Software Verification and Abstraction

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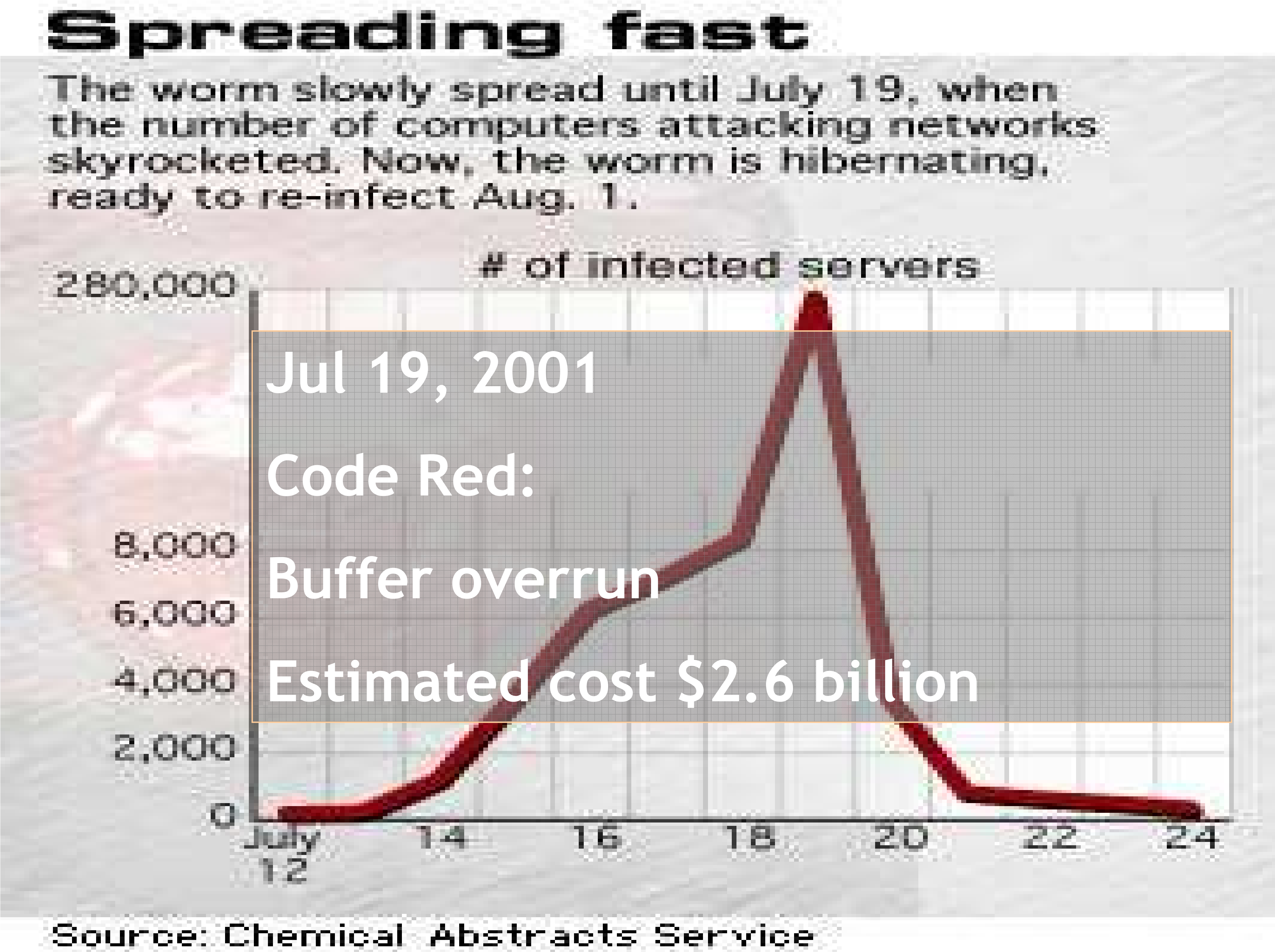


Jul 19, 2001

Code Red:

Buffer overrun

Estimated cost $2.6 billion

Software is Unreliable

The Premise

Reliability is important

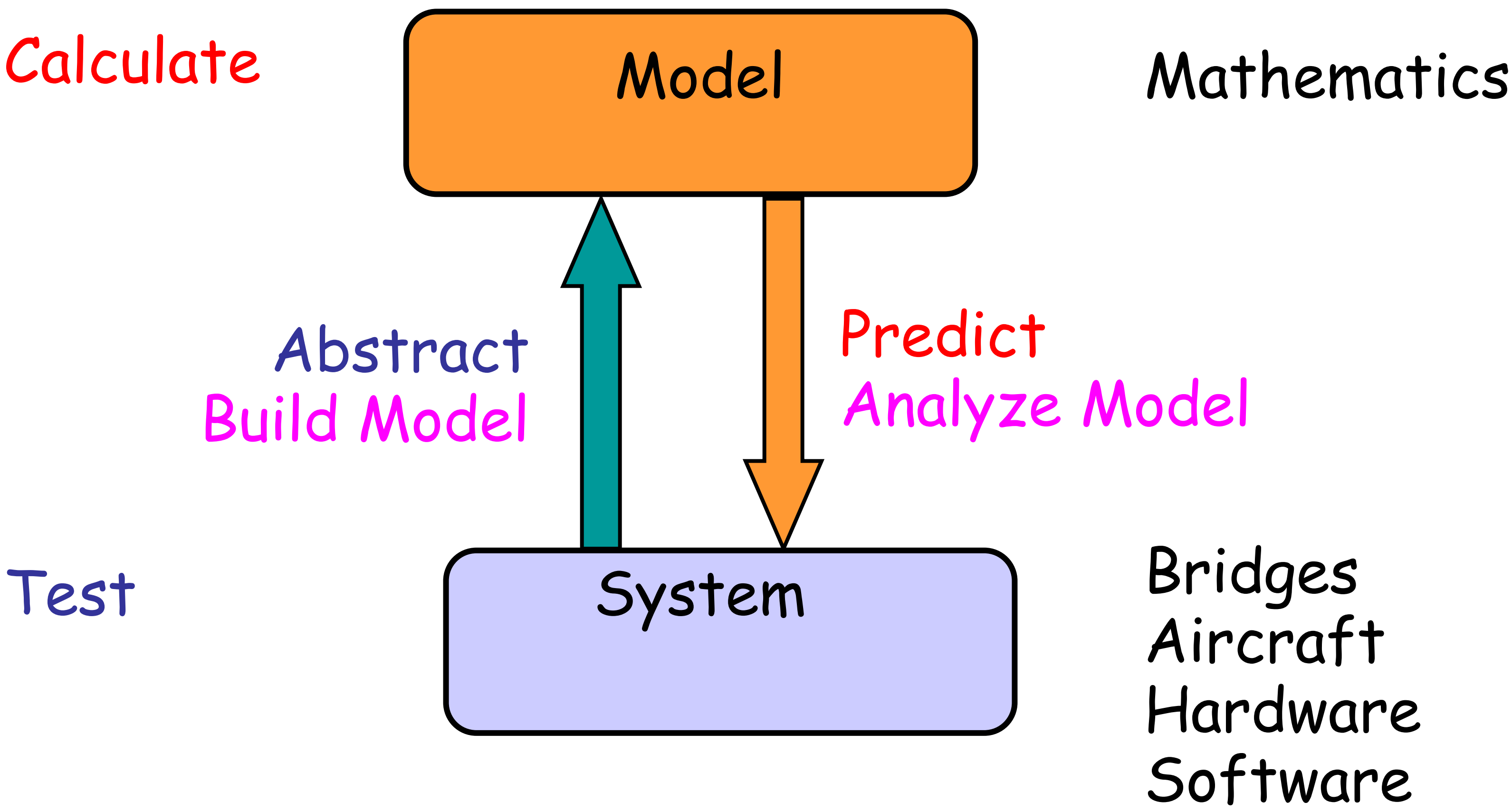
- More computers are used in embedded, networked, safety critical applications
- Cost of failure or fixing bug is high

Building large, complex, yet reliable systems is hard

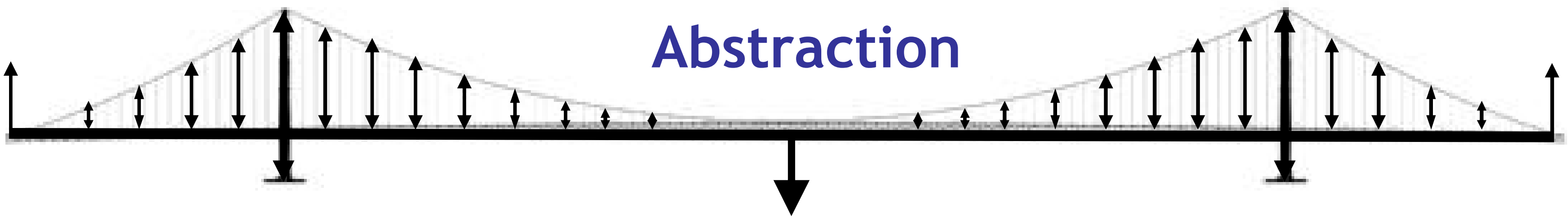
These lectures: What can we do about it?

- Survey research on automatic tools for ensuring a system is “correct”

Systems and Models



Mathematical Abstractions



	Bridges	Programs
Building Blocks	Mechanics	Logic
1. Relevant facts*	Mass, Tensile Strength	?
2. Model	ODEs/PDEs	?
3. Analysis	Solve Equations	?

* w.r.t. property of interest

System Verification Problem

$$I \quad \models \quad S$$

“Implementation”
System model

“Specification”
System properties

“models” “implements” “refines
Satisfaction relation

Example

C code for device driver $\models$ “Driver does not deadlock“

“Implementation”
System model

“Specification”
System properties

“models” “implements” “refines
Satisfaction relation

Example

Microprocessor design $\models$ ISA

"Implementation"
System model

"Specification"
System properties

"models" "implements" "refines
Satisfaction relation

Example

Electronic Control Unit $\models$ "Controller is stable"

"Implementation"
System model

"Specification"
System properties

"models" "implements" "refines
Satisfaction relation

Example

Transactional memory $\models$ Strict serializability

“Implementation”
System model

“Specification”
System properties

“models” “implements” “refines
Satisfaction relation

Lecture 1: Model Checking Basic Concepts

Rupak Majumdar

Model checking, generally interpreted:

Automatic algorithmic techniques for system verification which operate on a system model (semantics)

Somewhat General View

Model checking, narrowly interpreted:

Decision procedures for checking if a given Kripke structure is a model for a given formula of a modal logic (CTL or LTL).

Our view includes

- Dataflow analysis in compilers
- Symbolic execution based methods

Our view excludes

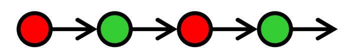
- Language design for ensuring properties
- Proof calculi and interactive theorem proving

There are many different model checking algorithms, depending on

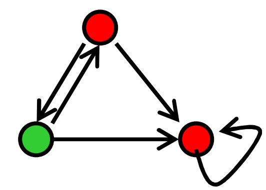
- The system model
- The specification formalism

Discrete Systems Theory

Trajectory: dynamic evolution of state
sequence of states



Model: generates a set of trajectories
transition graph



Property: assigns boolean values to trajectories
temporal logic formula

"red and green alternate"

Algorithm: compute values of the trajectories generated by a model

Paradigmatic Example: Mutual Exclusion

```
loop
   out:  x1 := 1; last := 1
   req:  await  x2 = 0  or  last = 2
   in:    x1 := 0
end loop.
```

P1

||

```
loop
   out:  x2 := 1; last := 2
   req:  await  x1 = 0  or  last = 1
   in:    x2 := 0
end loop.
```

P2

Property: It is never the case that P1 and P2 are both at `in’

System Modeling

- Various factors influence choice of model
  - State based vs event based
  - Concurrency model

- While the choice of system model is important for ease of modeling in a given situation,

the only thing that is important for model checking is that the system model can be translated into some form of state-transition graph.

- So: Will not focus much on syntactic constructs

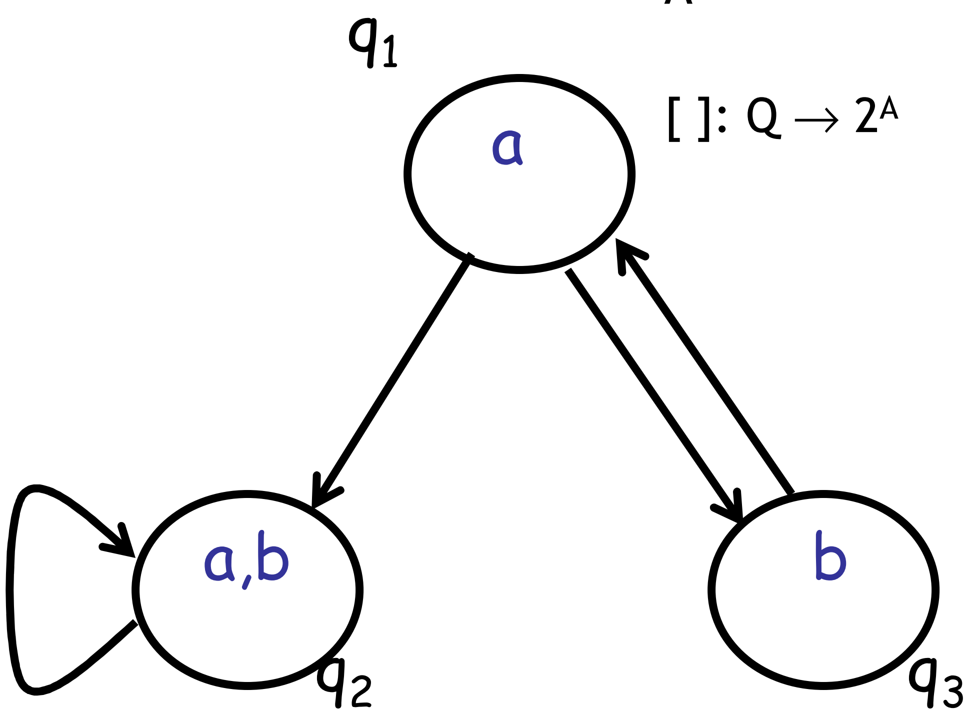
Syntax: Finite State Programs

- Parallel composition of C programs, without function calls
- Each variable has a finite range

- We'll write such programs as guarded commands

Semantics: State Transition Graph

Q	set of states	$\{q_1,q_2,q_3\}$
q_0	initial state	q_1
$\rightarrow \subseteq Q \times Q$	transition relation	$q_1 \rightarrow q_2$
A	set of atomic observations	$\{a,b\}$
$[\ ]: Q \rightarrow 2^A$	observation function	$[q_1] = \{a\}$



Important Restriction

Until notified, restrict attention to

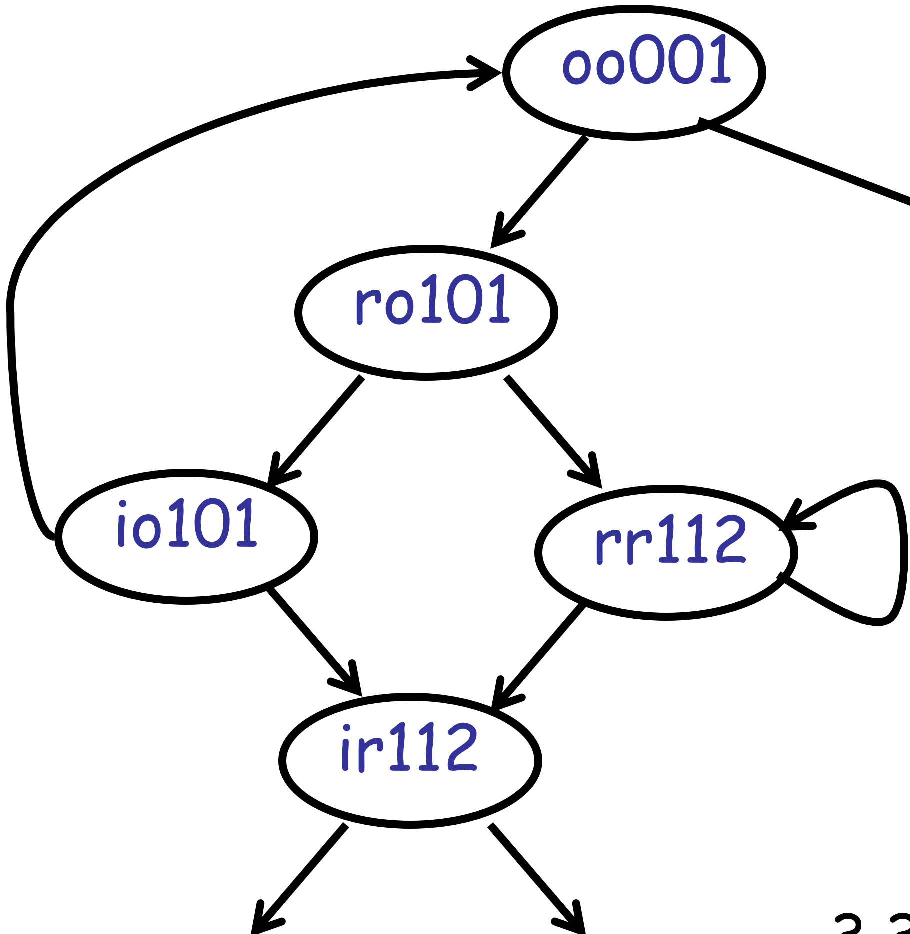
finite-state transition systems

Q is finite

Example: Mutual Exclusion

```
loop                                              ||  loop
   out:  x1 := 1; last := 1                              out:  x2 := 1; last := 2
   req:  await  x2 = 0  or  last = 2                     req:  await  x1 = 0  or  last = 1
   in:     x1 := 0                                       in:     x2 := 0
end loop.                                             end loop.

      P1                                                     P2
```



pc1: {o,r,I}
pc2: {o,r,i}
x1: {0,1}
x2: {0,1}
last: {1,2}

$3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 = 72$ states

State Explosion Problem

The translation from a system description to a state-transition graph usually involves an exponential blow-up !!!

e.g., n boolean variables $\Rightarrow$ 2^n states

System Verification Problem

$$I \models S$$

"Implementation"
System model

"models" "implements" "refines
Satisfaction relation

"Specification"
System properties

System Properties

Some orthogonal dimensions in choosing specification formalisms

1 operational vs. declarative:

automata vs. logic

2 may vs. must:

branching vs. linear time

2 prohibiting bad vs. desiring good behavior:

safety vs. liveness

The three decisions are orthogonal, and they lead to substantially different model-checking problems

Safety vs Liveness

- Safety: Something "bad" will never happen
  - Program does not produce bad result

    "partial correctness"
  - *Example*: *Mutual exclusion*

- Liveness: Something "good" eventually happens
  - The program produces a result "termination"
  - *Example*: *A process wanting to go to the critical section eventually gets in*

Safety vs Liveness Contd.

