Analysis of algorithms

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Programming and Data Structures with Python

Lecture 16, 11 Oct 2022

Measuring performance

- Example of validating SIM cards against Aadhaar data
  - Naive approach takes thousands of years
  - Smarter solution takes a few minutes

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- Typically, we focus on time rather than space

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Example 1 SIM cards vs Aadhaar cards

- $n \approx 10^9$ — number of cards
- Naive algorithm: $t(n) \approx n^2$
- Clever algorithm: $t(n) \approx n \log_2 n$
  - $\log_2 n$ — number of times you need to divide n by 2 to reach 1
  - $\log_2(n) = k \Rightarrow n = 2^k$

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- Asymptotic complexity
  - What happens in the limit, as n becomes large
- Typical growth functions
  - Is $t(n)$ proportional to $\log n, \ldots, n^2, n^3, \ldots, 2^n$?
    - Note: $\log n$ means $\log_2 n$ by default
  - Logarithmic, polynomial, exponential, ...

Orders of magnitude

Input size	Values of $t(n)$					
	$\log n$	n	$n \log n$	n^2	n^3	2^n	$n!$
10	3.3	10	33	100	1000	1000	10^6
100	6.6	100	66	10^4	10^6	10^{30}	10^{157}
1000	10	1000	10^4	10^6	10^9		
10^4	13	10^4	10^5	10^8	10^{12}		
10^5	17	10^5	10^6	10^{10}			
10^6	20	10^6	10^7	10^{12}			
10^7	23	10^7	10^8				
10^8	27	10^8	10^9				
10^9	30	10^9	10^{10}				
10^{10}	33	10^{10}	10^{11}				

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  - Compare two values
  - Assign a value to a variable
- Exchange a pair of values?

```
(x,y) = (y,x)          t = x
                       x = y
                       y = t
```

  - If we ignore constants, focus on orders of magnitude, both are within a factor of 3
  - Need not be very precise about defining basic operations

What is the input size

- Typically a natural parameter
  - Size of a list/array that we want to search or sort
  - Number of objects we want to rearrange
  - Number of vertices and number edges in a graph
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  - Magnitude of n is not the correct measure
  - Arithmetic operations are performed digit by digit
    - Addition with carry, subtraction with borrow, multiplication, long division ...
  - Number of digits is a natural measure of input size
    - Same as $\log_b n$, when we write n in base b

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  - Input that forces algorithm to take longest possible time
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  - We mainly focus on time
- Running time $t(n)$ is a function of input size n
  - Interested in orders of magnitude
  - Asymptotic complexity, as n becomes large
- From running time, we can estimate feasible input sizes
- We focus on worst case inputs
  - Pessimistic, but easier to calculate than average case
  - Upper bound on worst case gives us an overall guarantee on performance

Searching in a list

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- Worst case complexity is $O(n)$

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  - If `v` less than midpoint, search the first half
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  - Stop when the interval to search becomes empty

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def binarysearch(v,l):
  if l == []:
    return(False)

  m = len(l)//2

  if v == l[m]:
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  else:
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- $O(\log n)$ steps

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Alternative calculation

- $T(n)$: the time to search a list of length n
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$T(n//8) + 1 + 1 + 1$

$T(n/2^3)$ $\underbrace{1+1+1}_{3}$

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linked list

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- For a sorted list, binary search takes time $O(\log n)$
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- In a sorted list, we can determine that v is absent by examining just $\log n$ values!

Naïve Sorting Algorithms

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  - Checking for duplicates
  - Building a frequency table of values
- How do we sort a list?
- You are the TA for a course
  - Instructor has a pile of evaluated exam papers
  - Papers in random order of marks
  - Your task is to arrange the papers in descending order of marks

Strategy 1

- Scan the entire pile and find the paper with minimum marks
- Move this paper to a new pile
- Repeat with the remaining papers
  - Add the paper with next minimum marks to the second pile each time

Sorting a list

- Sorting a list makes many other computations easier
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  - Finding the median
  - Checking for duplicates
  - Building a frequency table of values
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Strategy 1

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- Move this paper to a new pile
- Repeat with the remaining papers
  - Add the paper with next minimum marks to the second pile each time
- Eventually, the new pile is sorted in descending order

