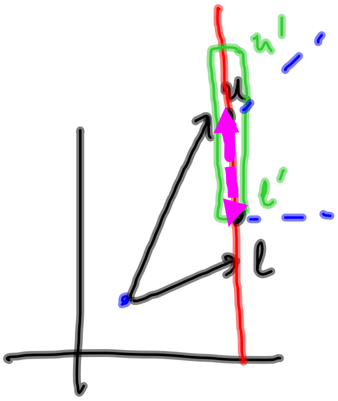
$\downarrow$

Initialized Singular

$\dot{x} \in [l, u]$

$\dot{x}_l = l$ $\dot{x}_u = u$

"Compact" rectangles
"
bounded & closed



$x_l \leq u'$

$x_u \geq l'$