- Safety: those properties whose violation always has a finite witness
  - "if something bad happens on an infinite run, then it happens already on some finite prefix" --- Can be checked on finite runs
- Liveness: those properties whose violation never has a finite witness
  - "no matter what happens along a finite run, something good could still happen later" --- Must be checked on infinite runs

Two Remarks

1. The vast majority of properties to be verified are safety

2. While nobody will ever observe the violation of a true liveness property, liveness is a useful abstraction that turns complicated safety into simple liveness

Accordingly, we focus on safety for most of the lectures

Safety Model Checking

- Requirement: The system should always stay within some safe region

- Input: A state transition graph
- Input: A set of good states “invariants”

- Output: “Safe” if all executions maintain the invariant, “Unsafe” otherwise (and a trace)

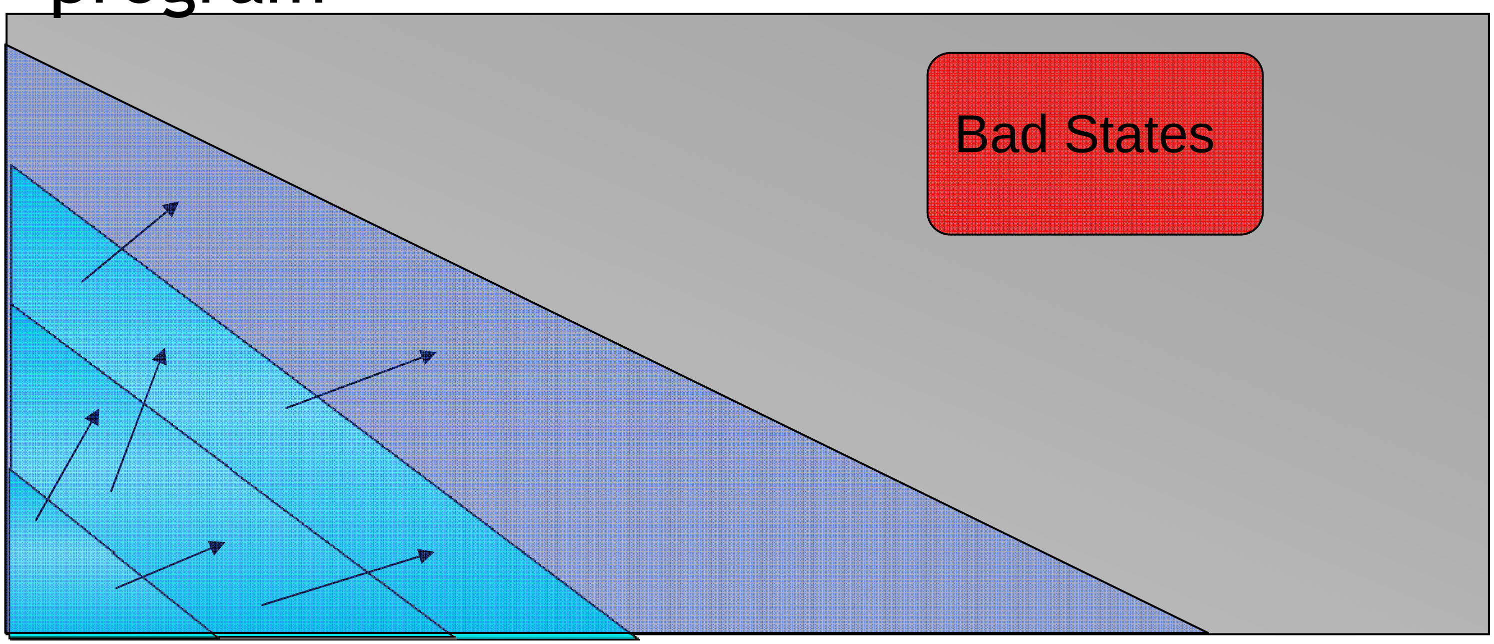
From Safety to Reachability

- Input: A state transition graph
- Input: A set of bad states

- Output: “Safe” if there is no run from an initial state to any bad state, “Unsafe” otherwise (and a trace)

Model Checking Algorithm

- Graph Search
  - Linear time in the size of the graph
  - Exponential time in the size of the program



Enumerative Model Checking

- Provide access to each state
- For each state, provide access to neighboring states

- Implement classical graph algorithms
  - Depth-first or breadth-first search
  - Starting from initial states and searching forward for bad states
  - Or starting from bad states and searching backward for initial states

State Space Explosion

- Biggest problem is state space explosion
  - N bits $\Rightarrow 2^N$ states
- Many heuristics
  - Search on-the-fly,
  - partial order and symmetry reduction
  - Do not store dead variables

- Many successful implementations
    - Spin, Murphi, Verisoft, … [Protocol verification]

Symbolic Model Checking

- Idea: Work with sets of states, rather than individual states

Given: Transition graph G, target states σ^T
begin
- σ^R = set of Initial states
- repeat forever
  if $\sigma^R \cap \sigma^T \neq \emptyset$ then return "yes"
  if $Post(\sigma^R) \subseteq \sigma^R$ then return "no"
  $\sigma^R := \sigma^R \cup Post(\sigma^R)$
  end

Here, $Post(\sigma) = \{s' \mid \exists s \in \sigma.\ s \to s'\}$

Encoding Sets through Formulas

- Idea: Represent sets of states symbolically, using constraints

- E.g., $1 \leq x \leq 100$ represents the 100 states $x = 1$, $x = 2$, …, $x = 100$

- Represent both sets of initial states and transition relation implicitly

Representing States as *Formulas*

$[F]$ states satisfying F $\{s \mid s \models F\}$	F FO fmla over prog. vars
$[F_1] \cap [F_2]$	$F_1 \wedge F_2$
$[F_1] \cup [F_2]$	$F_1 \vee F_2$
$\overline{[F]}$	$\neg F$
$[F_1] \subseteq [F_2]$	F_1 implies F_2

i.e. $F_1 \wedge \neg F_2$ unsatisfiable

Symbolic Transition Graph

- A transition graph
  - A Formula Init(x) representing initial states
  - A Formula TR(x,x') representing the transition relation

- Example: C program

x:=e	TR(x,x'): $loc=pc \wedge loc'=pc' \wedge x' = e \wedge \{ y'=y \mid y \neq x\}$
Assume(p)	TR(x,x'): $loc=pc \wedge loc'=pc' \wedge p$

Symbolic Transition Graph

- Operations:
  - $\text{Post}(X) = \{s' \mid \exists s \in X.\ s \rightarrow s'\}$
    $= \exists s.\ X(s) \wedge TR(s,s')$
  - $\text{Pre}(X) = \{s \mid \exists s' \in X.\ s \rightarrow s'\}$
    $= \exists s'.\ TR(s,s') \wedge X(s')$
- Can implement using formula manipulations

Symbolic Model Checking

Given: Transition graph G, target states σ^T

begin

- σ^R = Formula representing set of Initial states
- repeat forever

  if $\sigma^R \wedge \sigma^T$ is satisfiable then return "yes"

  if $Post(\sigma^R) \Rightarrow \sigma^R$ then return "no"

  $\sigma^R := \sigma^R \vee Post(\sigma^R)$

  end

Here, $Post(\sigma)(s') = \exists s.\ \sigma(s) \wedge TR(s,s')$

Can be implemented using decision procedures for the language of formulas

Finite State Systems

- Symbolic representation in propositional logic
- State described by n bits X
- A region is a propositional formula with free variables in X

- Can implement symbolic operations using propositional formula manipulations

Example: Mutual Exclusion

```
loop                                      ||   loop
  out:  x1 := 1; last := 1                       out:  x2 := 1; last := 2
  req:  await  x2 = 0  or  last = 2              req:  await  x1 = 0  or  last = 1
  in:     x1 := 0                                in:     x2 := 0
end loop.                                      end loop.
```

Symbolic representation has variables

pc1,pc2, x1, x2, last

Initial states:

$pc1=out \wedge pc2=out \wedge x1=0 \wedge x2=0$

No constraint on last

Transition relation:

$pc1=out \wedge x1'=1 \wedge last'=1 \wedge pc2'=pc2 \wedge x2'=x2$

$\vee$ ...

Additional Desirable Properties

- All operations must be efficient in practice
- Should maintain compactness whenever possible
- Canonical representations

- Representing initial states and transition relation from the program description should be efficient

Binary Decision Diagrams

- Efficient representations of boolean functions

[Bryant86]

- Share commonalities
- Ordered BDDs:
  - Fix a linear ordering of the variables in X
  - BDD = DAG, with nodes labeled with boolean variables
  - Each variable occurs 0 or 1 times along a path
  - Paths in the DAG encode assignments to variables

- Extremely successful in hardware verification

More on Safety Properties

- Not all safety properties can be written as invariants on the program state space
- For example, if correctness depends on the order of events
  - Locks can be acquired and released in alternation, it is an error to acquire/release a lock twice in succession without an intermediate release / acquire

Monitors

- Write the ordering of events as an automaton (called the monitor)

- Take the product of the system with the monitor
  - The monitor tracks the sequence of events
  - It goes to a special “bad” state if a bad sequence occurs

- Now we can express the property as an invariant: the monitor state is never bad

Symbolic Search

- Guaranteed to terminate for finite state systems
- And can be applied to infinite state systems as well
  - Although without guarantees of termination in general
  - Application to infinite state requires richer languages for formulas and associated decision procedures

What about Software?

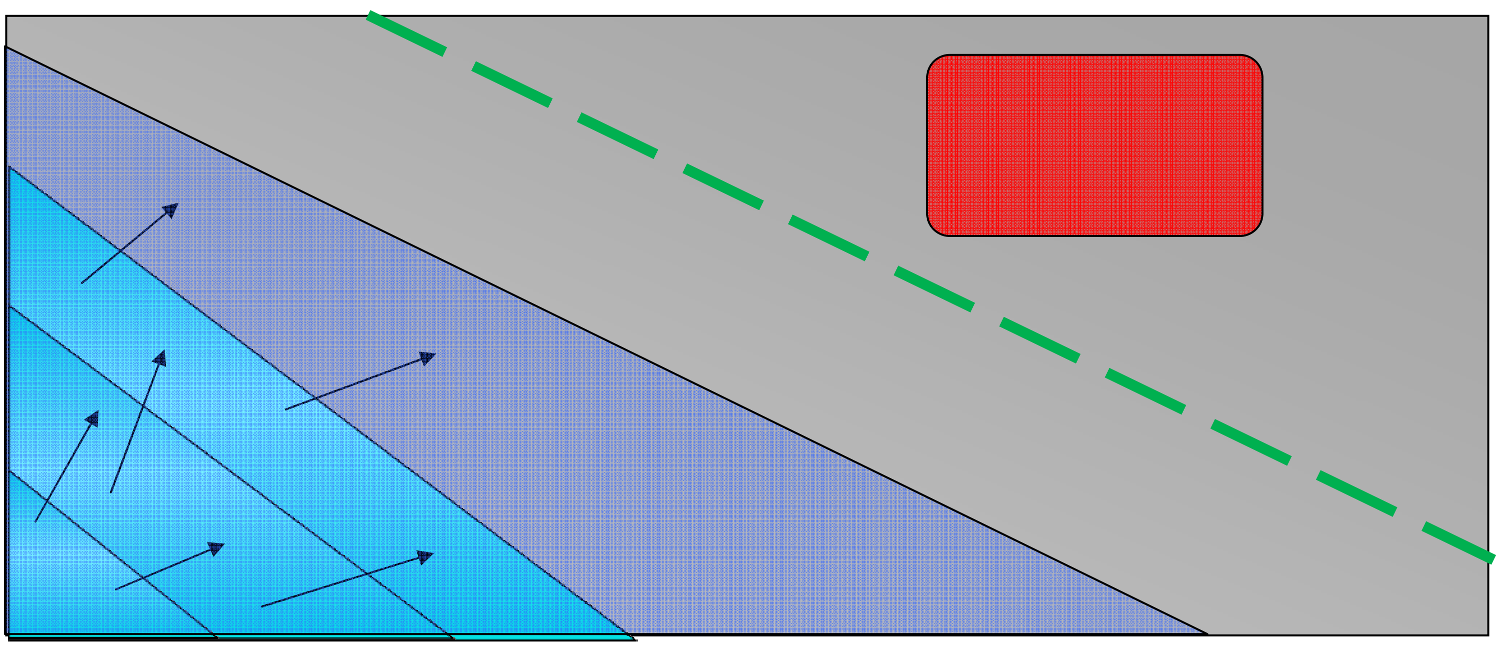
- Can construct an infinite state transition system from a program
- States: The state of the program
  - (stack, heap, pc location)
- Transitions: $q \rightarrow q'$ iff in the operational semantics, there is a transition of the program from q to q'
- Initial state: Initial state of the program

Termination

- Each operation can be computed
- But iterating Pre or Post operations may not terminate
- What do we do now?

Observation

- Often, we do not need the exact set of reachable states
  - We need a set of states that separates the reachable states from the bad states



One Possibility

- User gives an estimate (inductive invariant)

  A set of states Inv such that

  - Init $\subseteq$ Inv   - Inv $\cap$ bad $= \emptyset$ - Post(Inv) $\subseteq$ Inv

* Can show that this implies system is safe (How?)

* Given Inv, and decision procedures, this procedure is guaranteed to terminate

- This is the idea of classical loop invariants
  - Problem: In general, it can be hard to manually construct Inv

Before we proceed

- What is the sign of the following product:

  - 12433454628 * 94329545771 ?

Idea

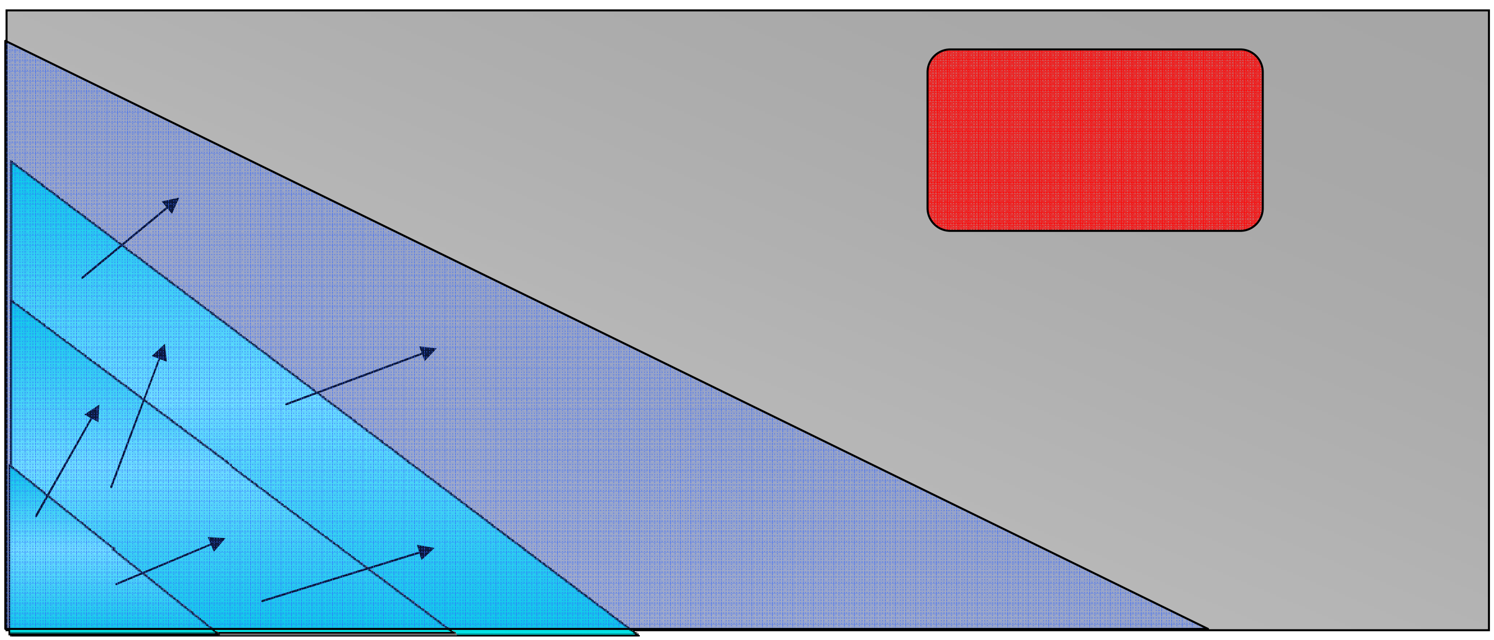
- One can “abstract” the behavior of the system, and yet reason about certain aspects of the program

- Abstraction:

    -ve * +ve = -ve

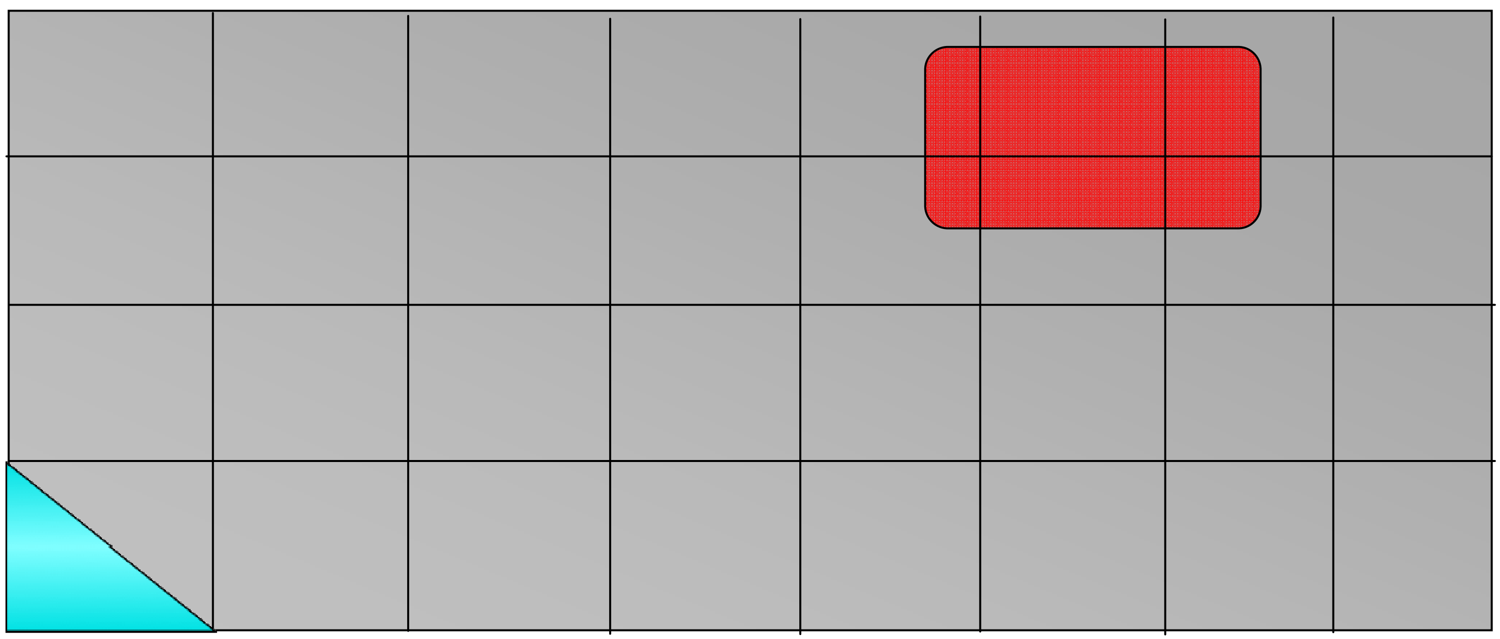
Model Checking Algorithm

- Graph Search



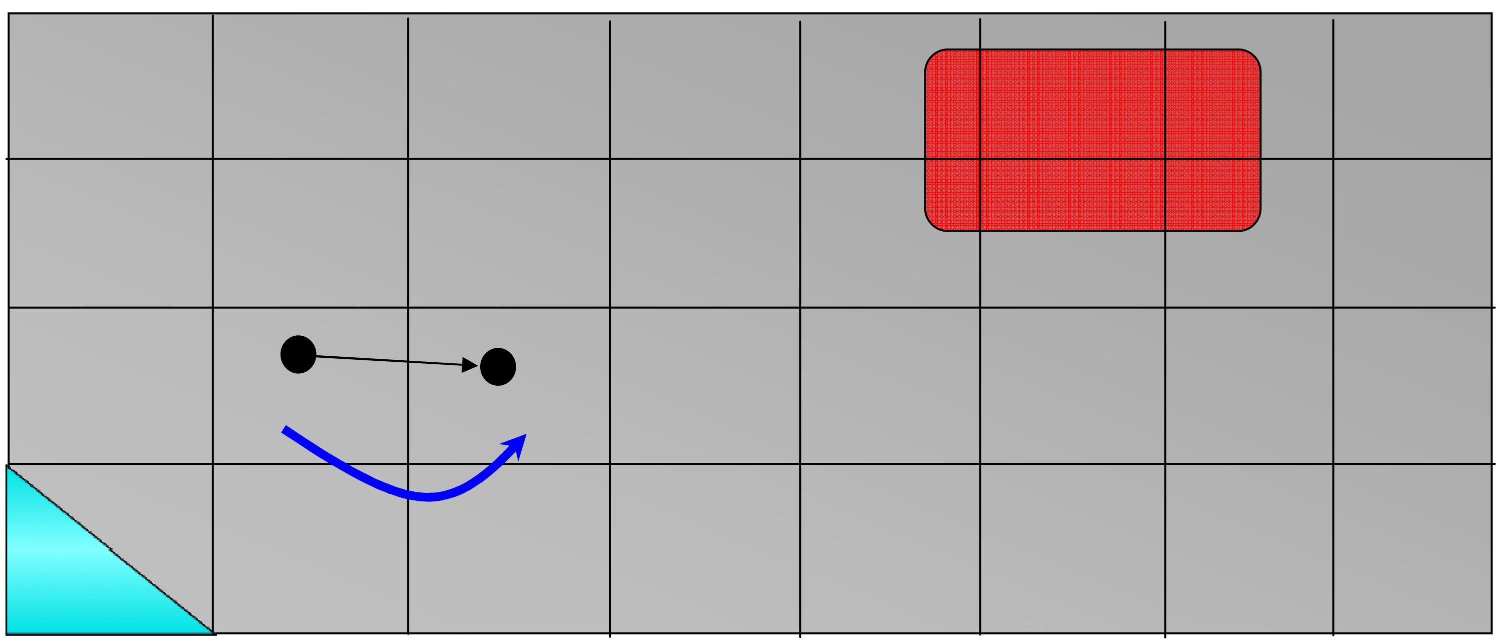
Abstract Interpretation

- The state transition graph is large/infinite
- Suppose we put a finite grid on top