Sorting a list

Top → Bottom

74 32 89 55 21 64

Sorting a list

74 32 89 55 ~~21~~ 64

21

Sorting a list

74	~~32~~	89	55	~~21~~	64

21	32

Sorting a list

74	~~32~~	89	~~55~~	~~21~~	64

21	32	55

Sorting a list

74	~~32~~	89	~~55~~	~~21~~	~~64~~

21	32	55	64

Sorting a list

~~74~~	~~32~~	89	~~55~~	~~21~~	~~64~~
21	32	55	64	74	

Sorting a list

~~74~~ ~~32~~ ~~89~~ ~~55~~ ~~21~~ ~~64~~

21 32 55 64 74 89

Selection sort

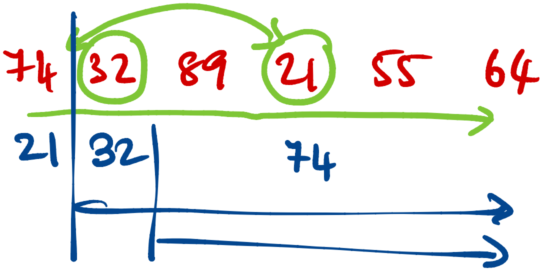
- Select the next element in sorted order

Selection sort

- Select the next element in sorted order
- Append it to the final sorted list

Selection sort

- Select the next element in sorted order
- Append it to the final sorted list
- Avoid using a second list
    - Swap the minimum element into the first position
    - Swap the second minimum element into the second position
    - ...



Selection sort

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  - ...
- Eventually the list is rearranged in place in ascending order

```
def SelectionSort(L):
  n = len(L)
  if n < 1:
    return(L)
  for i in range(n):
    # Assume L[:i] is sorted
    mpos = i
    # mpos: position of minimum in L[i:]
    for j in range(i+1,n):
      if L[j] < L[mpos]:
        mpos = j
    # L[mpos] : smallest value in L[i:]
    # Exchange L[mpos] and L[i]
    (L[i],L[mpos]) = (L[mpos],L[i])
    # Now L[:i+1] is sorted
  return(L)
```

$n=1$

$i=1$

Analysis of selection sort

- Correctness follows from the invariant

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- Efficiency

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  - Outer loop iterates n times

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  - Inner loop: $n - i$ steps to find minimum in `L[i:]`

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  - Outer loop iterates n times
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  - $T(n) = n + (n-1) + \cdots + 1$

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  - Outer loop iterates n times
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  - $T(n) = n(n+1)/2$

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Analysis of selection sort

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  - Outer loop iterates n times
  - Inner loop: $n - i$ steps to find minimum in `L[i:]`
  - $T(n) = n + (n-1) + \cdots + 1$
  - $T(n) = n(n+1)/2$
- $T(n)$ is $O(n^2)$

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Strategy 2

Sorting a list

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Strategy 2

- Move the first paper to a new pile

Sorting a list

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Strategy 2

- Move the first paper to a new pile
- Second paper
  - Lower marks than first paper? Place below first paper in new pile
  - Higher marks than first paper? Place above first paper in new pile

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Strategy 2

- Move the first paper to a new pile
- Second paper
  - Lower marks than first paper? Place below first paper in new pile
  - Higher marks than first paper? Place above first paper in new pile
- Third paper
  - Insert into correct position with respect to first two

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Strategy 2

- Move the first paper to a new pile
- Second paper
  - Lower marks than first paper? Place below first paper in new pile
  - Higher marks than first paper? Place above first paper in new pile
- Third paper
  - Insert into correct position with respect to first two
- Do this for the remaining papers
  - Insert each one into correct position in the second pile

Sorting a list

74	32	89	55	21	64

Sorting a list

~~74~~ 32 89 55 21 64

74

Sorting a list

~~74~~ ~~32~~ 89 55 21 64

32 74

Sorting a list

~~74~~	~~32~~	~~89~~	55	21	64
32	74	89			

Sorting a list

~~74~~	~~32~~	~~89~~	~~55~~	21	64
32	55	74	89		

Sorting a list

~~74~~ ~~32~~ ~~89~~ ~~55~~ ~~21~~ 64

21 32 55 74 89

Sorting a list

~~74~~	~~32~~	~~89~~	~~55~~	~~21~~	~~64~~
21	32	55	64	74	89

Insertion sort

- Start building a new sorted list

Insertion sort

- Start building a new sorted list
- Pick next element and insert it into the sorted list

Insertion sort

- Start building a new sorted list
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- An iterative formulation
  - Assume `L[:i]` is sorted
  - Insert `L[i]` in `L[:i]`

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- Pick next element and insert it into the sorted list
- An iterative formulation
  - Assume `L[:i]` is sorted
  - Insert `L[i]` in `L[:i]`

```
def InsertionSort(L):
    n = len(L)
    if n < 1:
       return(L)
    for i in range(n):
       # Assume L[:i] is sorted
       # Move L[i] to correct position in L
       j = i
       while(j > 0 and L[j] < L[j-1]):
         (L[j],L[j-1]) = (L[j-1],L[j])
         j = j-1
       # Now L[:i+1] is sorted
    return(L)
```