# Searching in a List

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Programming and Data Structures with Python

Lecture 15, 15 Nov 2021

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  return(False)
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- Worst case complexity is $O(n)$

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  - If midpoint is `v`, the value is found
  - If `v` less than midpoint, search the first half
  - If `v` greater than midpoint, search the second half
  - Stop when the interval to search becomes empty

```
def binarysearch(v,l):
  if l == []:
    return(False)

  m = len(l)//2

  if v == l[m]:
    return(True)

  if v < l[m]:
    return(binarysearch(v,l[:m]))
  else:
    return(binarysearch(v,l[m+1:]))
```

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# Binary search

- How long does this take?
  - Each call halves the interval to search
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- $\log n$ — number of times to divide $n$ by $2$ to reach $1$
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  - $1 // 2 = 0$, so next call reaches empty interval
- $O(\log n)$ steps

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# Alternative calculation

- $T(n)$ : the time to search a list of length $n$
  - If $n = 0$, we exit, so $T(n) = 1$
  - If $n > 0$, $T(n) = T(n // 2) + 1$

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def bsearch(v,l):
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  m = len(l)//2

  if v == l[m]:
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  - Need to scan the entire list
  - Worst case is when the value is not present in the list
- For a sorted list, binary search takes time $O(\log n)$
  - Halve the interval to search each time
- In a sorted list, we can determine that $v$ is absent by examining just $\log n$ values!

# Selection Sort

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- Sorting a list makes many other computations easier
  - Binary search
  - Finding the median
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  - Add the paper with next minimum marks to the second pile each time
- Eventually, the new pile is sorted in descending order

# Sorting a list

74 32 89 55 21 64

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21

# Sorting a list

| | | | | | |
|---|---|---|---|---|---|
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| | |
|---|---|
| 21 | 32 |

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  - Swap the second minimum element into the second position
  - ...

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  - ...
- Eventually the list is rearranged in place in ascending order

```
def SelectionSort(L):
  n = len(L)
  if n < 1:
    return(L)
  for i in range(n):
    # Assume L[:i] is sorted
    mpos = i
    # mpos: position of minimum in L[i:]
    for j in range(i+1,n):
      if L[j] < L[mpos]:
        mpos = j
    # L[mpos] : smallest value in L[i:]
    # Exchange L[mpos] and L[i]
    (L[i],L[mpos]) = (L[mpos],L[i])
    # Now L[:i+1] is sorted
  return(L)
```

# Analysis of selection sort

- Correctness follows from the invariant

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  - Outer loop iterates $n$ times

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# Analysis of selection sort

- Correctness follows from the invariant
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  - Outer loop iterates $n$ times
  - Inner loop: $n - i$ steps to find minimum in `L[i:]`

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  - Outer loop iterates $n$ times
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  - $T(n) = n + (n-1) + \cdots + 1$

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  - $T(n) = n(n+1)/2$

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  - $T(n) = n(n+1)/2$
- $T(n)$ is $O(n^2)$

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def SelectionSort(L):
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- Selection sort is an intuitive algorithm to sort a list
- Repeatedly find the minimum (or maximum) and append to sorted list
- Worst case complexity is $O(n^2)$
  - Every input takes this much time
  - No advantage even if list is arranged carefully before sorting

# Insertion Sort

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- Move the first paper to a new pile

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## Strategy 2

- Move the first paper to a new pile
- Second paper
  - Lower marks than first paper? Place below first paper in new pile
  - Higher marks than first paper? Place above first paper in new pile

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  - Your task is to arrange the papers in descending order of marks

## Strategy 2

- Move the first paper to a new pile
- Second paper
  - Lower marks than first paper? Place below first paper in new pile
  - Higher marks than first paper? Place above first paper in new pile
- Third paper
  - Insert into correct position with respect to first two

# Sorting a list

- You are the TA for a course
    - Instructor has a pile of evaluated exam papers
    - Papers in random order of marks
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## Strategy 2

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- Third paper
    - Insert into correct position with respect to first two
- Do this for the remaining papers
    - Insert each one into correct position in the second pile

# Sorting a list

| | | | | | |
|---|---|---|---|---|---|
| 74 | 32 | 89 | 55 | 21 | 64 |

# Sorting a list

~~74~~ 32 89 55 21 64

74

# Sorting a list

~~74~~ ~~32~~ 89 55 21 64

32 74

# Sorting a list

~~74~~ ~~32~~ ~~89~~ 55 21 64

32 74 89

# Sorting a list

| | | | | | |
|---|---|---|---|---|---|
| ~~74~~ | ~~32~~ | ~~89~~ | ~~55~~ | 21 | 64 |
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# Sorting a list

