Analysis of algorithms

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Programming and Data Structures with Python

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A real world problem

- Every SIM card needs to be linked to an Aadhaar card
- Validate Aadhaar number for each SIM card
- Simple nested loop
- How long will this take?
  - M SIM cards, N Aadhaar cards
  - Nested loops iterate $M \cdot N$ times
- What are M and N
  - Almost everyone in India has an Aadhaar card: $N > 10^9$
  - Number of SIM cards registered is similar: $M > 10^9$

```
for each SIM card S:
  for each Aadhaar number A:
     check if Aadhaar number in S
     matches A
```

A real world problem

- Assume $M = N = 10^9$
- Nested loops execute 10^{18} times
- We calculated that Python can perform 10^7 operations in a second
- This will take at least 10^{11} seconds
  - $10^{11}/60 \approx 1.67 \times 10^9$ minutes
  - $(1.67 \times 10^9)/60 \approx 2.8 \times 10^7$ hours
  - $(2.8 \times 10^7)/24 \approx 1.17 \times 10^6$ days
  - $(1.17 \times 10^6)/365 \approx 3200$ years!
- How can we fix this?

```
for each SIM card S:
  for each Aadhaar number A:
     check if Aadhaar number in S
     matches A
```

Guess my birthday

- You propose a date
- I answer, *Yes*, *Earlier*, *Later*
- Suppose my birthday is 12 April
- A possible sequence of questions
  - September 12? Earlier
  - February 23? Later
  - July 2? Earlier
  - ...
- What is the best strategy?

- Interval of possibilities
- Query midpoint — halves the interval
  - June 30? Earlier
  - March 31? Later
  - May 15? Earlier
  - April 22? Earlier
  - April 11? Later
  - April 16? Earlier
  - April 13? Earlier
  - April 12? Yes
- Interval shrinks from $365 \rightarrow 182 \rightarrow 91 \rightarrow 45 \rightarrow 22 \rightarrow 11 \rightarrow 5 \rightarrow 2 \rightarrow 1$
- Under 10 questions

A real world problem

- Assume Aadhaar details are sorted by Aadhaar number
- Use the halving strategy to check each SIM card
- Halving 10 times reduces the interval by a factor of 1000, because $2^{10} = 1024$
- After 10 queries, interval shrinks to 10^6
- After 20 queries, interval shrinks to 10^3
- After 30 queries, interval shrinks to 1
- Total time $\approx 10^9 \times 30$

```
for each SIM card S:
  probe sorted Aadhaar list to
  find a match with S
```

- 3000 seconds, or 50 minutes
- From 3200 years to 50 minutes!
- Of course, to achieve this we have to first sort the Aadhaar cards
- Arranging the data results in a much more efficient solution
- Both algorithms and data structures matter