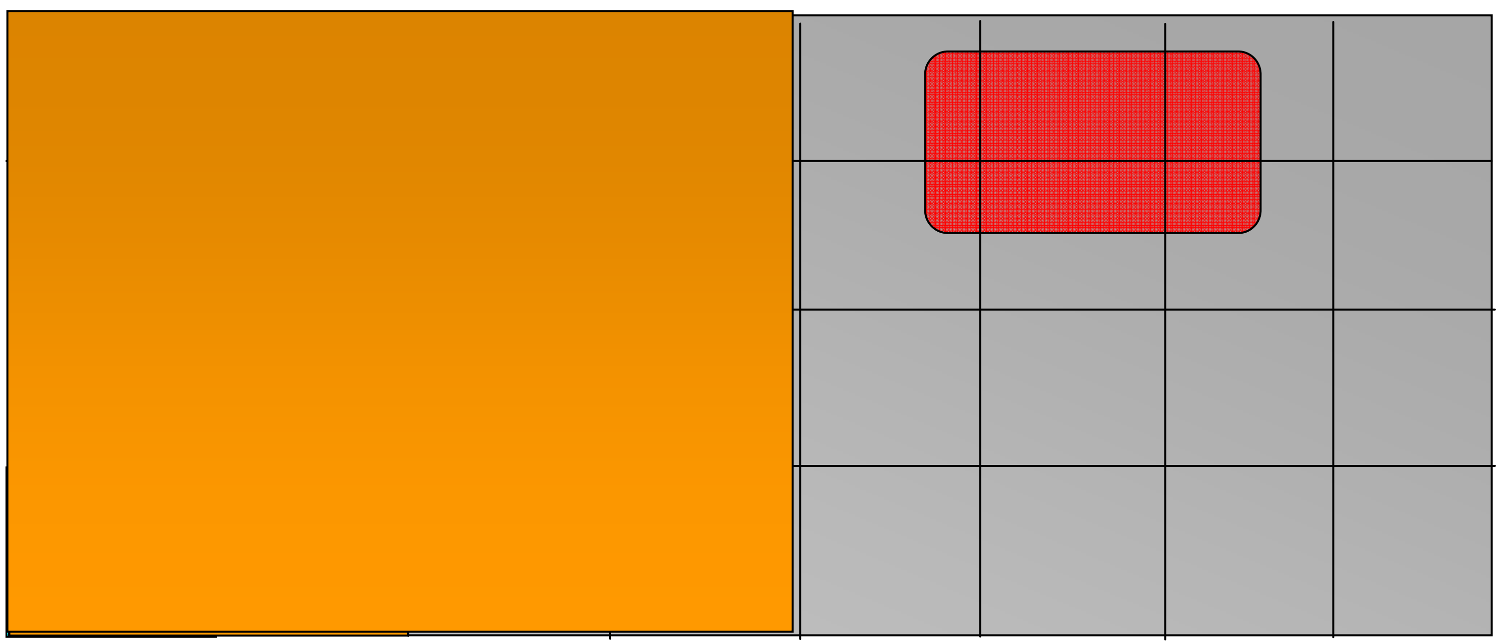
Existential Abstraction

- Every time s $\rightarrow$ s', we put [s] $\rightarrow$ [s']
- This allows more behaviors



Abstract Model Checking

- Search the abstract graph until fixpoint
  - Can be much smaller than original graph
  - Can be finite, when original is infinite



Simulation Relations

- A relation $\preceq \subseteq Q \times Q$ is a simulation relation if $s \preceq s'$ implies
  - Observation(s) = Observation(s')
  - For all t such that $s \rightarrow t$

    there exists t' such that $s' \rightarrow t'$

    and $s' \preceq t'$

Formally captures notion of "more behaviors"

Implies containment of reachable behaviors

Main Theorem

- $s \preceq [s]$ is a simulation relation

- If an error is unreachable in Abs(G) then it is unreachable in G

- Plan:

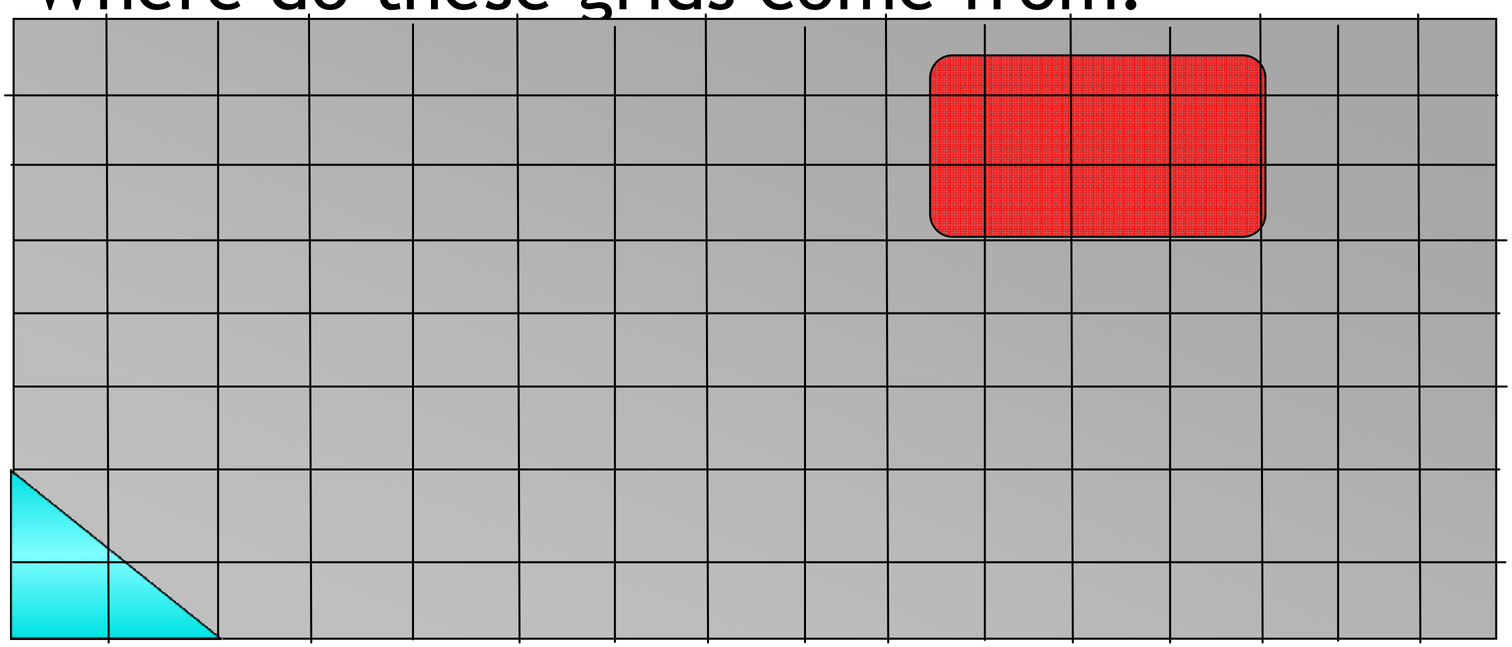
1. Find a suitable grid to make the graph finite state
2. Run the finite-state model checking algorithm on this abstract graph
3. If abstract graph is safe, say “safe” and stop

What if the Abstract Graph says Unsafe?

- The error may or may not be reachable in the actual system
  - Stop and say "Don't know"

What if the Abstract Graph says Unsafe?

- Or, put a finer grid on the state space
- And try again
  - The set of abstract reachable states is smaller
  - Where do these grids come from?

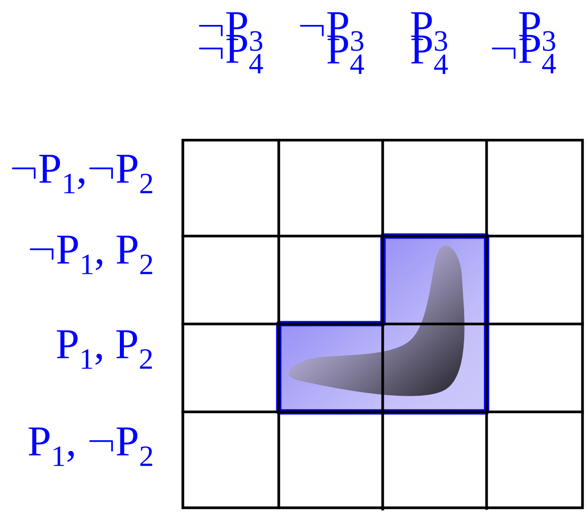


Grids: Predicate Abstraction

- Suppose we fix a set of facts about program variables
  - E.g., old = new, lock = 0, lock = 1
- Grid: Two states of the program are equivalent if they agree on the values of all predicates
  - N predicates = 2^N abstract states
- How do we compute the grid from the program?

Predicate Abstraction

Region Representation: formulas over predicates



Karnaugh Map

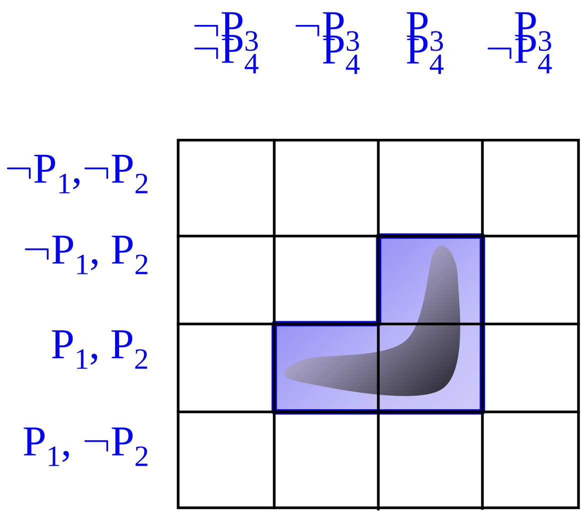
$P_1 : x = y$ $P_2 : z = t + y$

$P_3 : x \leq z+1$ $P_4 : *u = x$

Set of states

Abstract Set: $P_1P_2P_4 \vee \neg P_1 P_2 P_3 P_4$

Predicate Abstraction



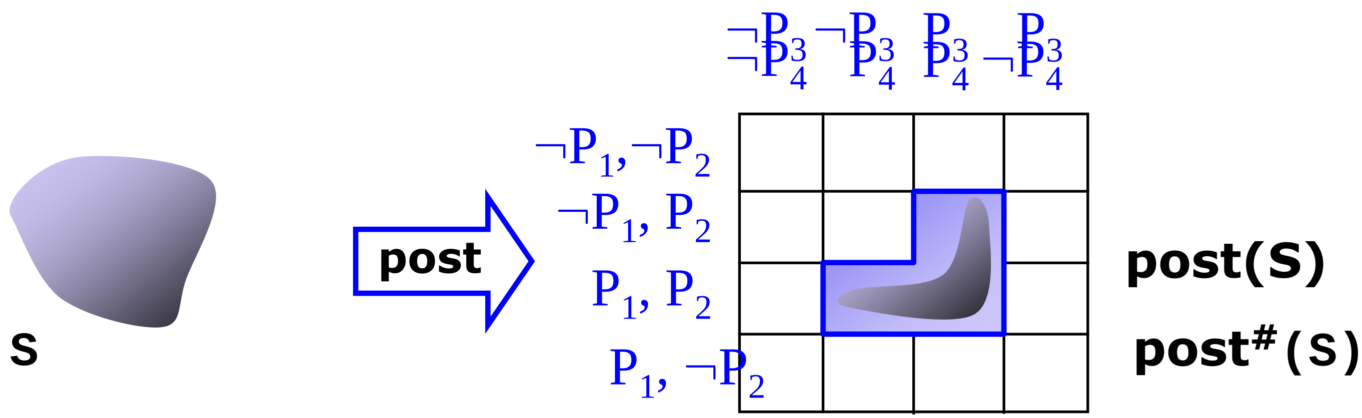
Karnaugh Map

$P_1 : x = y$ $\quad$ $P_2 : z = t + y$

$P_3 : x \leq z{+}1$ $\quad$ $P_4 : {*}u = x$

- Box: abstract variable valuation
- BoxCover(S): Set of boxes covering S
- Theorem prover used to compute BoxCover

Post#, Pre

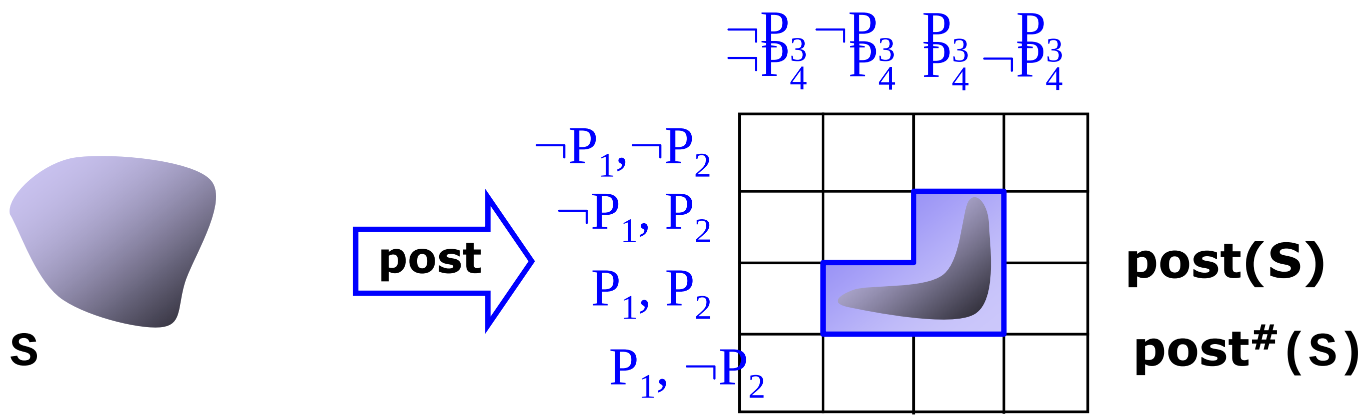


- pre(S,op) = { s | $\exists$s'$\in$S. s $\rightarrow^{op}$ s'} (Weakest Precondition)
- post(S,op) = { s | $\exists$s'$\in$S. s' $\rightarrow^{op}$ s} (Strongest Postcondition)

- Abstract Operators: post#

  post(S,op) $\subseteq$ post#(S,op)

Computing Post$^{\#}$



- For each predicate p, check if
  - S⇒ Pre(p, op) then have a conjunct p
  - S⇒ Pre(¬ p, op) then have a conjunct ¬ p
  - Else have no conjunct corresponding to p

- Use a theorem prover for these queries

Example

- I have predicates
  - lock=0, new=old, lock=1
- My current region is lock = 0 $\wedge$ new= old
- Consider the assignment new = new+1

- What is abstract post?

Example

- WP(new:=new+1, lock=0) is lock=0
- WP(new:=new+1, lock=1) is lock=1
- WP(new:=new+1, new=old) is new+1=old

- lock=0$\wedge$ new=old $\Rightarrow$ lock = 0 YES
- lock=0$\wedge$ new=old $\Rightarrow$ lock $\neq$ 0 NO
- lock=0$\wedge$ new=old $\Rightarrow$ lock = 1 NO
- lock=0$\wedge$ new=old $\Rightarrow$ lock $\neq$ 1 YES
- lock=0$\wedge$ new=old $\Rightarrow$ new+1=old NO
- lock=0$\wedge$ new=old$\Rightarrow$ new+1$\neq$ old YES

- So post is lock = 0 $\wedge$ lock$\neq$ 1 $\wedge$ new$\neq$ old

Symbolic Search with Predicates

Symbolic representation:
   Boolean formulas of (fixed set of) predicates

- Boolean operations: easy
- Emptiness check: Decision procedures

- Post: The abstract post computation algorithm

- Can now implement symbolic reachability search!

Big Question

- Who gives us these predicates?

- Answer 1: The user
  - Manual abstractions
    - Given a program and property, the user figures out what are the interesting predicates
  - Dataflow analysis
    - For "generic" properties, come up with a family of predicates that are likely to be sufficient for most programs

Abstract Interpretation

- Abstract model checking is formalized through abstract interpretation
  - Formalizes and unifies semantics-based program analysis

More Approximations

- Many program dataflow analyses do not perform exact reachability analysis on the abstract state space

- Instead, use the structure of the control flow graph to further approximate the result

Example: Flow Sensitive Analysis

- For each control flow node, keep track of the set of reachable states (along any program path) to that node
  - Information may be lost at merge points by abstracting $\vee$ by something coarser
- Assumption: All paths of the control flow graph can be executed
  - Ignore conditional statements

Flow Insensitive Analysis

- Even more approximate
- Disregard the order of operations in the program!
- Much faster analysis than abstract model checking
  - But results are much cruder of course!
  - Can still be useful: e.g., primary way to perform alias analysis

When I run a model checker, it goes to compute the result and never comes back. When I run a dataflow analysis, it comes back immediately and says “Don’t know”!

– Patrick Cousot

Questions?

Lecture 2:
Software Model Checking
and
Counterexample-Guided Refinement

Rupak Majumdar

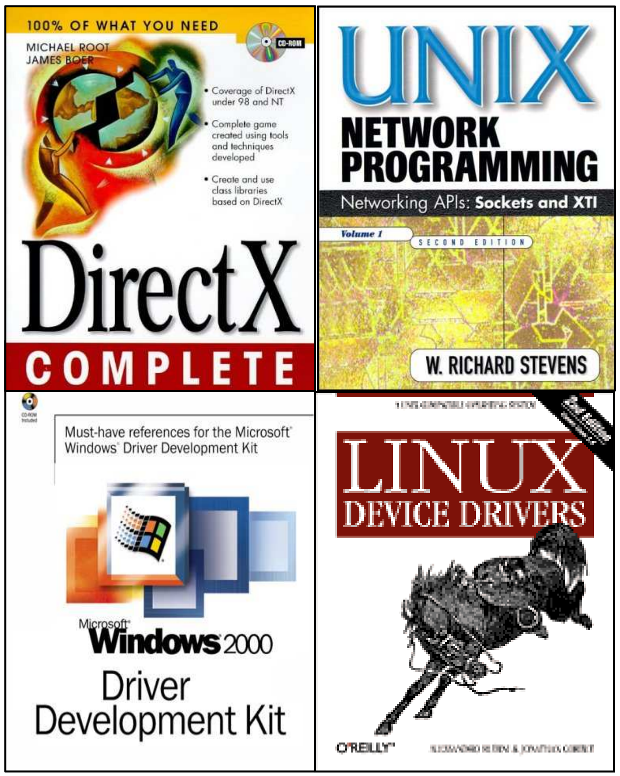
Recap

- Model checking is an algorithmic technique to verify properties of systems
- In conjunction with abstractions, can be effective in proving subtle properties
- Today: Consider the problem of abstract model checking of (sequential) software implementations

Setting: Property Checking

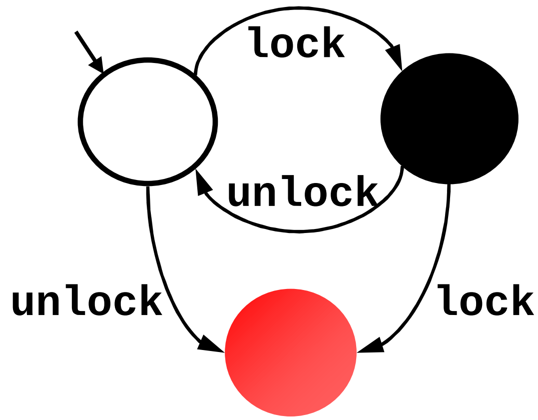
- Programmer gives partial specifications
- Code checked for consistency w/ spec
- Different from program correctness
  - Specifications are not complete
  - Is there a complete spec for Word ? Emacs ?