~~74~~ ~~32~~ ~~89~~ ~~55~~ ~~21~~ 64

21 32 55 74 89

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~~74~~ ~~32~~ ~~89~~ ~~55~~ ~~21~~ ~~64~~

21 32 55 64 74 89

# Insertion sort

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  - Assume `L[:i]` is sorted
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- An iterative formulation
  - Assume `L[:i]` is sorted
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def InsertionSort(L):
  n = len(L)
  if n < 1:
    return(L)
  for i in range(n):
    # Assume L[:i] is sorted
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    j = i
    while(j > 0 and L[j] < L[j-1]):
      (L[j],L[j-1]) = (L[j-1],L[j])
      j = j-1
    # Now L[:i+1] is sorted
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# Insertion sort

- Start building a new sorted list
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- An iterative formulation
  - Assume L[:i] is sorted
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def Insert(L,v):
   n = len(L)
   if n == 0:
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def ISort(L):
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# Analysis of iterative insertion sort

- Correctness follows from the invariant

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  - $T(n) = n(n-1)/2$
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# Analysis of recursive insertion sort

- For input of size $n$, let
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   n = len(L)
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# Analysis of recursive insertion sort

- For input of size $n$, let
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- Create a new sorted list
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- Insertion sort is another intuitive algorithm to sort a list
- Create a new sorted list
- Repeatedly insert elements into the sorted list
- Worst case complexity is $O(n^2)$
  - Unlike selection sort, not all cases take time $n^2$
  - If list is already sorted, `Insert` stops in 1 step
  - Overall time can be close to $O(n)$

# Merge Sort

Madhavan Mukund

https://www.cmi.ac.in/~madhavan

Programming and Data Structures with Python

Lecture 15, 15 Nov 2021

# Beating the $O(n^2)$ barrier

- Both selection sort and insertion sort take time $O(n^2)$
- This is infeasible for $n > 10000$

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Strategy 3

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- Separately sort the left and right half

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## Strategy 3

- Divide the list into two halves
- Separately sort the left and right half
- Combine the two sorted halves to get a fully sorted list

# Combining two sorted lists

- Combine two sorted lists `A` and `B` into a single sorted list `C`

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| | | |
|---|---|---|
| 32 | 74 | 89 |
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|---|---|---|
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21

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~~32~~ ~~74~~ ~~89~~

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- Combine two sorted lists `A` and `B` into a single sorted list `C`
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  - Move the smaller of the two to `C`
  - Repeat till you exhaust `A` and `B`
- Merging `A` and `B`

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# Merge sort

- Let $\texttt{n}$ be the length of $L$

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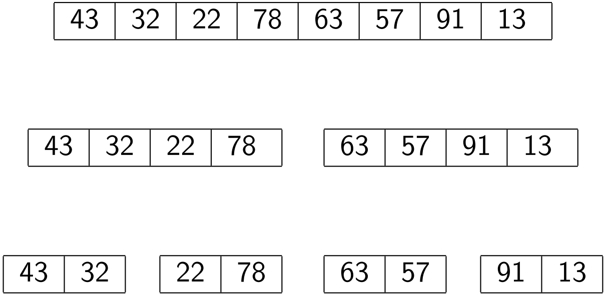

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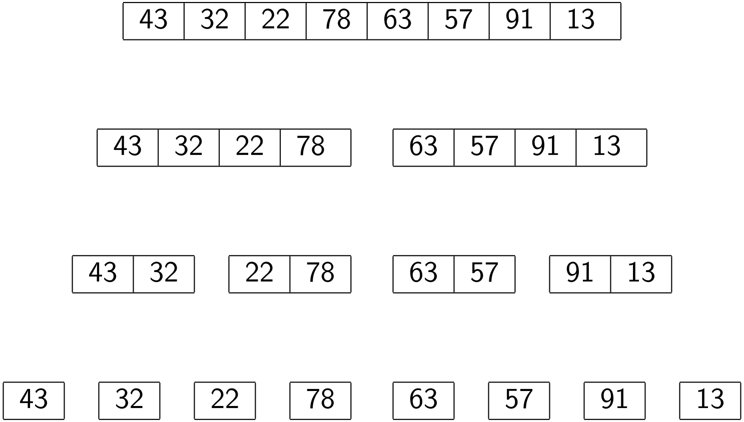

