

Quicksort

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Programming and Data Structures with Python

Lecture 18, 20 Oct 2022

Shortcomings of merge sort

- Merge needs to create a new list to hold the merged elements
 - No obvious way to efficiently merge two lists in place
 - Extra storage can be costly
- Inherently recursive
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 - Consider an input of the form [0,2,4,6,1,3,5,9]
- Can we divide the list so that everything on the left is smaller than everything on the right?
 - No need to merge!

Divide and conquer without merging

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- How do we find the median?
 - Sort and pick up the middle element
 - But our aim is to sort the list!
 - Instead pick some value in L — **pivot**
 - Split L with respect to the pivot element

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High level view of quicksort

- Input list

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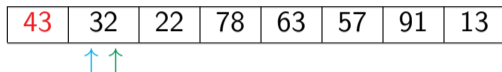
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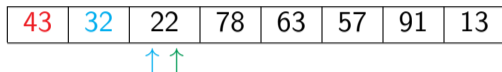
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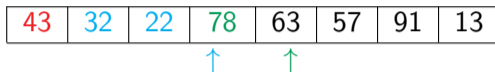
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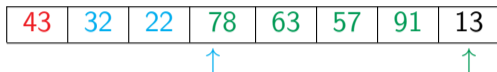
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Quicksort code

- Scan the list from left to right
- Four segments: **Pivot**, **Lower**, **Upper**, Unclassified
- Classify the first unclassified element
 - If it is larger than the pivot, extend **Upper** to include this element
 - If it is less than or equal to the pivot, exchange with the first element in **Upper**. This extends **Lower** and shifts **Upper** by one position.

```
def quicksort(L,l,r): # Sort L[l:r]
    if (r - l <= 1):
        return(L)
    (pivot,lower,upper) = (L[l],l+1,l+1)
    for i in range(l+1,r):
        if L[i] > pivot: # Extend upper segment
            upper = upper+1
        else: # Exchange L[i] with start of upper segment
            (L[i], L[lower]) = (L[lower], L[i])
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            (lower,upper) = (lower+1,upper+1)
    # Move pivot between lower and upper
    (L[l],L[lower-1]) = (L[lower-1],L[l])
    lower = lower-1
    # Recursive calls
    quicksort(L,l,lower)
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- We can also provide an iterative implementation to avoid the cost of recursive calls
- The partitioning strategy we described is not the only one used in the literature
 - Can build the lower and upper segments from opposite ends and meet in the middle
- Need to analyse the complexity of quick sort

Analysis

- Partitioning with respect to the pivot takes time $O(n)$

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- Worst case? Pivot is maximum or minimum
 - Partitions are of size 0, $n - 1$
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- Already sorted array: worst case!

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- Can explicitly keep track of left and right endpoints of each segment to be sorted

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    (L[l],L[lower-1]) = (L[lower-1],L[l])
    lower = lower-1
    # Recursive calls
    quicksort(L,l,lower)
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    return(L)
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Quicksort in practice

- In practice, quicksort is very fast

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def quicksort(L,l,r): # Sort L[l:r]
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    for i in range(l+1,r):
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Quicksort in practice

- In practice, quicksort is very fast
- Very often the default algorithm used for in-built sort functions
 - Sorting a column in a spreadsheet
 - Library sort function in a programming language

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- However, the average case is $O(n \log n)$
- Randomly choosing the pivot is a good strategy to beat worst case inputs
- Quicksort works in-place and can be implemented iteratively
- Very fast in practice, and often used for built-in sorting functions
 - Good example of a situation when the worst case upper bound is pessimistic

Stable sorting

- Often list values are tuples
 - Rows from a table, with multiple columns / attributes
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 - Swapping values while partitioning can disturb existing sorted order
- Merge sort is stable if we merge carefully
 - Do not allow elements from the right to overtake elements on the left
 - While merging, prefer the left list while breaking ties

- Minimizing data movement
 - Imagine each element is a heavy carton
 - Reduce the effort of moving values around

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- Other $O(n \log n)$ algorithms exist — heapsort
- Sometimes hybrid strategies are used
 - Use divide and conquer for large n
 - Switch to insertion sort when n becomes small (e.g., $n < 16$)