Interface Usage Rules



- Rules in documentation
  - Order of operations & data access
  - Resource management
  - Incomplete, unenforced, wordy
- Violated rules $\Rightarrow$ bad behavior
  - System crash or deadlock
  - Unexpected exceptions
  - Failed runtime checks

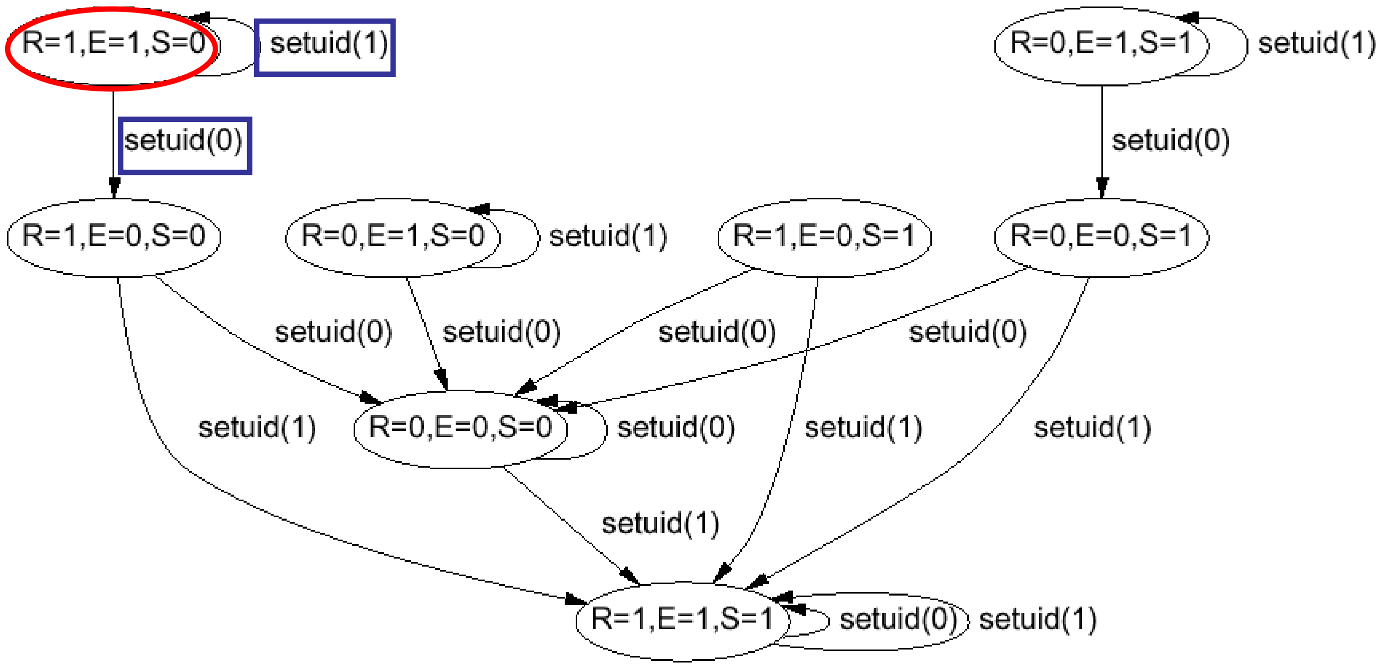
Property 1: Double Locking



"An attempt to re-acquire an acquired lock or release a released lock will cause a ***deadlock****."*

Calls to **lock** and **unlock** must **alternate**.

Property 2: Drop Root Privilege

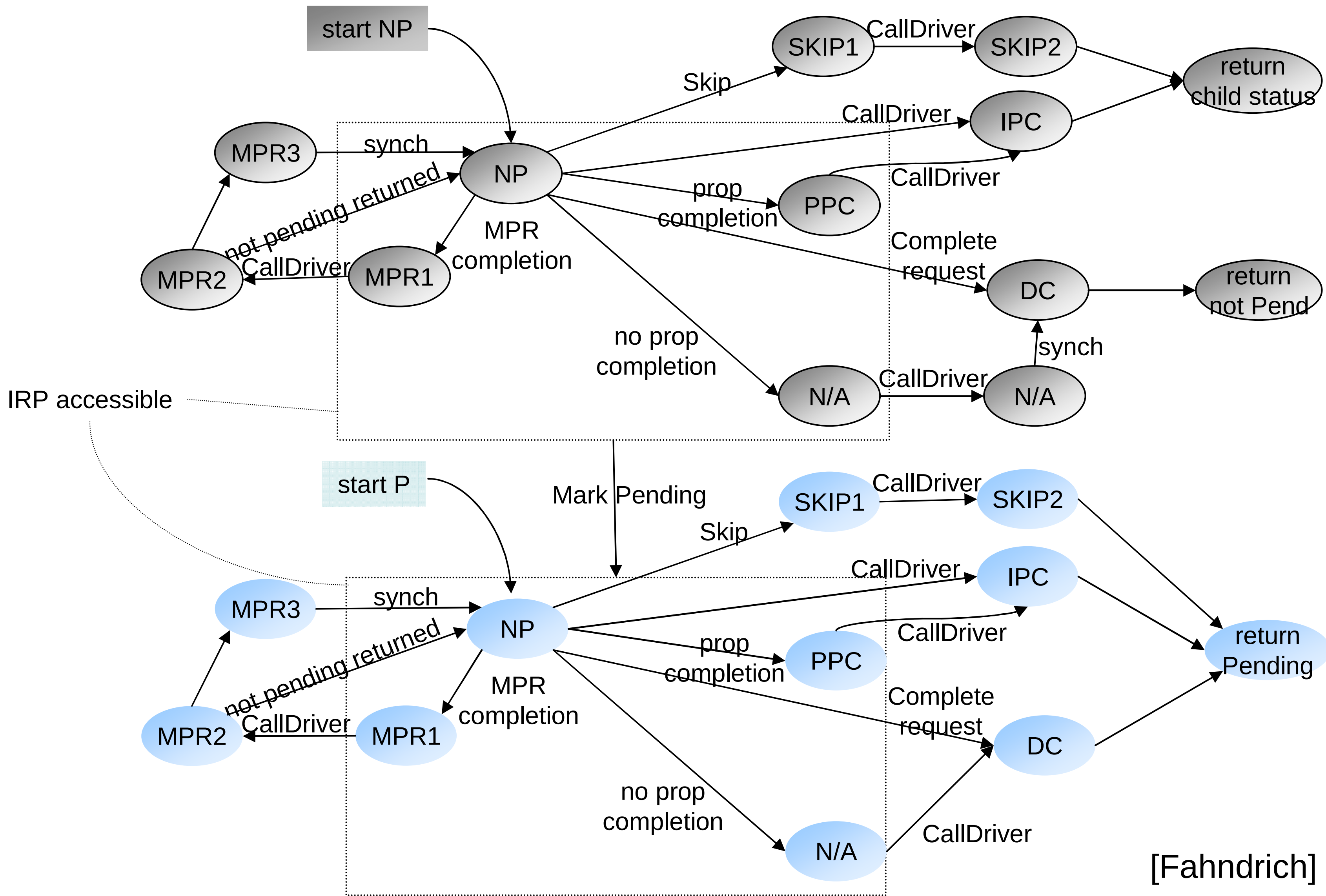


[Chen-Dean-Wagner '02]

"User applications must not run with root privilege"

When **execv** is called, must have **suid ≠ 0**

Property 3 : IRP Handler



[Fahndrich]

Does a given usage rule hold?

- Undecidable!
  - Equivalent to the halting problem
- Restricted computable versions are prohibitively expensive (PSPACE)
- Why bother ?
  - Just because a problem is undecidable, it doesn’t go away!

Example

```
Example ( ) {
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        lock();
        old = new;
        q = q->next;
2:      if (q != NULL){
3:          q->data = new;
            unlock();
            new ++;
        }
4: } while(new != old);
5:   unlock ();
     return;
}
```
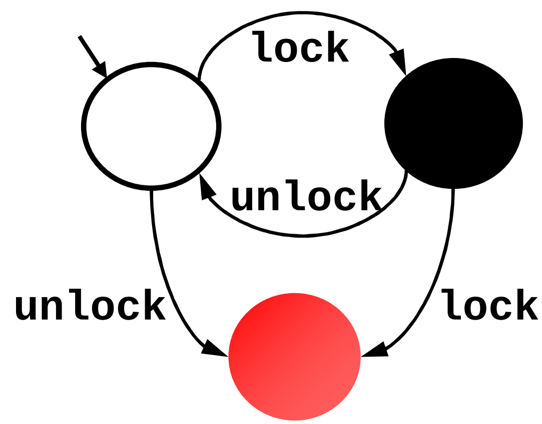
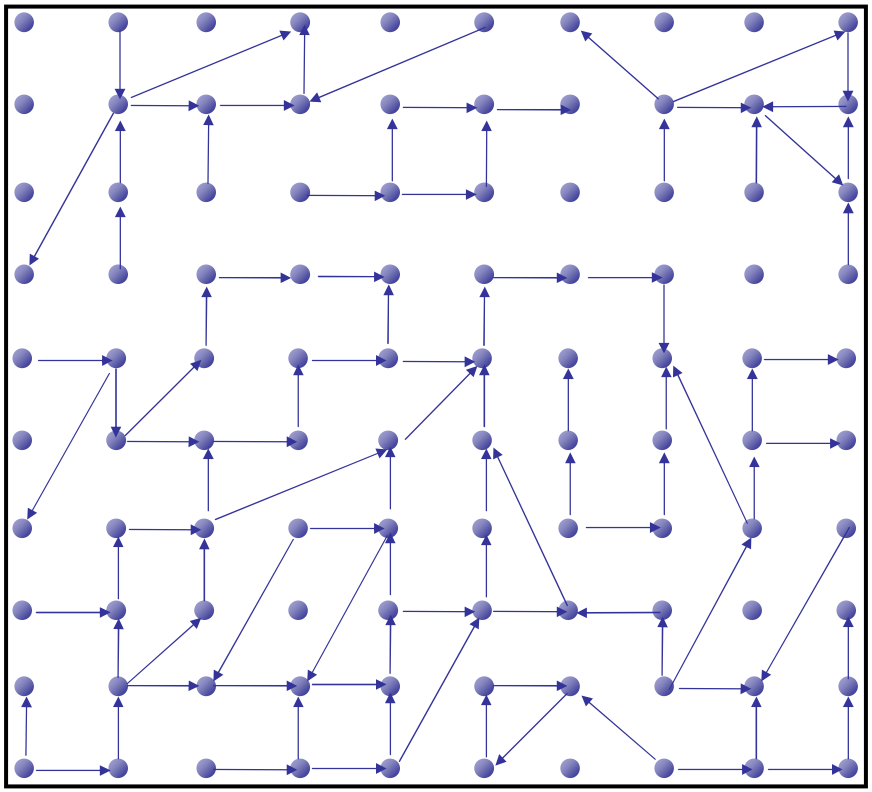



