Quantative Automata Theory, January–April 2015

Problem sheet on Weighted Automata

Due Monday, April 27, 2015

Page numbers and other references are with respect to the MPRI lecture notes (2011) on Weighted Automata by Benedikt Bollig and Marc Zeitoun.

1. Prove Lemma 3.4, page 11:

   Lemma 3.4 Given two probabilistic automata $\mathcal{A}_1$ and $\mathcal{A}_2$, one can build automata $\mathcal{B}_1$, $\mathcal{B}_2$ and $\mathcal{B}_3$ such that:

   (1) $\|\mathcal{B}_1\| = 1 - \|\mathcal{A}_1\|$.

   (2) $\|\mathcal{B}_2\| = 1 - \|\mathcal{A}_1\| \cdot \|\mathcal{A}_2\|$.

   (3) $\|\mathcal{B}_3\|(w) = \begin{cases} 0, & \text{if } w = \varepsilon \\ \alpha\|\mathcal{A}_1\|(w) + \beta\|\mathcal{A}_2\|(w), & \text{otherwise} \end{cases}$

   where $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$.

2. Prove Theorem 3.6, page 12:

   Theorem 3.6: For every regular language L, there is a probabilistic automaton $\mathcal{A}$ such that $L = L_{>0}(\mathcal{A})$.

3. Exercise 3.6, page 23:

   Is the value 0 problem decidable?