Remarks:

- In the distance graphs illustrated below, if there is no edge $x \to y$, then it denotes $y - x < \infty$.
- Normally edges $x \to y$ in a distance graph are of the form $(\leq, c)$ or $(<, c)$. For convenience, in this tutorial we only write constants c on edges and assume that it denotes $(\leq, c)$, that is $y - x \leq c$.

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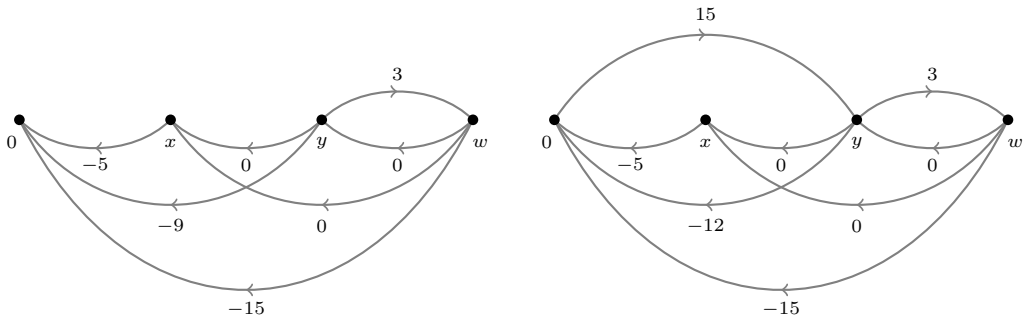
1. Provide an example of an automaton with a single state, $n \geq 2$ clocks and with maximum constant $M = 1$, such that the zone graph computed by algorithm discussed in class (also Algorithm 1.3 in the notes) gives at least 2^n nodes.

2. Let Z be the zone defined by $-3 \leq y - x \leq 4$. Can you construct an automaton whose zone graph contains a node (q, Z)?

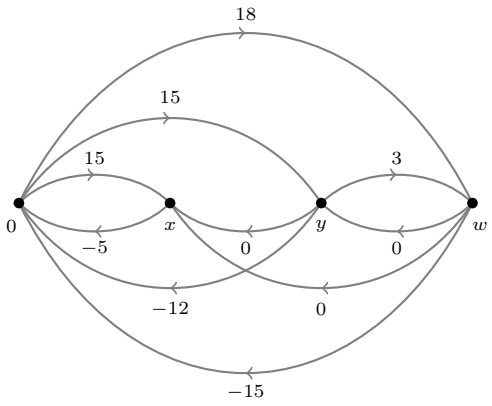
3. Which of the following distance graphs are in canonical form? If not, canonicalize them.



4. Is the set of solutions represented by the above graphs non-empty?

5. Construct a distance graph in canonical form with 3 clocks, in which at most 2 edges have weight 0.

6. Consider the distance graph G_Z below that represents some zone Z. Find the distance graph of $\overrightarrow{Z}$. Recall that $\overrightarrow{Z}$ denotes the zone obtained by elapsing time from Z, i.e., $\overrightarrow{Z} = \{v + \delta \mid v \in Z \text{ and } \delta \geq 0\}$



G_Z

7. Let G be a distance graph in canonical form that has no negative cycles. Suppose the edge $0 \to x$ in G is reduced to a new value so that adding this new value does not create negative cycles. Let G' be this new graph. Is G' necessarily canonical? If not, characterize the set of edges that need to be changed (reduced) in G'.

8. Same question as above, but now instead of $0 \to x$, the edge $x \to 0$ is reduced.

9. Consider a distance graph. Suppose some edges of the form $0 \to x$ and some of the form $y \to 0$ are reduced. Provide a quadratic algorithm to canonicalize this graph.

10. Consider the zone Z represented by the distance graph G_Z in the Question 4. Let Z' be the zone obtained by resetting x in all valuations of Z. Give the distance graph of Z'.

11. Let n be the number of clocks, and (q, Z) be a node in the zone graph. Assume that you are given a canonical graph G_Z representing Z. Give an $\mathcal{O}(n^2)$ algorithm for computing the successor of (q, Z) with respect to a transition (q, g, λ, q') where g is a guard, and λ is the set of clocks to be reset.