What a program *really* is…



pc	□	3
lock	□	●
old	□	5
new	□	5
q	□	0x133a

```
3: unlock();
   new++;
4:} …
```

pc	□	4
lock	□	O
old	□	5
new	□	6
q	□	0x133a

```
Example ( ) {
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      }
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```
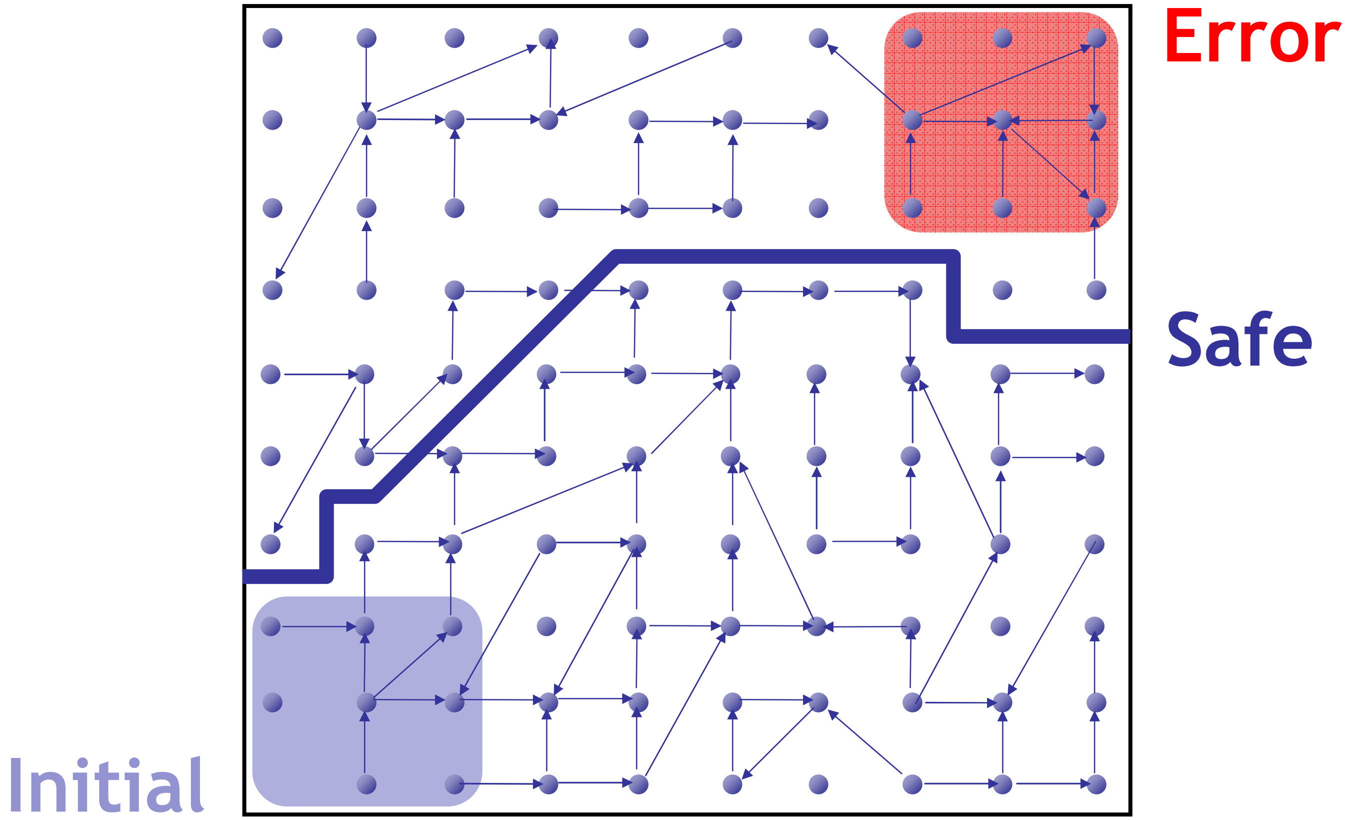

The Safety Verification Problem

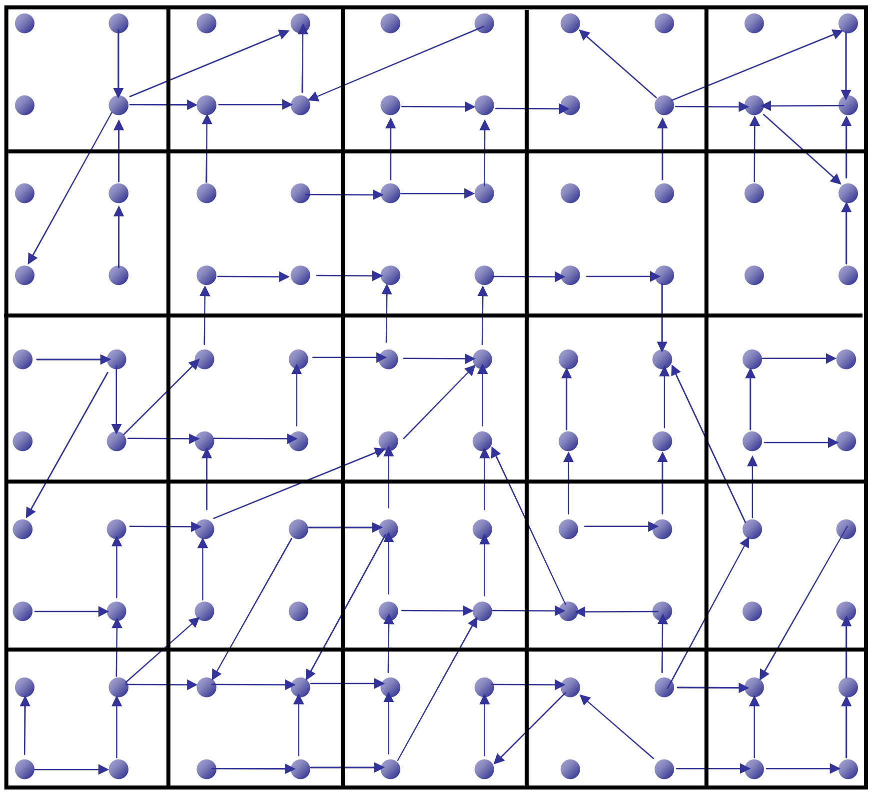


Is there a **path** from an **initial** to an **error** state ?

Problem: **Infinite** state graph

Solution : **Set** of states $\simeq$ logical **formula**

Idea 1: Predicate Abstraction

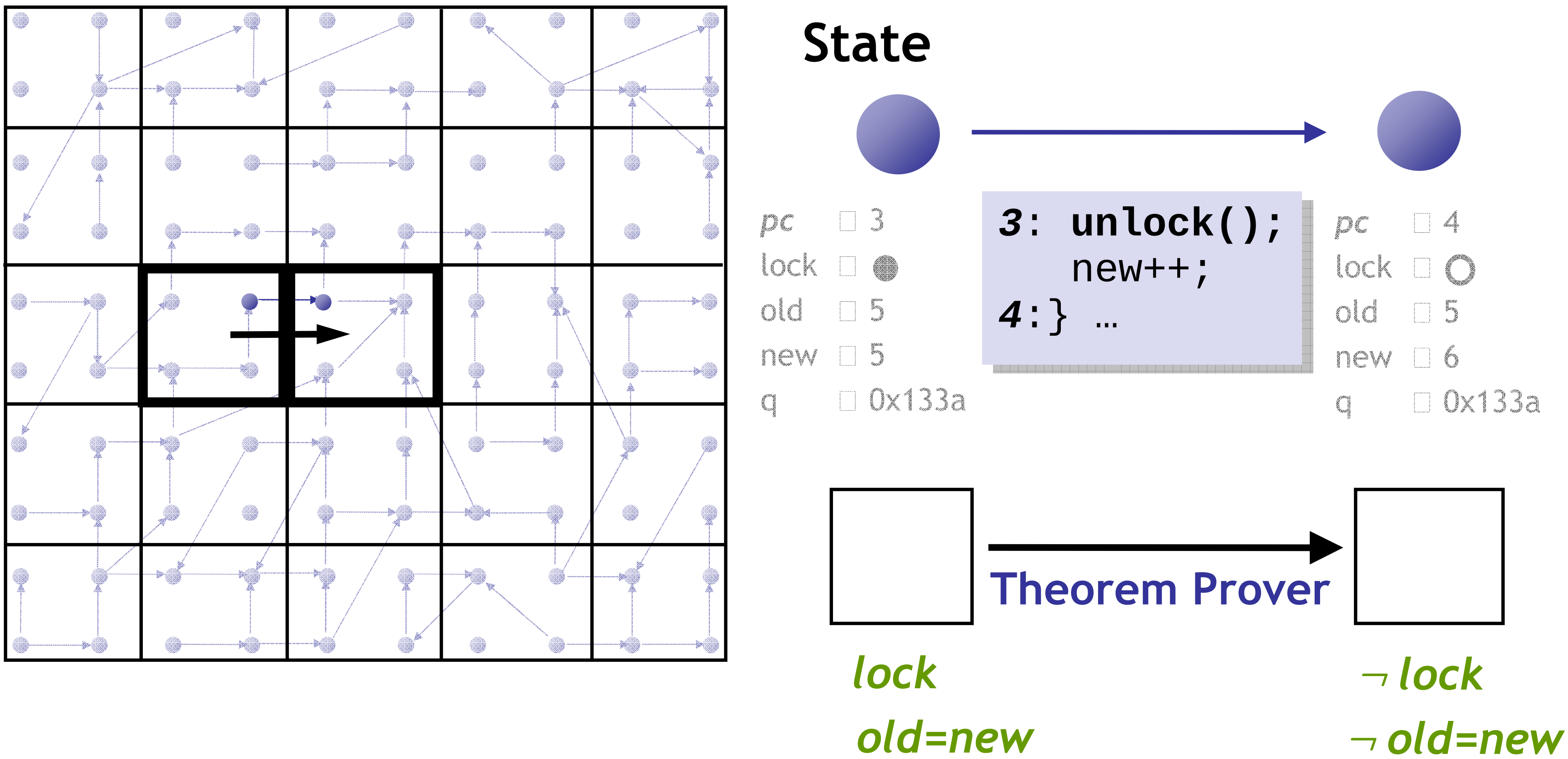


- **Predicates** on program state:
  lock
  old = new

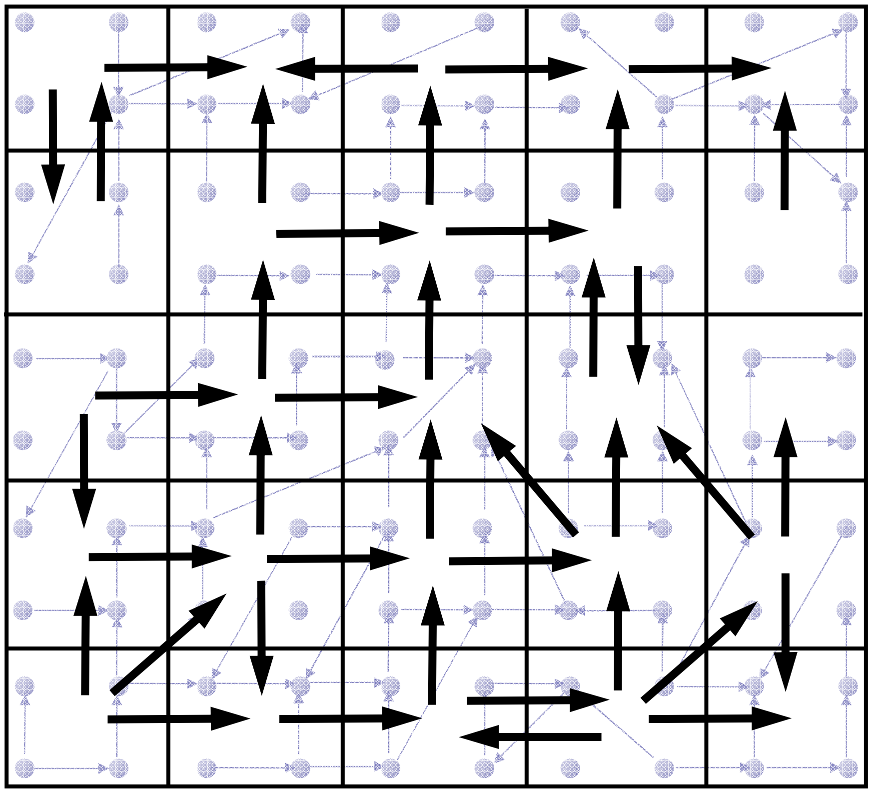
- States satisfying **same** predicates are **equivalent**
  - **Merged** into one **abstract state**

- #abstract states is **finite**

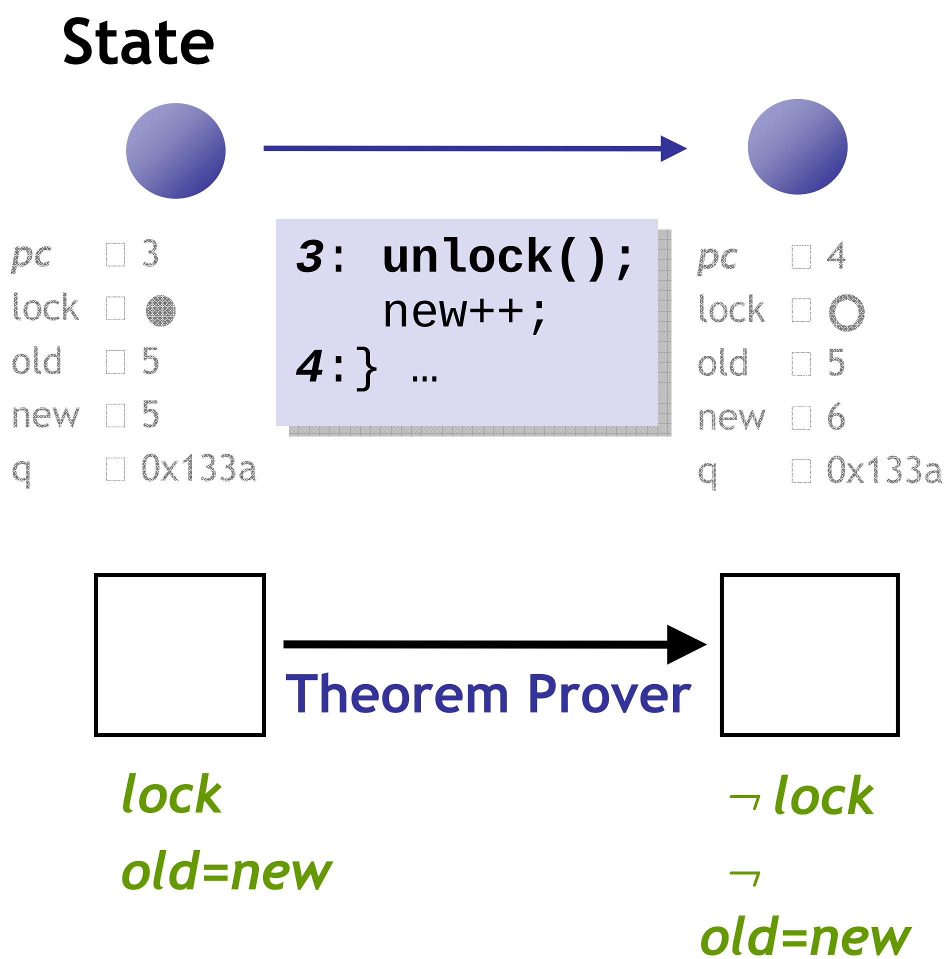
Abstract States and Transitions



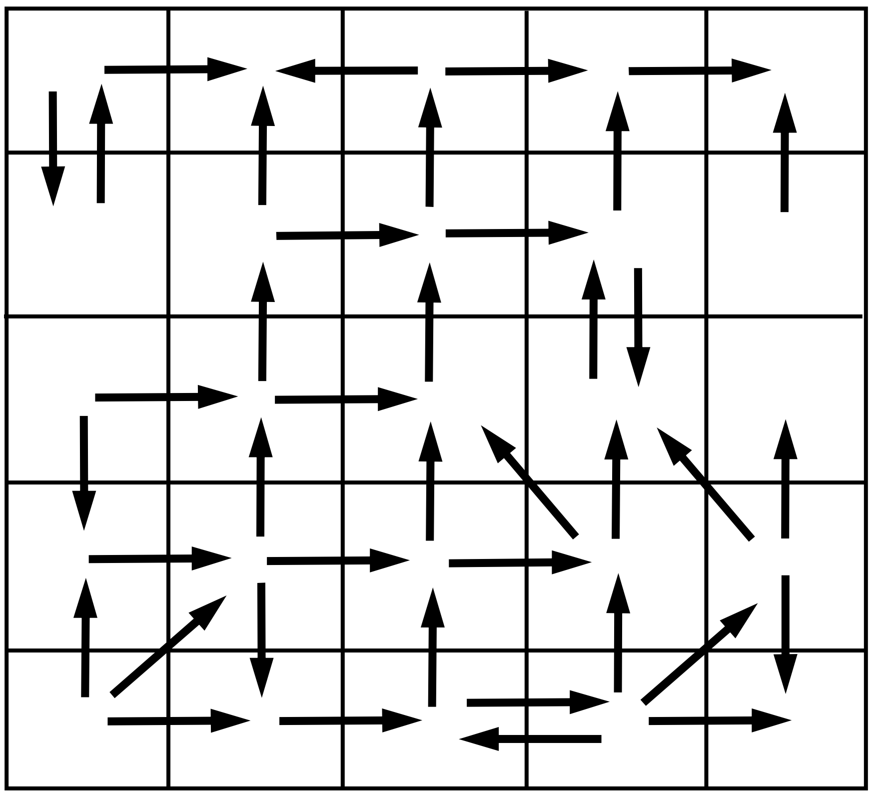
Abstraction

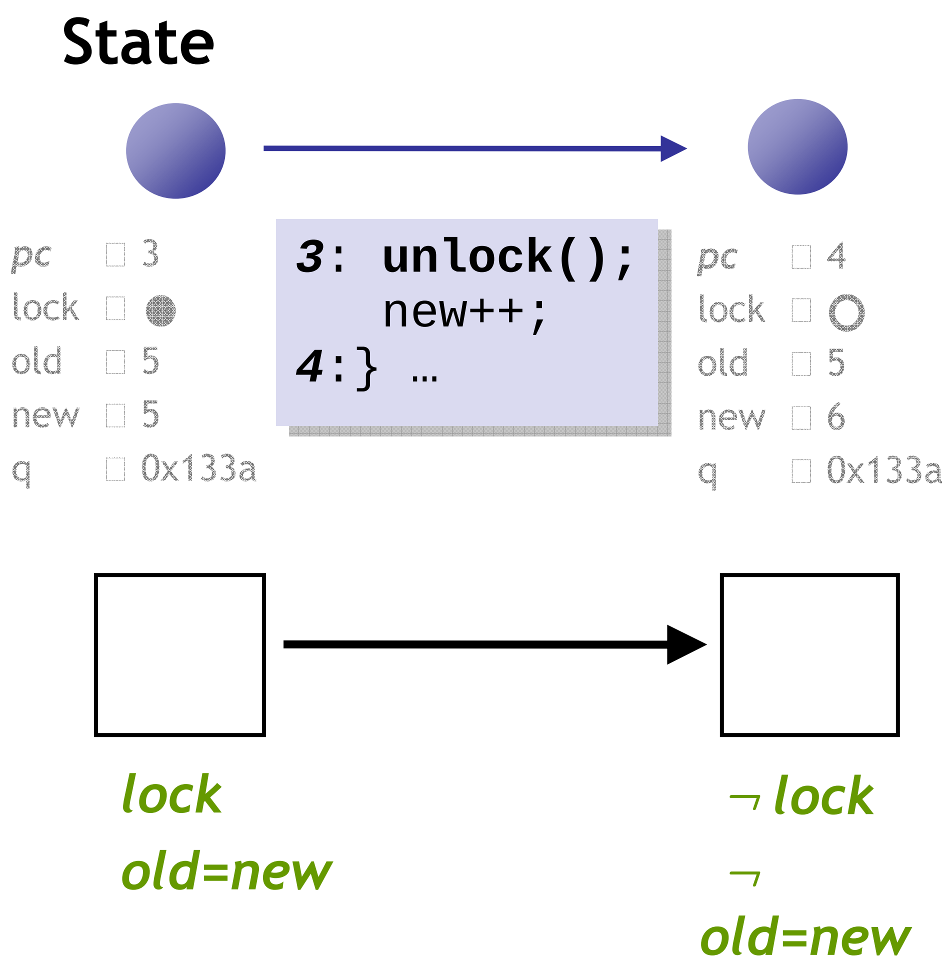


Existential Lifting

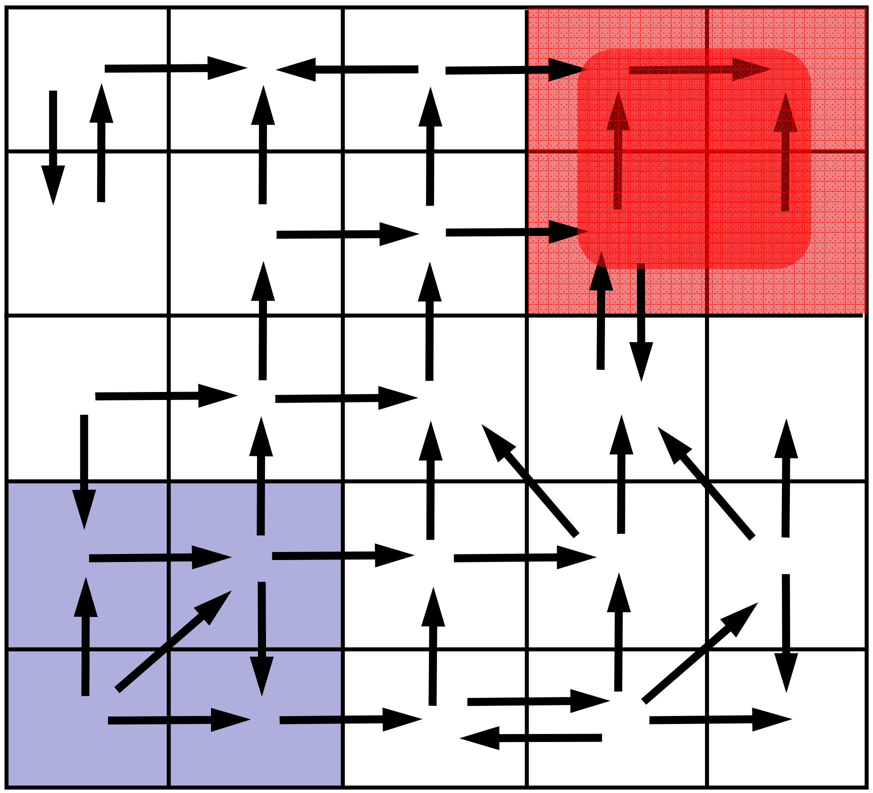


Abstraction





Analyze Abstraction



Analyze finite graph

Over Approximate:

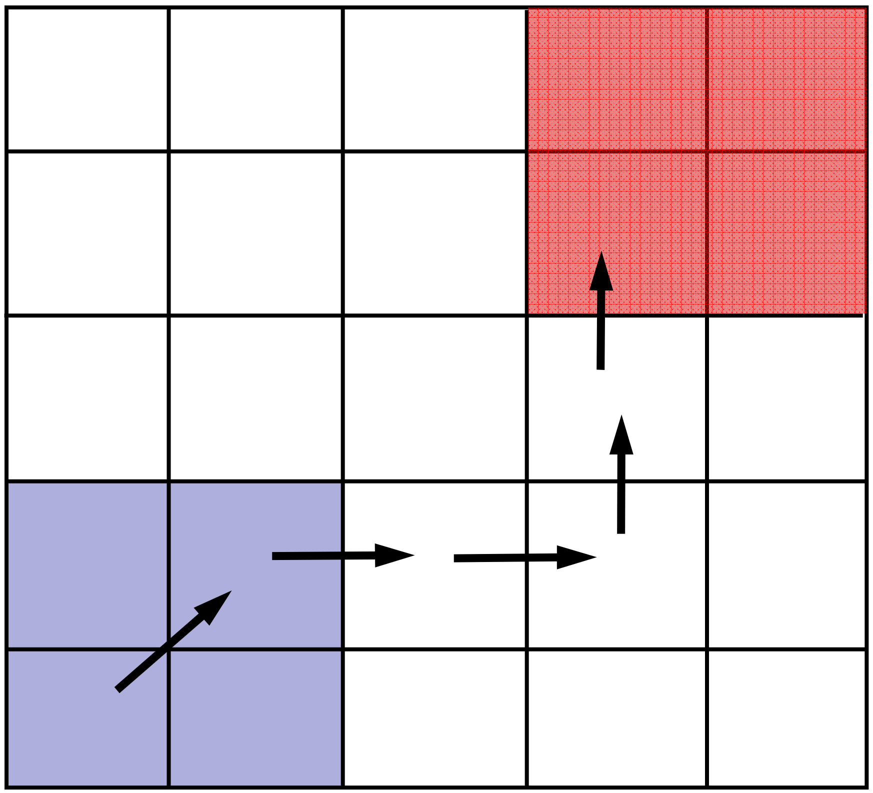
Safe $\Rightarrow$ System Safe

No **false negatives**

Problem

Spurious **counterexamples**

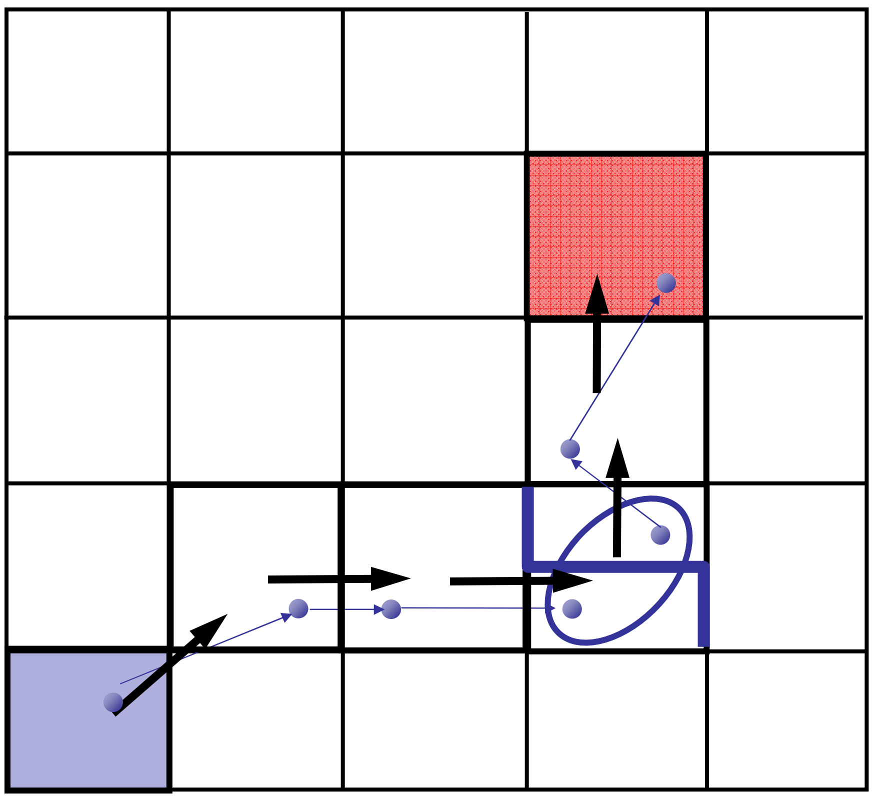
Idea 2: Counterex.-Guided Refinement



Solution

Use spurious **counterexamples** to **refine** abstraction !

Idea 2: Counterex.-Guided Refinement



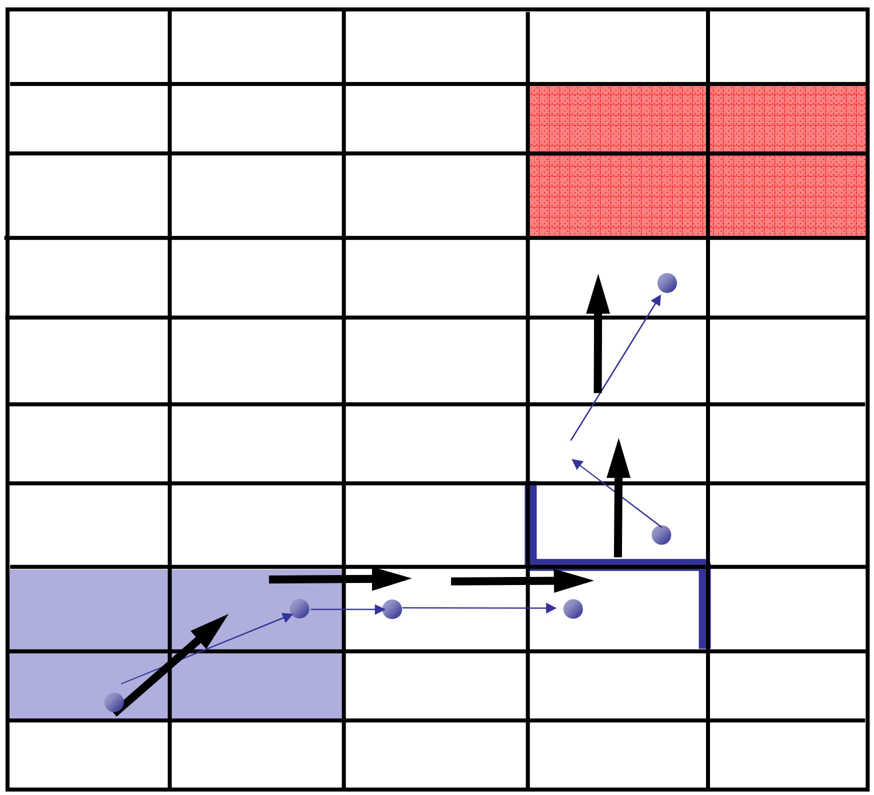
Solution

Use spurious **counterexamples** to **refine** abstraction

1. **Add predicates** to distinguish states across **cut**
2. Build **refined** abstraction

Imprecision due to **merge**

Iterative Abstraction-Refinement



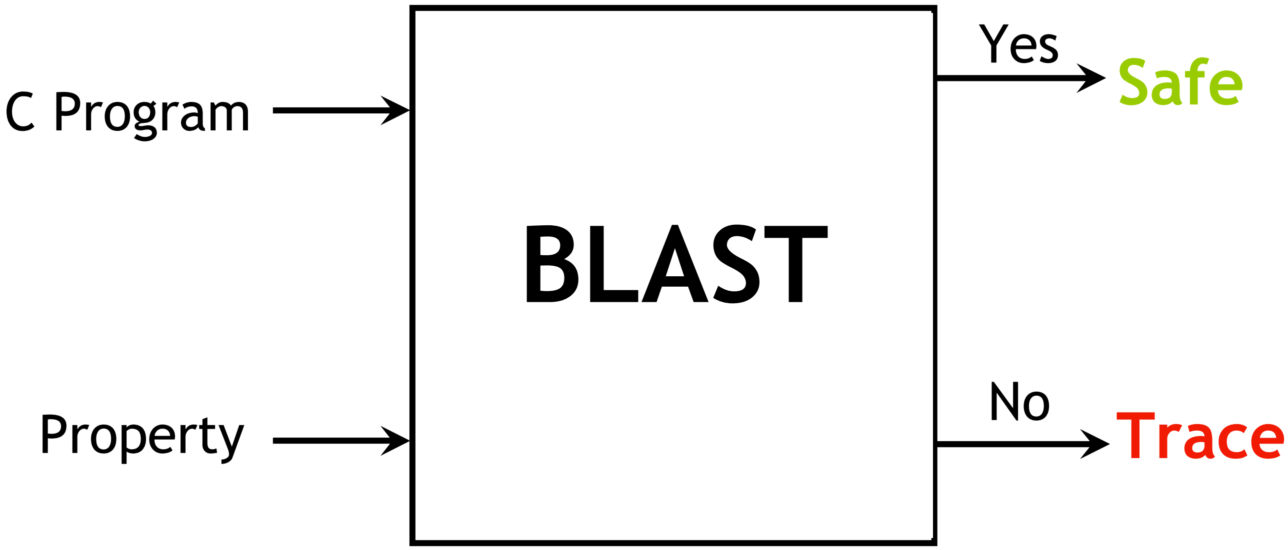
[Kurshan et al 93] [Clarke et al 00]
[Ball-Rajamani 01]

Solution

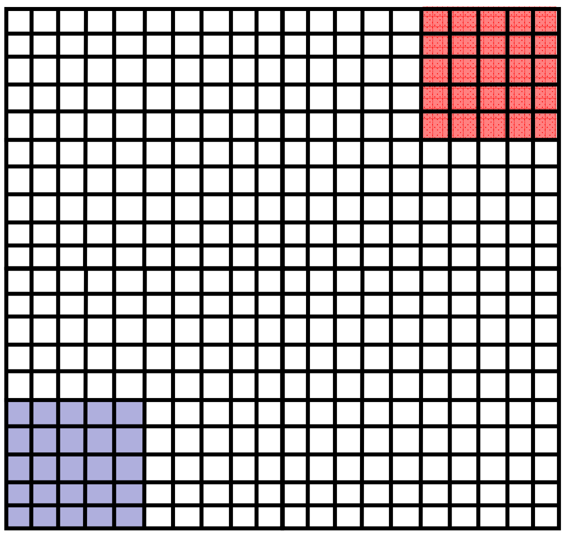
Use spurious **counterexamples** to **refine** abstraction

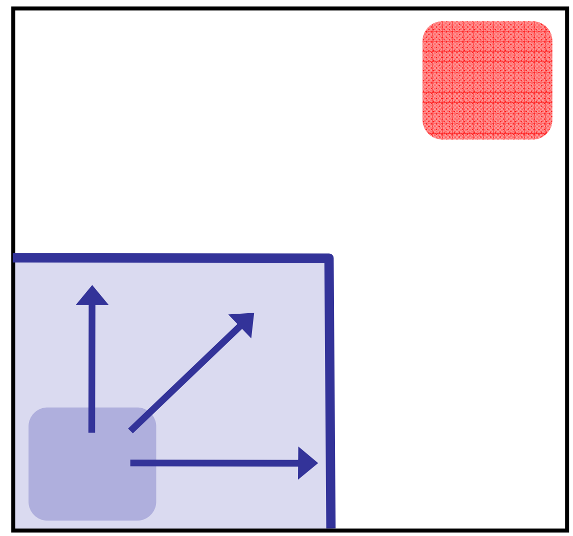
1. Add predicates to distinguish states across **cut**
2. Build **refined** abstraction
   -eliminates counterexample
3. **Repeat** search
   Till real counterexample or system proved safe

Lazy Abstraction



Problem: Abstraction is Expensive





Reachable

Problem

#abstract states = $2^{\#predicates}$

Exponential Thm. Prover queries

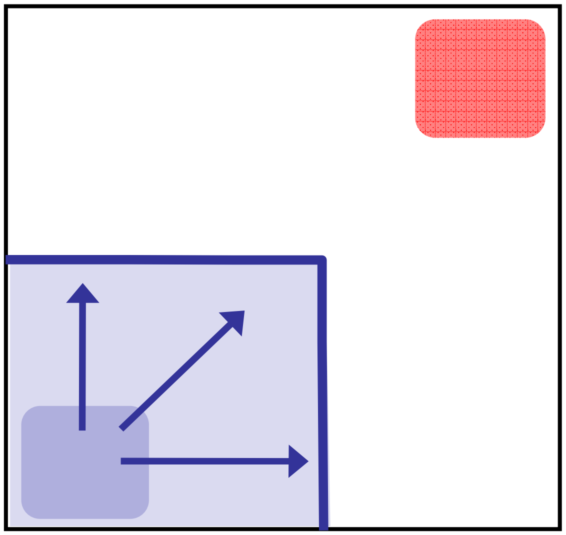
Observe

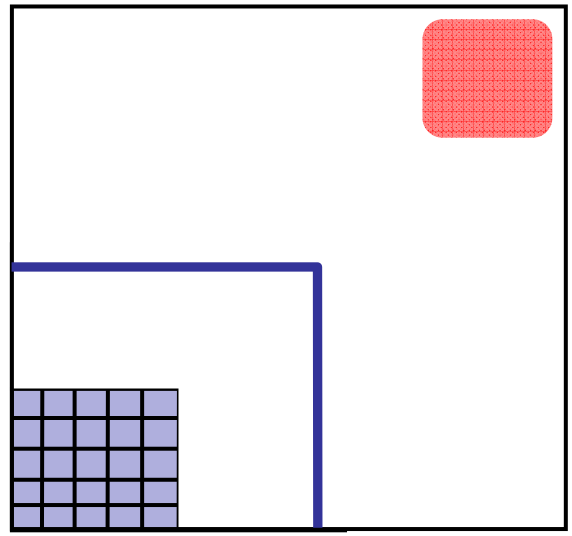
Fraction of state space reachable

#Preds ~ 100's, #States ~ 2^{100} ,

#Reach ~ 1000's

Solution1: Only Abstract Reachable States





Safe

Problem

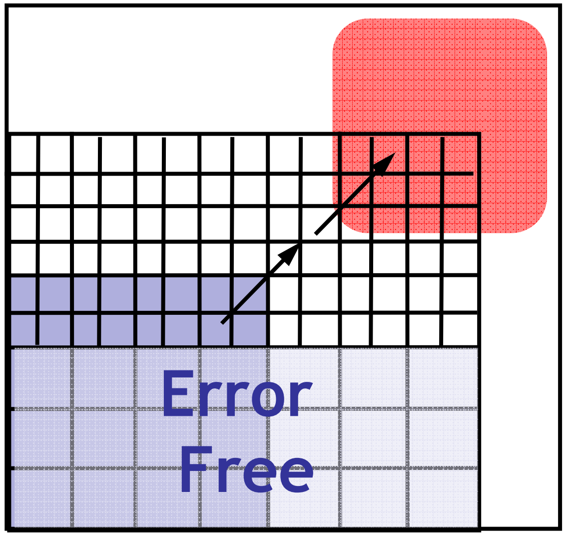
#abstract states = $2^{\#predicates}$

Exponential Thm. Prover queries

Solution

Build abstraction **during** search

Solution2: Don’t Refine Error-Free Regions



Problem

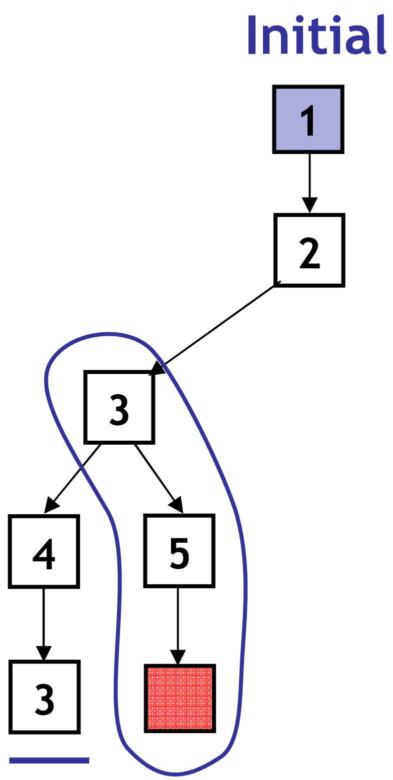
#abstract states = $2^{\#\text{predicates}}$

Exponential Thm. Prover queries

Solution

Don’t refine error-free regions

Key Idea: Reachability Tree



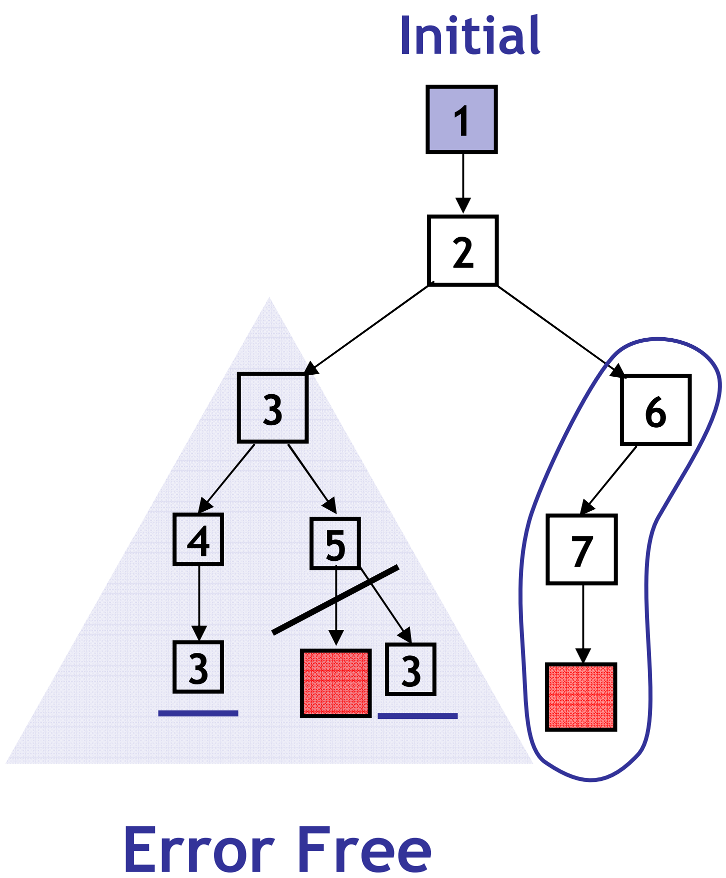
Unroll Abstraction

1. Pick tree-node (=abs. state)
2. Add children (=abs. successors)
3. On re-visiting abs. state, cut-off

Find min infeasible suffix

- Learn new predicates
- Rebuild subtree with new preds.

Key Idea: Reachability Tree



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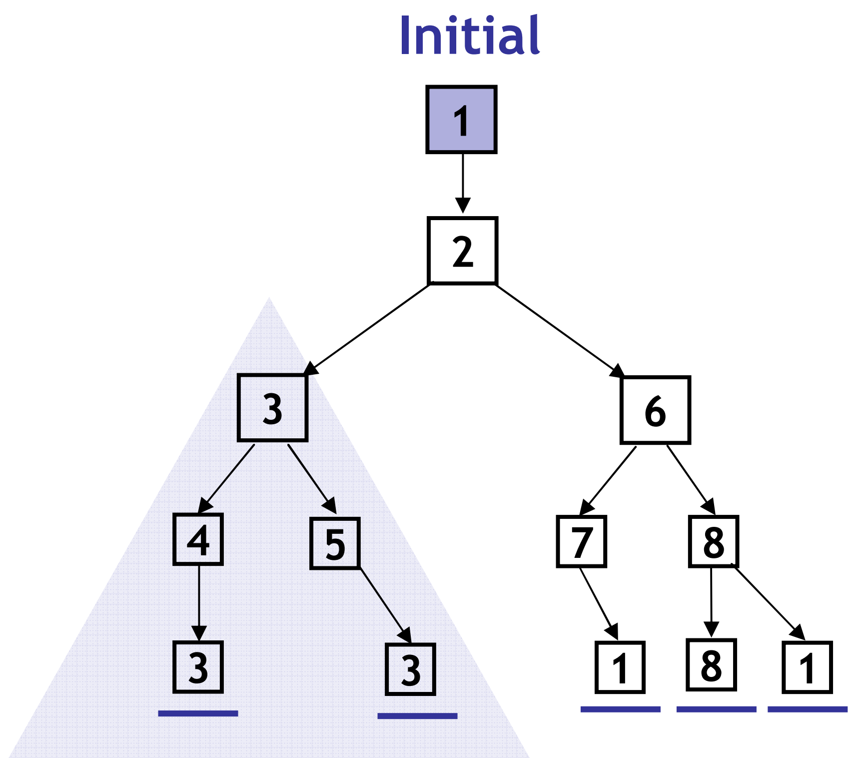

Unroll

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Find min spurious suffix

- Learn new predicates
- Rebuild subtree with new preds.

Error Free

SAFE

S1: Only Abstract Reachable States

S2: Don't refine error-free regions

Build-and-Search

