## Evaluation

Programming Assignments $\approx 50\%$

Quiz $\approx 10\%$

Final exam $\approx 40\%$

## Books

No fixed book

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What is programming?

Algorithm – systematic procedure

Chop onions < fine / coarse > size?

Precise notation to convey algorithm

Who is executing?

Prepare a classroom for a guest lecture

- How many people, time
- Checklist – projector, audio, pointer ---
- How to align chairs

Greatest common divisor - gcd (hcf)

gcd(m,n) def largest d that divides m & n

integers $m, n > 0$

How? Find systematic solution - may or may not be clever

Brute Force $d \mid n$ d divides n

tied to efficiency

$\gcd(m,n) \leq m \leq \min(m,n)$
$\leq n$

$n \div d$ -
no remainder
$d \leq n$

Try every d between 1 and min(m,n)

Find largest divisor ] "Remember" largest divisor seen

gcd(72, 90)

1 2 3 4 ... 9 10 - 18 72

✓ ✓ ✓ × ✓

18

Compute divisors of m, divisors n

72 — {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}

90 — {1, 2, 3 ... 9, 10 ... 30, 45, 90}

Common divisors = {1, 2, 3, 9, 18}

Remembering a single number

Remembering two sets of numbers

list

in sequence

$\{1,2\} = \{2,1\} = \{2,1,2\} = \{1,1,2\}$

$[1,2] \neq [2,1]$ list - order matters

$[1,2,1] \neq [1,2]$ Must match by value & position

Keep track of intermediate quantities - use names

$l = \not{X}\ \not{2}$ last_divisor $= \not{X}\ \not{2}\ \not{3}$ .. 18

By equation (12.2) we know

(*)

factors-m = [ ——— ]

factors-n = [ ——— ]

Names are also called variables

I bought some pens. 2 have broken. I have 7 left. How many did I buy?

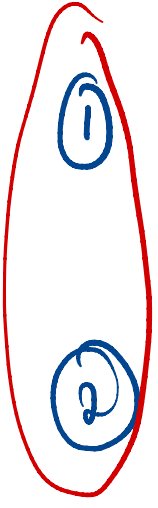
Let $x$ be # pens I bought.

Unknown but fixed quantity

$x - 2 = 7 \Rightarrow x = 9$

Names — values are periodically updated

Variables, but not in the same sense as maths



① gcd — scanning $i = 1$ to $\min(m, n)$ & checking for divisors of both numbers

② compute two lists of factors

↳ Directly compute cf

# Largest common divisor

Run from $i = \min(m,n)$ down to 1

1 always divides every number

$\gcd(m,n)$ is at least 1

$\gcd(m,n) = 1$ "co-prime"

go all the way to $\min(m,n)$

stop at first one

3rd but perhaps better brute force.

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How long do these take?

Worst-case $\gcd(m,n) = 1$

Forwards always checks $\min(m,n)$ divisors

Backwards also checks $\min(m,n)$ divisors

Time proportional to $\min(m,n)$

$\gcd(256, 729) = 1$

$2^8 \quad 3^6$

$\approx 256$ steps

$\gcd(1024, 2187)$ $\simeq 1024$ steps

Divide 256 by 2

1024 by 2

No of steps = No of digit

Need some cleverness to get there

"Reduce" the problem.

$d$ divides $m, n$ $\quad m > n$

$m = a \cdot d$

$n = b \cdot d$

$d$ divides $m, n$
$\Rightarrow d$ divides $m - n$

$$m - n = a \cdot d - b \cdot d = (a - b) \cdot d$$

$m - n = cd \qquad m = n + cd$

$gcd(m,n) \rightsquigarrow gcd(m-n,n)$ $\quad m > n$

$gcd(m,m) = m$

$gcd(256,729) = gcd(256, 473)$

$= gcd(256, 217)$

$= gcd(217, 39)$ ...

$gcd(1,1)$

Faster

$gcd(101, 2) = gcd(99, 2)$

$97, 2)$

$\vdots$

$91, 2)$

$\vdots$

$(3, 2)$

$(2, 1)$

$(1, 1) \longrightarrow 1$

$\frac{101}{2}$ steps!