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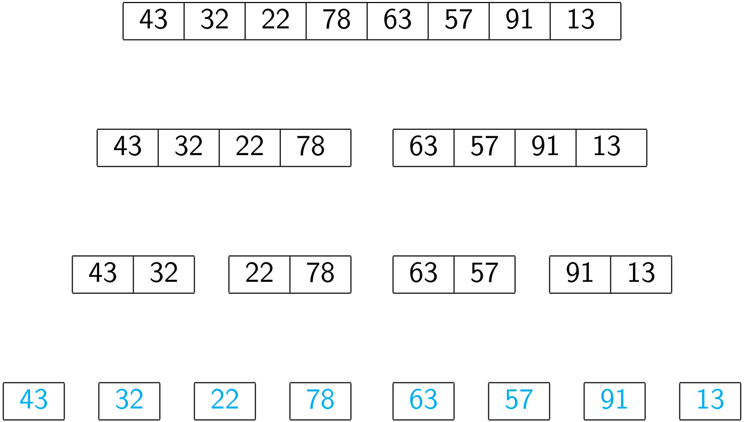

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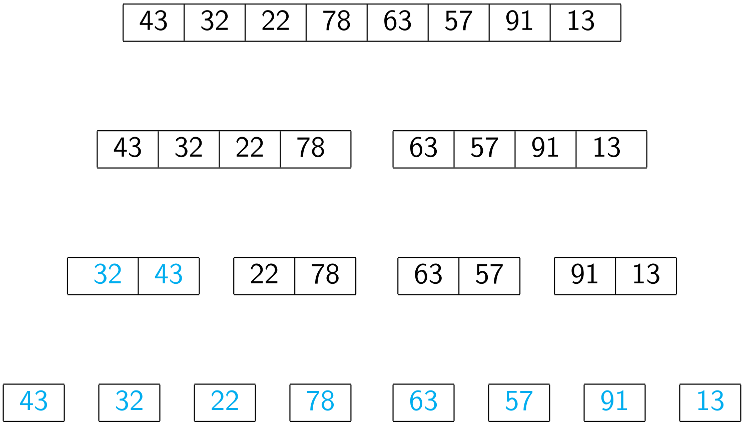

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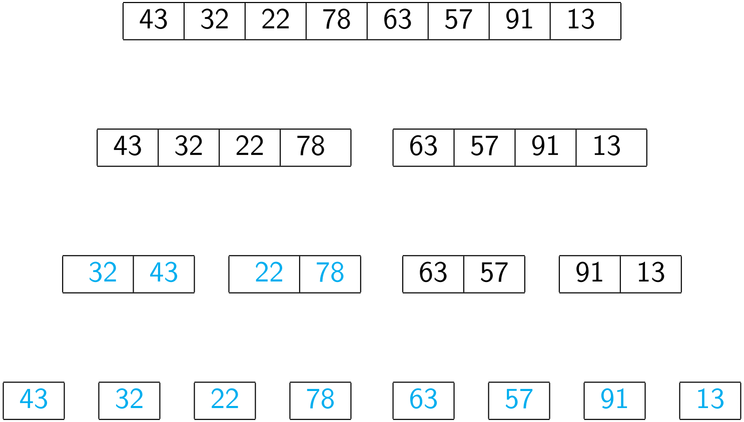

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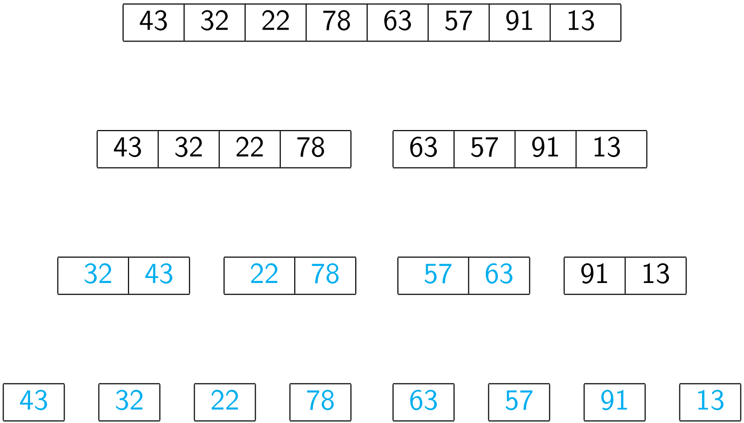