```
Example ( ) {
1: do{
     lock();
     old = new;
     q = q->next;
2:   if (q != NULL){
3:     q->data = new;
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5: unlock ();
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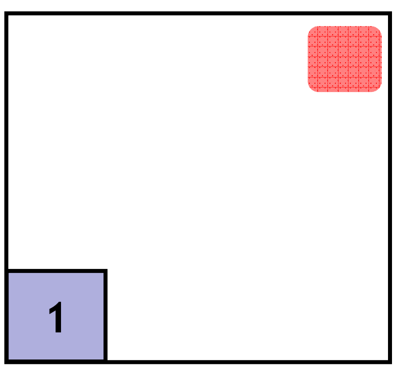

Predicates: *LOCK*

Reachability Tree

Build-and-Search

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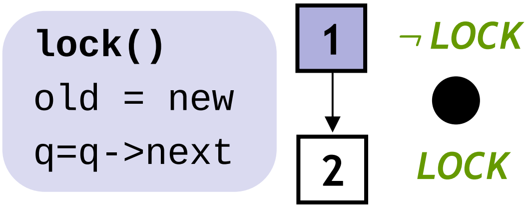

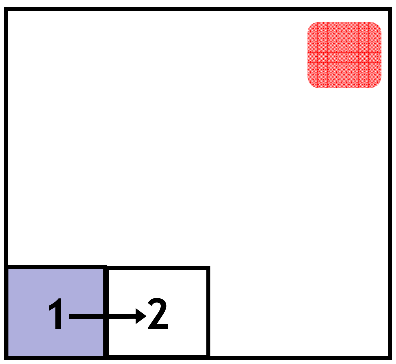

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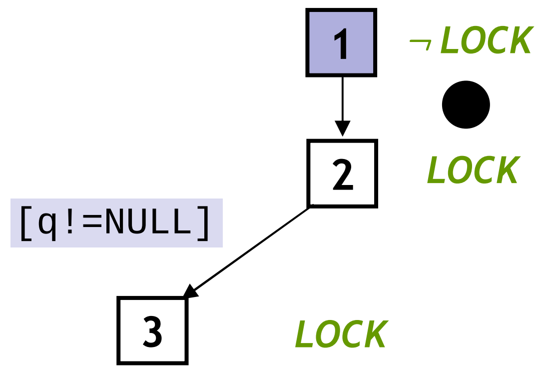

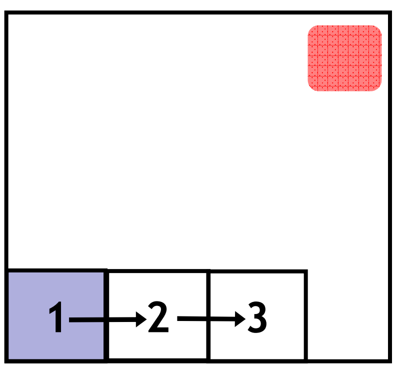

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Reachability Tree

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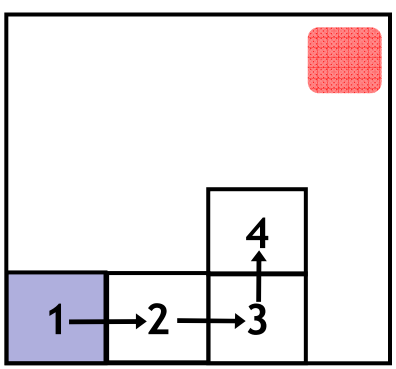

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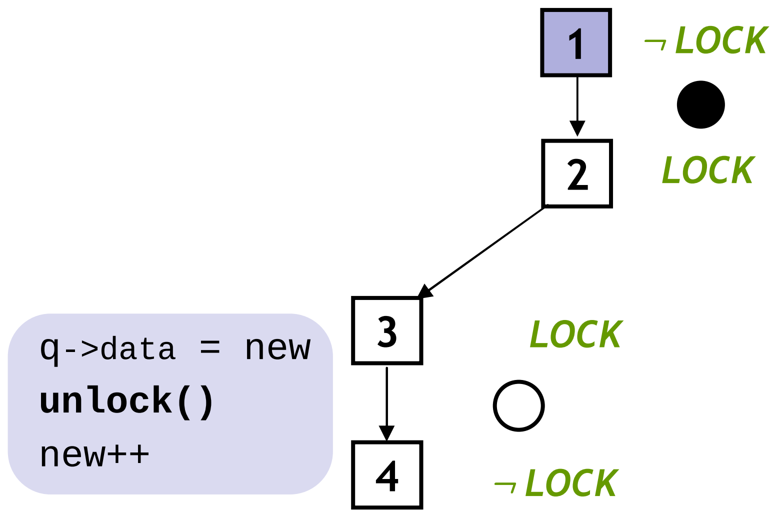

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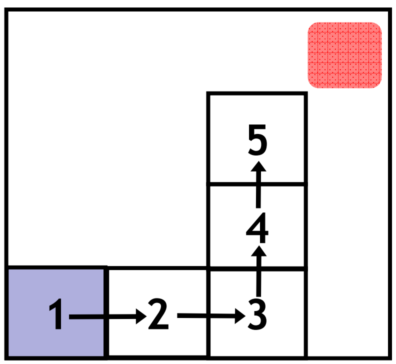

