

Lecture 13, 18 September 2025

- Recursive definition of factorial
- Add diagnostic `print()` to trace recursive calls

```
In [1]: def fact(n):  
        print("evaluating factorial of",n)  
        if n == 0:  
            return(1)  
        else:  
            return(n*fact(n-1))
```

```
In [2]: fact(8)
```

```
evaluating factorial of 8  
evaluating factorial of 7  
evaluating factorial of 6  
evaluating factorial of 5  
evaluating factorial of 4  
evaluating factorial of 3  
evaluating factorial of 2  
evaluating factorial of 1  
evaluating factorial of 0
```

```
Out[2]: 40320
```

- A negative argument creates an unending sequence of recursive calls
- Python has a built in `recursion limit` to abort such cases
 - This limit may deny some legitimate computation -- for instance `factorial(3000)` will also abort
 - We will see later that we can increase this limit

```
In [3]: fact(-1)
```

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```

-----
RecursionError                                Traceback (most recent call last)
Cell In[3], line 1
----> 1 fact(-1)

Cell In[1], line 6, in fact(n)
      4     return(1)
      5 else:
----> 6     return(n*fact(n-1))

Cell In[1], line 6, in fact(n)
      4     return(1)
      5 else:
----> 6     return(n*fact(n-1))

[... skipping similar frames: fact at line 6 (2970 times)]

Cell In[1], line 6, in fact(n)
      4     return(1)
      5 else:
----> 6     return(n*fact(n-1))

Cell In[1], line 2, in fact(n)
      1 def fact(n):
----> 2     print(
      3         if n == 0:
      4         return(1)

File ~/python-venv/lib/python3.13/site-packages/IPython/core/interactiveshell.py:3056, in InteractiveShell._tee.<locals>.write(data, *args, **kwargs)
    3054 if not data:
    3055     return result
-> 3056 execution_count = self.execution_count
    3057 output_stream = None
    3058 outputs_by_counter = self.history_manager.outputs

File ~/python-venv/lib/python3.13/site-packages/traitlets/traitlets.py:687, in TraitType.__get__(self, obj, cls)
    685     return self
    686 else:
-> 687     return t.cast(G, self.get(obj, cls))

File ~/python-venv/lib/python3.13/site-packages/traitlets/traitlets.py:666, in TraitType.get(self, obj, cls)
    664     raise TraitError("Unexpected error in TraitType: default value not set properly") from e
    665 else:
-> 666     return t.cast(G, value)

RecursionError: maximum recursion depth exceeded

```

- We can also induct on the structure of a datastructure
- We can break up a list into the first element (the *head*) and the rest (the *tail*) as a smaller list
- Here is a recursive definition of `len()`

```

In [4]: def mylen(l):
        print("Call with",l)
        if l == []:
            return(0)
        else:
            return(1+mylen(l[1:]))

```

```

In [5]: mylen([7,18,2,-1,3])

```

```

Call with [7, 18, 2, -1, 3]
Call with [18, 2, -1, 3]
Call with [2, -1, 3]
Call with [-1, 3]
Call with [3]
Call with []

```

```

Out[5]: 5

```

- A similar definition for `sum()`

```
In [6]: def mysum(l):
        print("Call with",l)
        if l == []:
            return(0)
        else:
            return(l[0]+mysum(l[1:]))
```

```
In [7]: mysum([7,18,2,-1,3])
```

```
Call with [7, 18, 2, -1, 3]
Call with [18, 2, -1, 3]
Call with [2, -1, 3]
Call with [-1, 3]
Call with [3]
Call with []
```

```
Out[7]: 29
```

- Insert a value `v` into a list `l` sorted in ascending order

```
In [8]: def sortedinsert(v,l):
        if l == []:
            return([v])
        if v <= l[0]:
            return([v]+l)
        else:
            return(l[:1] + sortedinsert(v,l[1:]))
```

```
In [9]: sortedinsert(14,list(range(1,20,2)))
```

```
Out[9]: [1, 3, 5, 7, 9, 11, 13, 14, 15, 17, 19]
```

- The built in recursion limit causes this function to fail on moderate size lists

```
In [10]: sortedinsert(3000,list(range(3000)))
```

```
-----
RecursionError                                Traceback (most recent call last)
Cell In[10], line 1
----> 1 sortedinsert(3000,list(range(3000)))

Cell In[8], line 7, in sortedinsert(v, l)
      5     return([v]+l)
      6 else:
----> 7     return(l[:1] + sortedinsert(v,l[1:]))

Cell In[8], line 7, in sortedinsert(v, l)
      5     return([v]+l)
      6 else:
----> 7     return(l[:1] + sortedinsert(v,l[1:]))

[... skipping similar frames: sortedinsert at line 7 (2974 times)]

Cell In[8], line 7, in sortedinsert(v, l)
      5     return([v]+l)
      6 else:
----> 7     return(l[:1] + sortedinsert(v,l[1:]))

RecursionError: maximum recursion depth exceeded
```

- Can manually increase the recursion limit
- Maximum is $2^{31} - 1$, the largest `int` value that can be stored in 32 bits

```
In [11]: import sys
        sys.setrecursionlimit(2**31-1)
```

```
In [12]: len(sortedinsert(3000,list(range(3000))))
```

Out[12]: 3001