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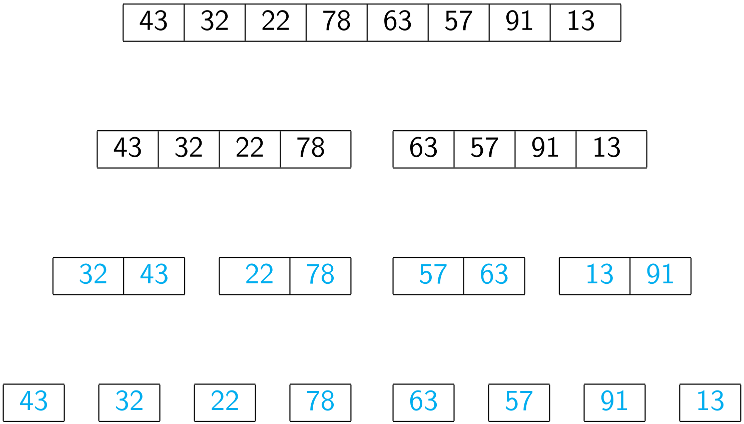

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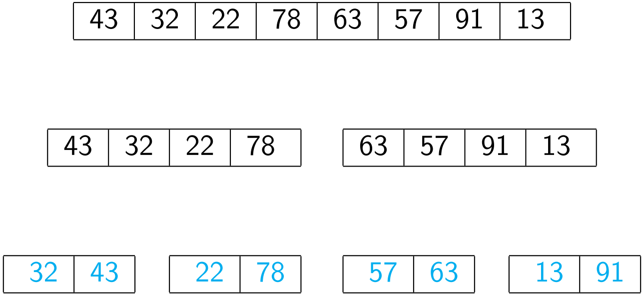

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| | | | |
|---|---|---|---|
| 63 | 57 | 91 | 13 |

| | |
|---|---|
| 32 | 43 |

| | |
|---|---|
| 22 | 78 |

| | |
|---|---|
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|---|---|
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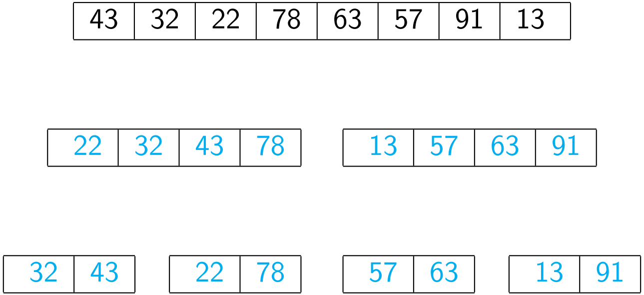

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Divide and Conquer

- Break up the problem into disjoint parts
- Solve each part separately
- Combine the solutions efficiently

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```
def merge(A,B):
  (m,n) = (len(A),len(B))
  (C,i,j,k) = ([],0,0,0)
  while k < m+n:
    if i == m:
      C.extend(B[j:])
      k = k + (n-j)
    elif j == n:
      C.extend(A[i:])
      k = k + (n-i)
    elif A[i] < B[j]:
      C.append(A[i])
      (i,k) = (i+1,k+1)
    else:
      C.append(B[j])
      (j,k) = (j+1,k+1)
  return(C)
```

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- To sort `A` into `B`, both of length $n$
- If $n \leq 1$, nothing to be done
- Otherwise
  - Sort `A[:n//2]` into `L`
  - Sort `A[n//2:]` into `R`

# Merge sort

- To sort `A` into `B`, both of length $n$
- If $n \leq 1$, nothing to be done
- Otherwise
  - Sort `A[:n//2]` into `L`
  - Sort `A[n//2:]` into `R`
  - Merge `L` and `R` into `B`

# Merge sort

- To sort `A` into `B`, both of length $n$
- If $n \leq 1$, nothing to be done
- Otherwise
  - Sort `A[:n//2]` into `L`
  - Sort `A[n//2:]` into `R`
  - Merge `L` and `R` into `B`

```
def mergesort(A):
  n = len(A)

  if n <= 1:
     return(A)

  L = mergesort(A[:n//2])
  R = mergesort(A[n//2:])

  B = merge(L,R)

  return(B)
```

# Summary

- Merge sort using divide and conquer to sort a list

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- Merge sort using divide and conquer to sort a list
- Divide the list into two halves
- Sort each half
- Merge the sorted halves
- Next, we have to check that the complexity is less than $O(n^2)$