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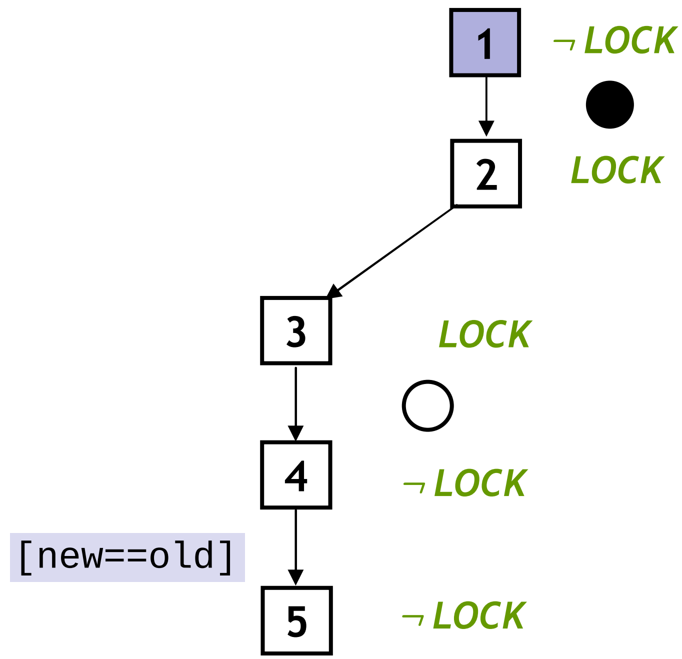

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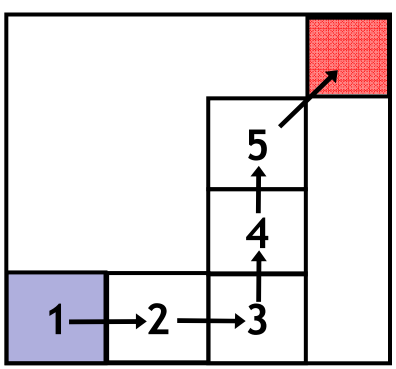

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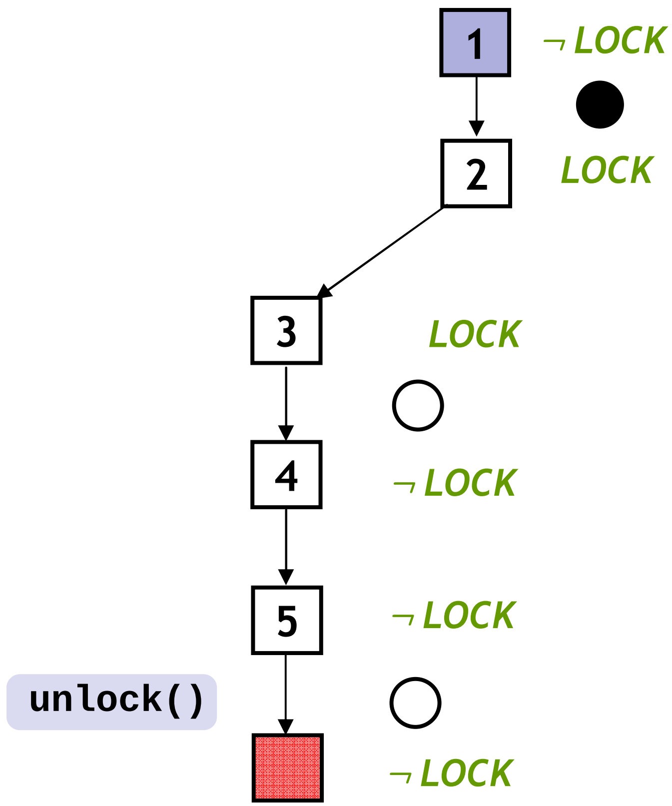

Reachability Tree

Analyze Counterexample

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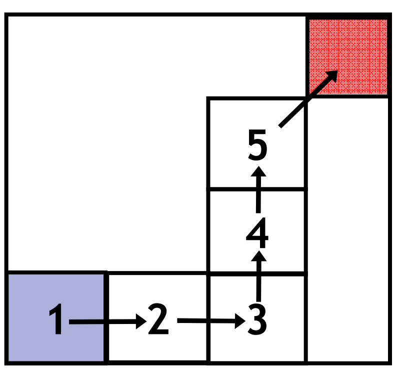

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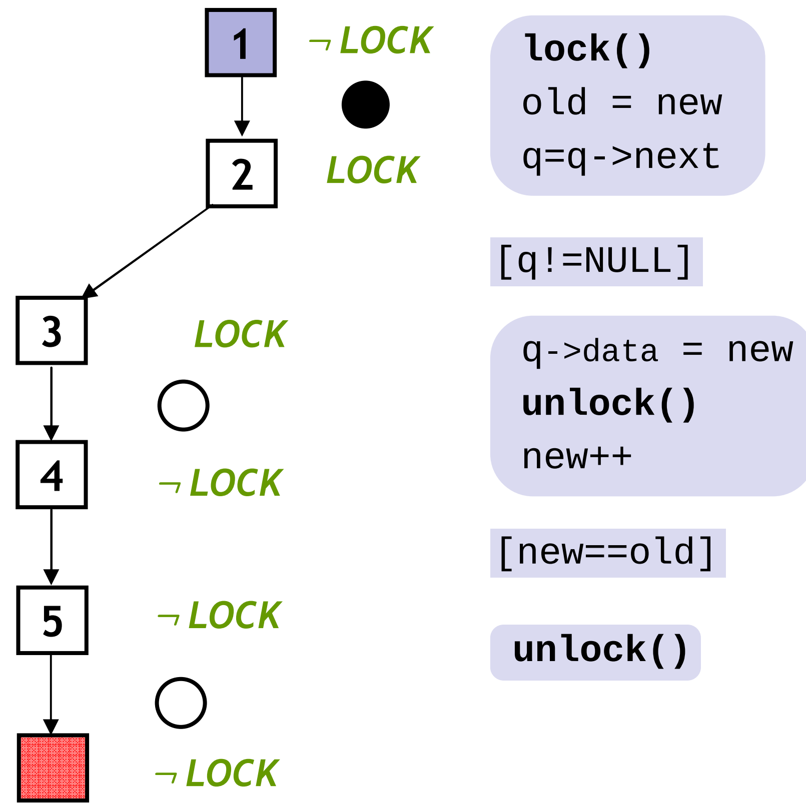

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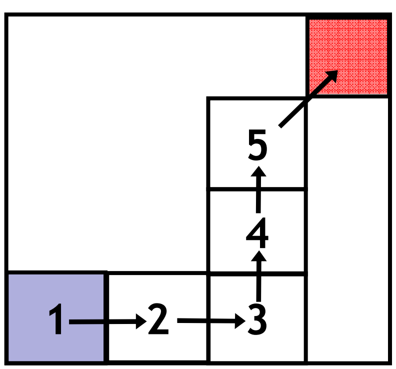

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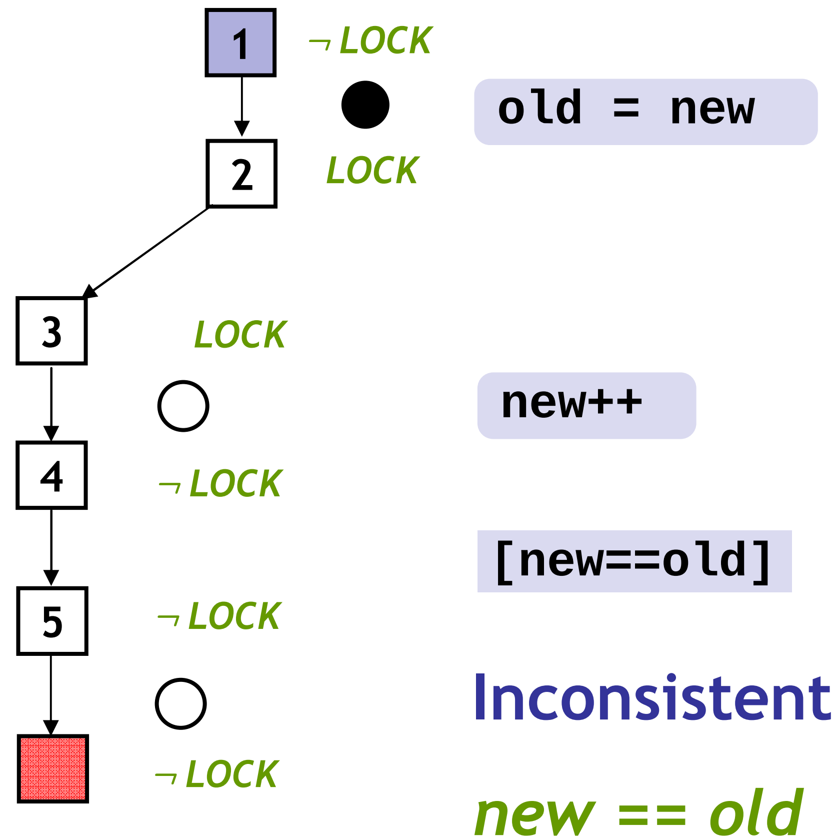

Reachability Tree

Repeat Build-and-Search

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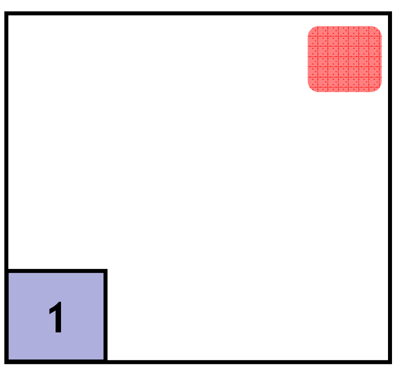

Reachability Tree

Predicates: *LOCK, new==old*

Repeat Build-and-Search

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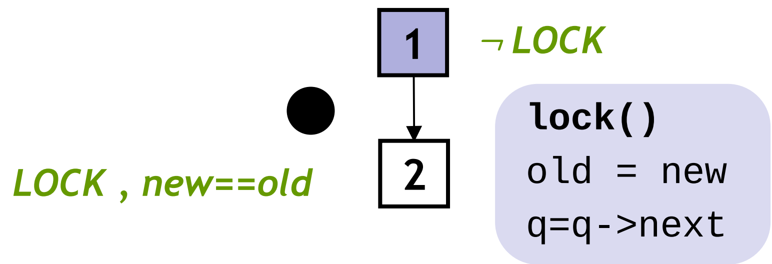

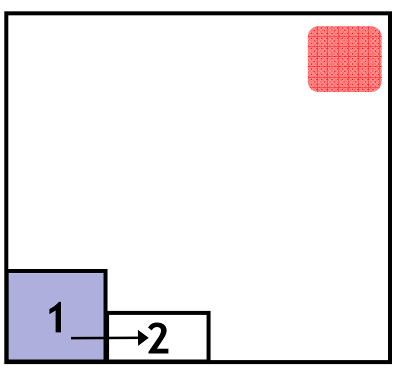

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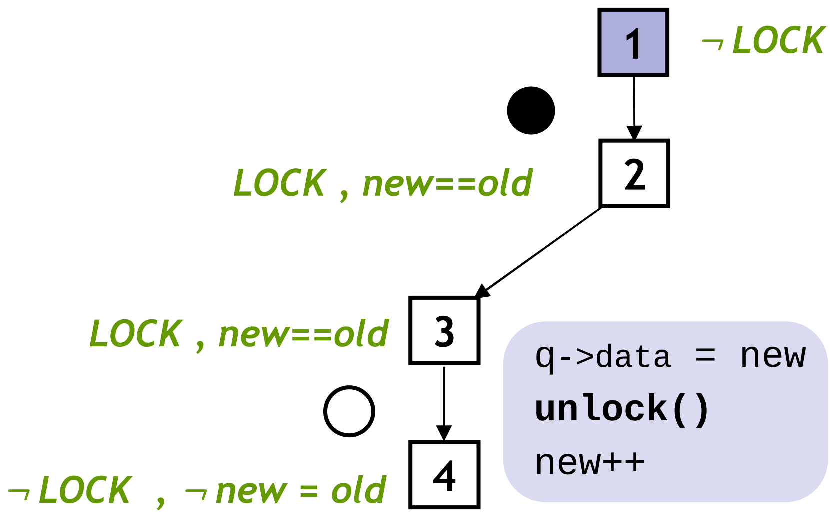

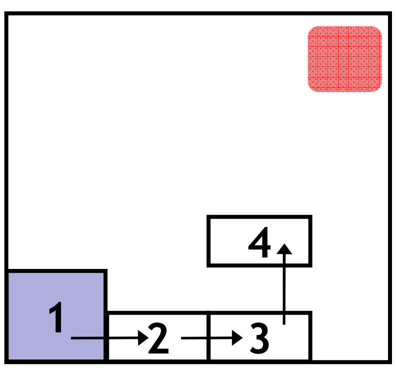

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Reachability Tree

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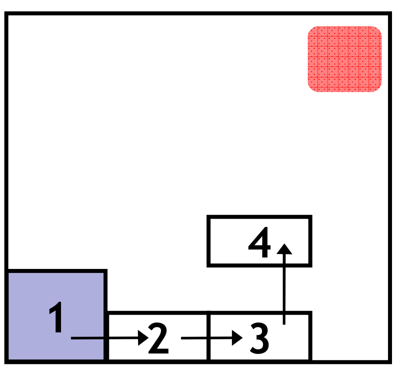

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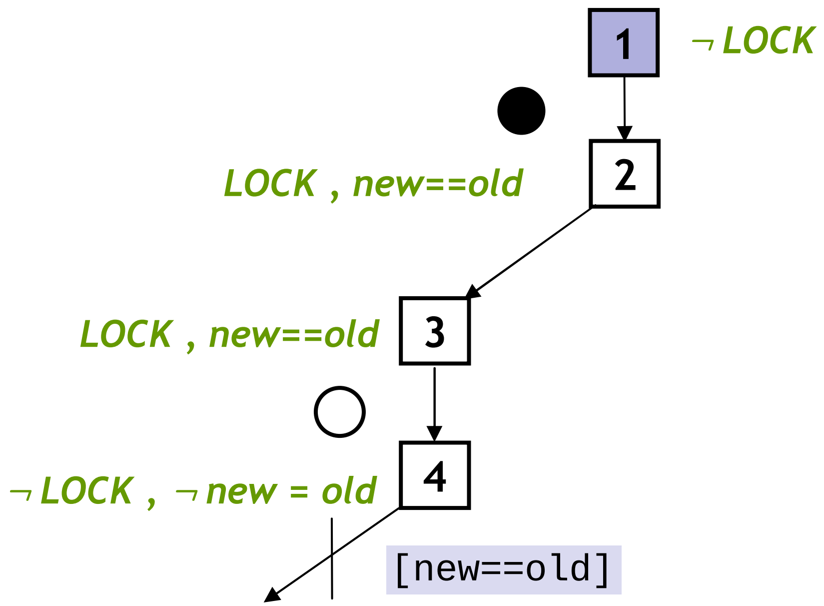

Reachability Tree

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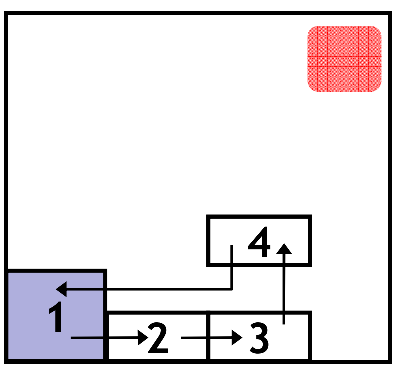

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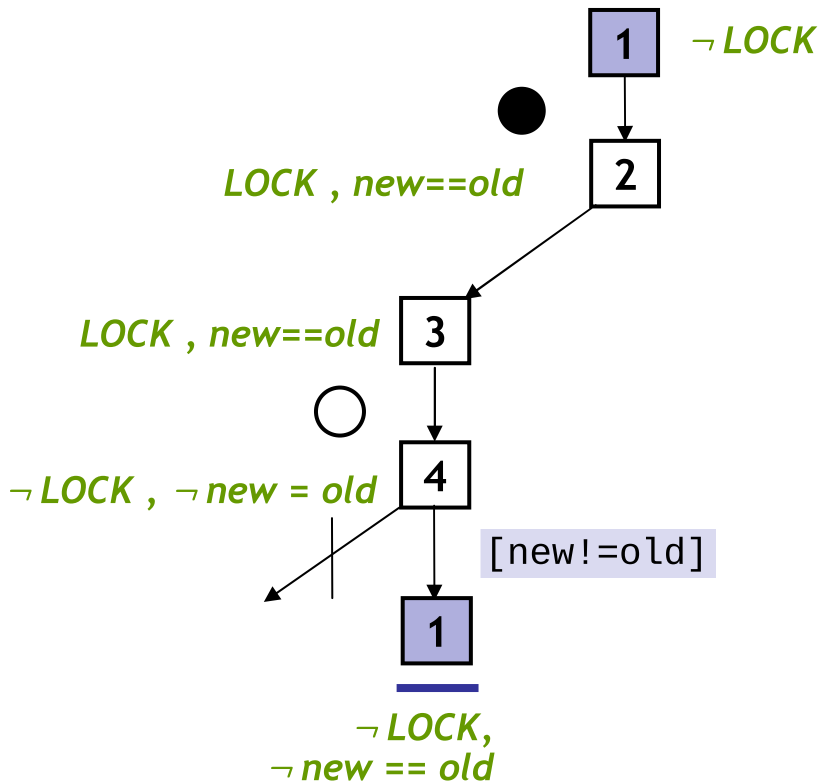

Reachability Tree

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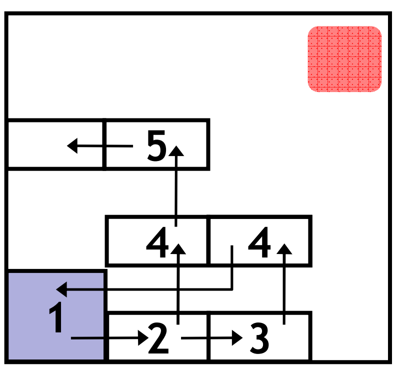

