Quicksort

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Programming and Data Structures with Python

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Shortcomings of merge sort

- Merge needs to create a new list to hold the merged elements
  - No obvious way to efficiently merge two lists in place
  - Extra storage can be costly
- Inherently recursive
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- Merging happens because elements in the left half need to move to the right half and vice versa
  - Consider an input of the form `[0,2,4,6,1,3,5,9]`

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- Merging happens because elements in the left half need to move to the right half and vice versa
  - Consider an input of the form `[0,2,4,6,1,3,5,9]`
- Can we divide the list so that everything on the left is smaller than everything on the right?
  - No need to merge!

Divide and conquer without merging

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- How do we find the median?
  - Sort and pick up the middle element
  - But our aim is to sort the list!
- Instead pick some value in `L` — pivot
  - Split `L` with respect to the pivot element

Quicksort [C.A.R. Hoare]

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- Maintain two indices to mark the end of the Lower and Upper segments
- After partitioning, exchange the pivot with the last element of the Lower segment

Quicksort code

- Scan the list from left to right
- Four segments: Pivot, Lower, Upper, Unclassified
- Classify the first unclassified element
  - If it is larger than the pivot, extend Upper to include this element
  - If it is less than or equal to the pivot, exchange with the first element in Upper. This extends Lower and shifts Upper by one position.

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def quicksort(L,l,r):  # Sort L[l:r]
  if (r - l <= 1):
    return(L)
  (pivot,lower,upper) = (L[l],l+1,l+1)
  for i in range(l+1,r):
    if L[i] > pivot:  # Extend upper segment
      upper = upper+1
    else: # Exchange L[i] with start of upper segment
      (L[i], L[lower]) = (L[lower], L[i])
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      (lower,upper) = (lower+1,upper+1)
  # Move pivot between lower and upper
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  # Recursive calls
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- The partitioning strategy we described is not the only one used in the literature
  - Can build the lower and upper segments from opposite ends and meet in the middle

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- We can also provide an iterative implementation to avoid the cost of recursive calls
- The partitioning strategy we described is not the only one used in the literature
  - Can build the lower and upper segments from opposite ends and meet in the middle
- Need to analyse the complexity of quick sort

Analysis

- Partitioning with respect to the pivot takes time $O(n)$

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- Worst case? Pivot is maximum or minimum
  - Partitions are of size 0, $n-1$
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- Already sorted array: worst case!

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Iterative quicksort

- Recursive calls work on disjoint segments
  - No recombination of results is required

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Iterative quicksort

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  - No recombination of results is required
- Can explicitly keep track of left and right endpoints of each segment to be sorted

```
def quicksort(L,l,r):  # Sort L[l:r]
  if (r - l <= 1):
    return(L)
  (pivot,lower,upper) = (L[l],l+1,l+1)
  for i in range(l+1,r):
    if L[i] > pivot:  # Extend upper segment
      upper = upper+1
    else: # Exchange L[i] with start of upper segment
      (L[i], L[lower]) = (L[lower], L[i])
      # Shift both segments
      (lower,upper) = (lower+1,upper+1)
  # Move pivot between lower and upper
  (L[l],L[lower-1]) = (L[lower-1],L[l])
  lower = lower-1
  # Recursive calls
  quicksort(L,l,lower)
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Quicksort in practice

- In practice, quicksort is very fast
- Very often the default algorithm used for in-built sort functions
  - Sorting a column in a spreadsheet
  - Library sort function in a programming language

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- However, the average case is $O(n \log n)$
- Randomly choosing the pivot is a good strategy to beat worst case inputs
- Quicksort works in-place and can be implemented iteratively
- Very fast in practice, and often used for built-in sorting functions
  - Good example of a situation when the worst case upper bound is pessimistic

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  - Swapping values while partitioning can disturb existing sorted order
- Merge sort is stable if we merge carefully
  - Do not allow elements from the right to overtake elements on the left
  - While merging, prefer the left list while breaking ties

Other criteria

- Minimizing data movement
  - Imagine each element is a heavy carton
  - Reduce the effort of moving values around

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- Other $O(n \log n)$ algorithms exist — heapsort
- Sometimes hybrid strategies are used
  - Use divide and conquer for large n
  - Switch to insertion sort when n becomes small (e.g., $n < 16$)