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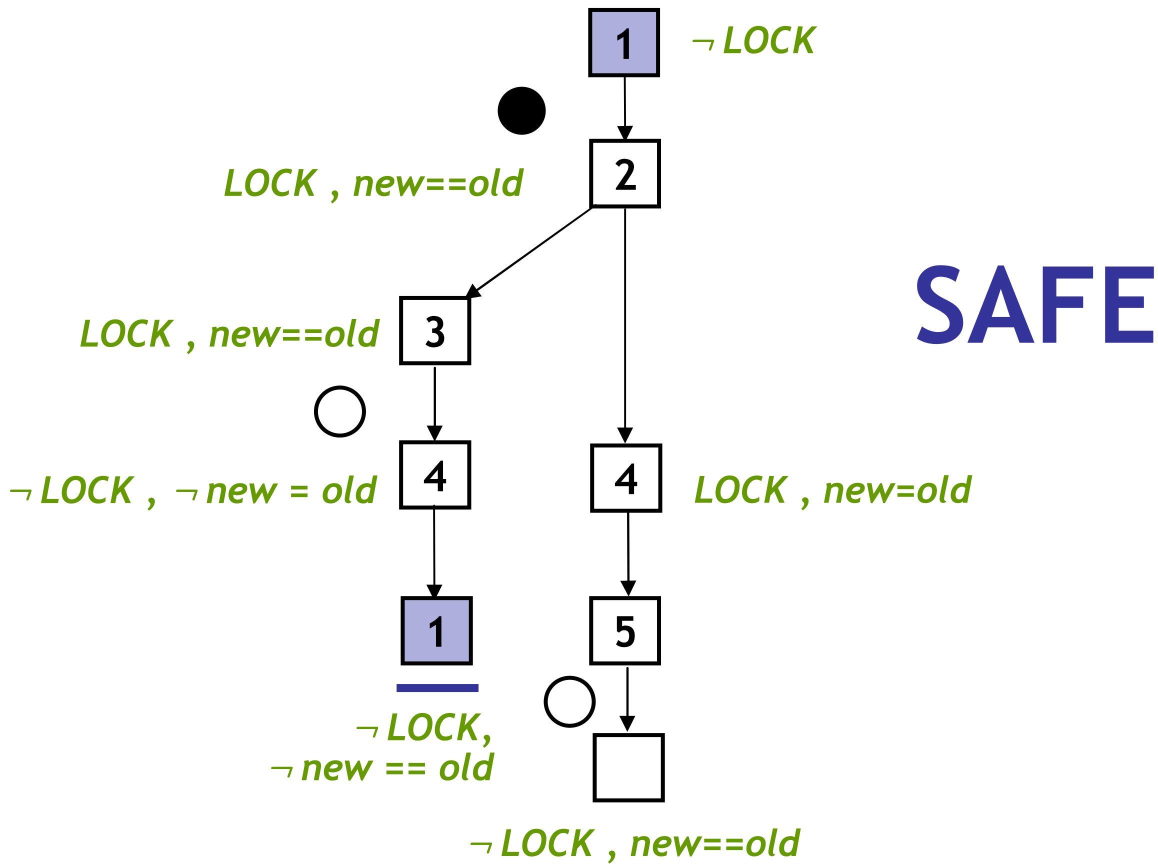
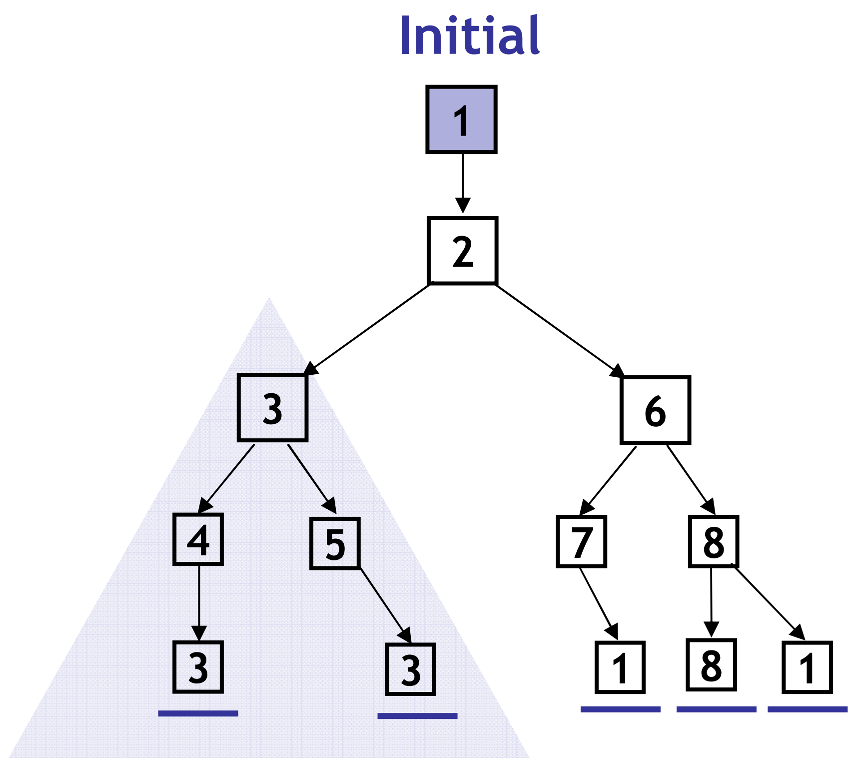


Reachability Tree

Key Idea: Reachability Tree



Unroll

1. Pick tree-node (=abs. state)
2. Add children (=abs. successors)
3. On re-visiting abs. state, cut-off

Find min spurious suffix

- Learn new predicates
- Rebuild subtree with new preds.

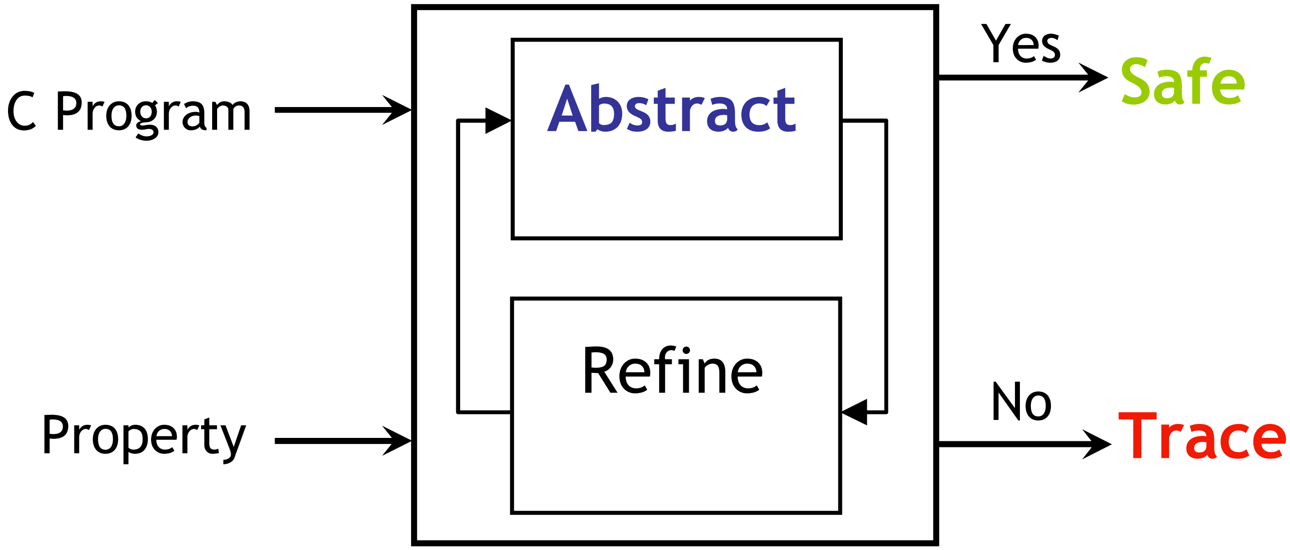
Error Free

SAFE

S1: Only Abstract Reachable States

S2: Don't refine error-free regions

Lazy Abstraction



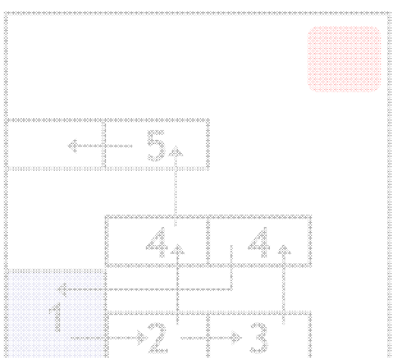
Problem: Abstraction is Expensive

Solution: 1. Abstract reachable states,
2. Avoid refining error-free regions

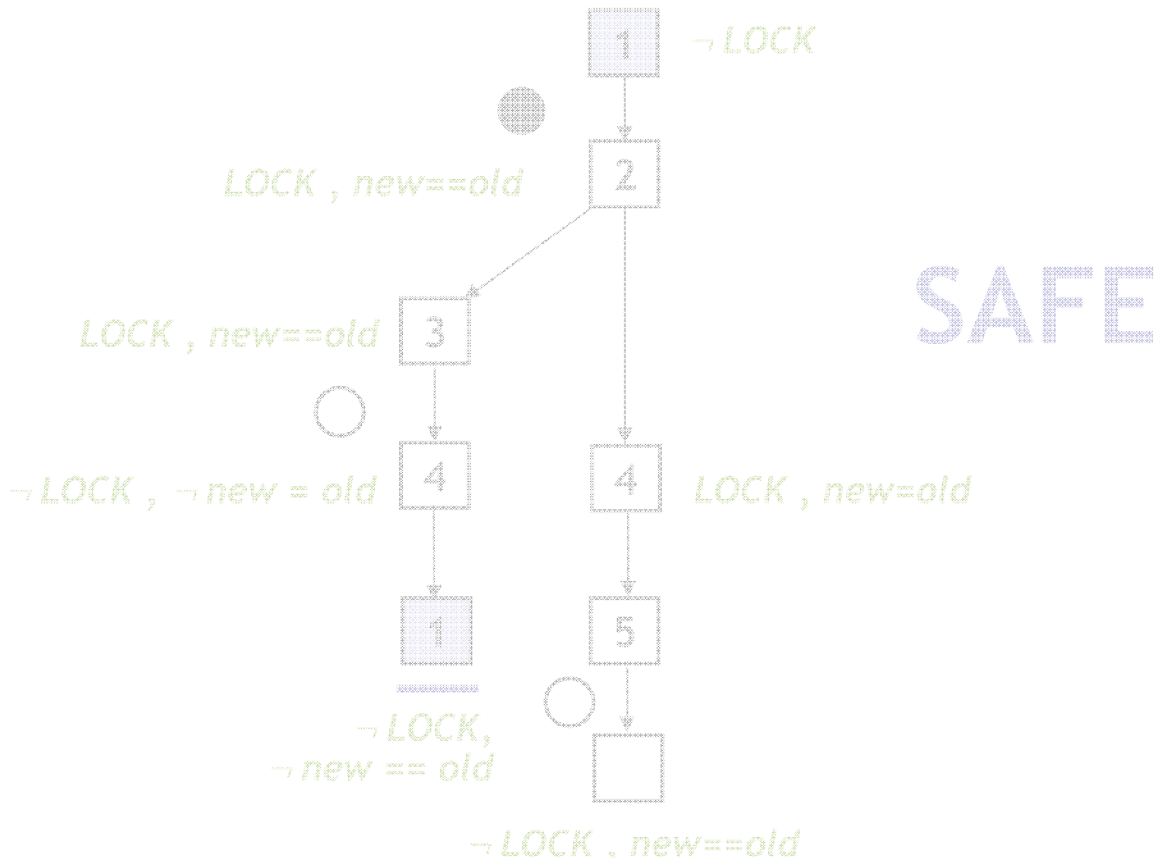
Key Idea: Reachability Tree

Technical Details

```
Example ( ) {
1: do{
       lock();
       old = new;
       q = q->next;
2:     if (q != NULL){
3:        q->data = new;
          unlock();
          new ++;
       }
4:}while(new != old);
5: unlock ();
}
```



Predicates: *LOCK, new==old*



Reachability Tree

Technical Details

- Q: How to compute “successors”?
- Q: How to find predicates ? [Interpolation]
- Q: How to analyze (recursive) procedures? [Context-free reachability]

Technical Details

Q. How to compute "successors" ?

Predicate Abstraction

- From last lecture

Technical Details

Q. How to find predicates ?

#Predicates grows with program size

```
     while(1){
T ●  1: if (p₁) lock() ;
F       if (p₁) unlock() ;
                …
T ●  2: if (p₂) lock() ;
        if (p₂) unlock() ;
                …
     n: if (pₙ) lock() ;
        if (pₙ) unlock() ;
     }
```

Tracking **lock** not enough

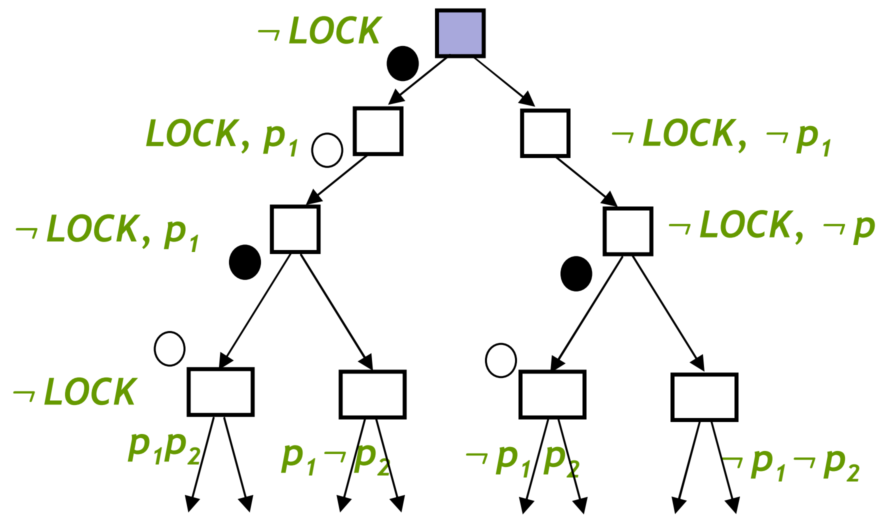
Problem:

$p_1, \ldots, p_n$ needed for verification

Exponential reachable abstract states

#Predicates grows with program size

```
while(1){
1: if (p1) lock() ;
   if (p1) unlock() ;
              …
2: if (p2) lock() ;
   if (p2) unlock() ;
              …
n: if (pn) lock() ;
   if (pn) unlock() ;
}
```



2^n Abstract States

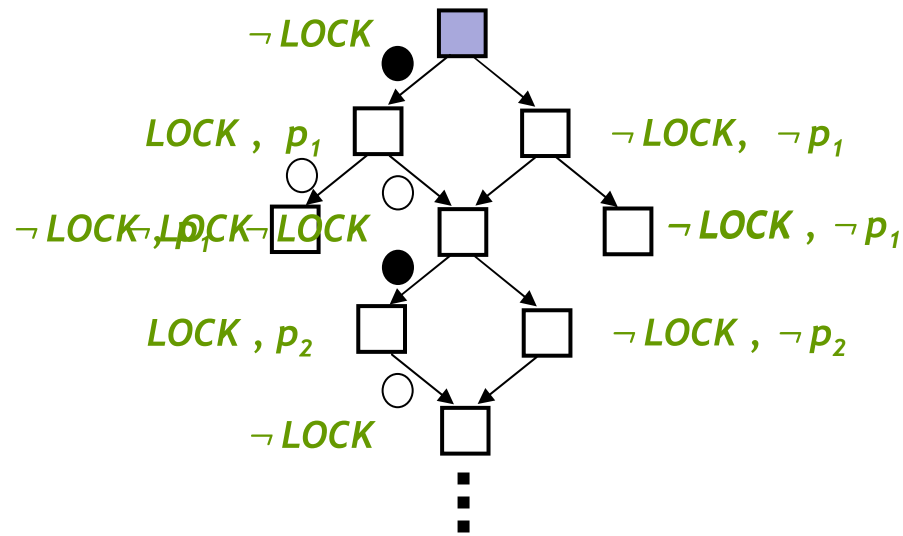
Problem:

$p_1,\ldots,p_n$ needed for verification

Exponential reachable abstract states

Predicates useful *locally*

```
     while(1){
p1   1: if (p1) lock() ;
        if (p1) unlock() ;
                …
p2   2: if (p2) lock() ;
        if (p2) unlock() ;
                …
pn   n: if (pn) lock() ;
        if (pn) unlock() ;
     }
```



2n Abstract States

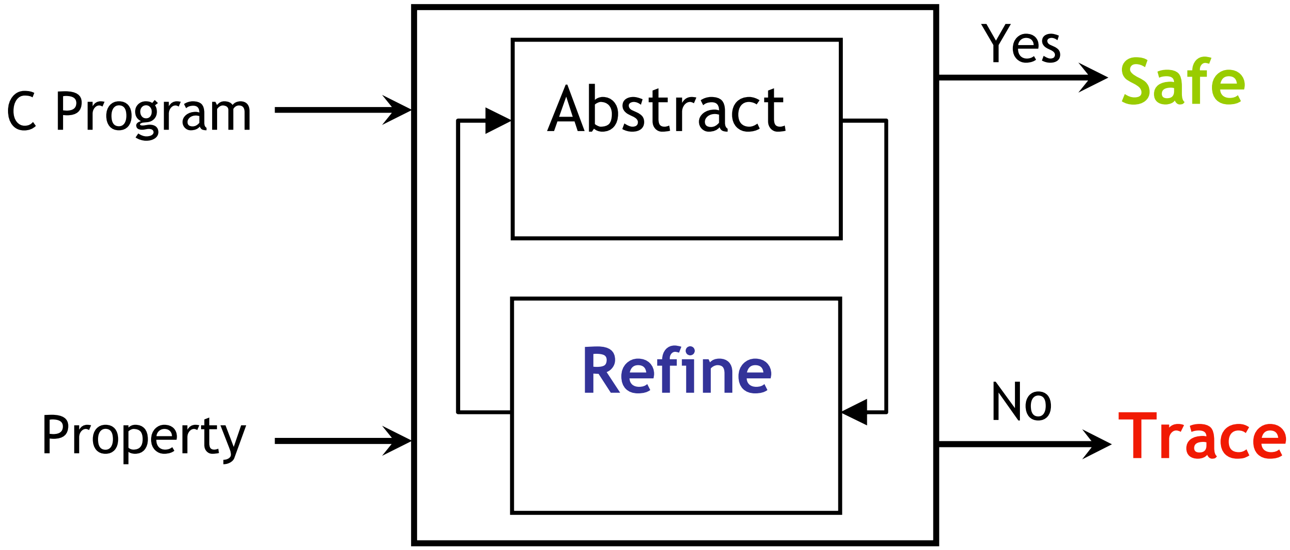
Solution: Use predicates **only where needed**

Using **Counterexamples:**

Q1. Find **predicates**

Q2. Find **where** predicates are needed

Lazy Abstraction



Problem: #Preds grows w/ Program Size

Solution: Localize pred. use, find where preds. needed

Counterexample Traces

```
lock()
old = new
q=q->next
```

```
[q!=NULL]
```

```
q->data = new
unlock()
new++
```

```
[new==old]
```

```
unlock()
```

$lock_1 = 1$
$old_1 = new_0$
$q_1 = q_0\text{->}next$
$assume(q_1 \text{ != } NULL)$
$(q_1 \text{ -> } data)_1 = new_0$
$lock_2 = 0$
$new_1 = new_0 + 1$
$assume(new_1 = old_1)$
$assert(lock_2 = 1)$

$lock_1 = 1 \;\wedge$
$old_1 = new_0 \;\wedge$
$q_1 = q_0\text{->}next \;\wedge$
$q_1 \text{ != } NULL \;\wedge$
$(q_1 \text{ -> } data)_1 = new_0 \;\wedge$
$lock_2 = 0 \;\wedge$
$new_1 = new_0 + 1 \;\wedge$
$new_1 = old_1$

Trace | SSA Trace | Trace Feasibility Formula

Thm: Trace is feasible $\Leftrightarrow$ TF is satisfiable

Proof of Unsatisfiability

$\texttt{lock}_1$ `= 1` $\wedge$
$\boxed{\text{old}_1 = \text{new}_0}$ $\wedge$
$\texttt{q}_1$ `=` $\texttt{q}_0$`->next` $\wedge$
$\texttt{q}_1$ `!= NULL` $\wedge$
$(\texttt{q}_1 \texttt{ -> data})_1$ `=` $\texttt{new}_0$ $\wedge$
$\texttt{lock}_2$ `= 0` $\wedge$
$\boxed{\texttt{new}_1 = \text{new}_0 \texttt{ +1}}$ $\wedge$

$\boxed{\texttt{new}_1\texttt{=old}_1}$

Trace Feasibility Formula

$$\cfrac{\cfrac{old_1 = new_0 \qquad new_1 = new_0 + 1}{new_1 = old_1 + 1} \qquad new_1 = old_1}{\emptyset}$$

Proof of Unsatisfiability

Predicates: old=new, new=new+1, new=old

Add: old=new

[HenzingerJhalaM.Sutre02]

Counterexample Traces: Take 2

```
1: x = ctr;
2: ctr = ctr + 1;
3: y = ctr;
4: if (x = i-1){
5:   if (y != i){
          ERROR: }
   }
```

```
1: x = ctr
2: ctr = ctr + 1
3: y = ctr
4: assume(x = i-1)
5: assume(y ≠ i)
```

$y = x + 1$

Trace Formulas

1: `x = ctr`	*1*: x_1 = ctr_0	$x_1 = ctr_0$
2: `ctr = ctr+1`	*2*: ctr_1 = ctr_0+1	$\wedge\ ctr_1 = ctr_0 + 1$
3: `y = ctr`	*3*: y_1 = ctr_1	$\wedge\ y_1 = ctr_1$
4: `assume(x=i-1)`	*4*: assume(x_1=i_0-1)	$\wedge\ x_1 = i_0 - 1$
5: `assume(y≠i)`	*5*: assume(y_1≠i_0)	$\wedge\ y_1 \neq i_0$
Trace	SSA Trace	Trace Feasibility Formula

Proof of Unsatisfiability

$$x_1 = ctr_0$$
$$\wedge \; ctr_1 = ctr_0 + 1$$
$$\wedge \; y_1 = ctr_1$$
$$\wedge \; x_1 = i_0 - 1$$
$$\wedge \; y_1 \neq i_0$$

Trace Formula

$$\cfrac{\cfrac{\cfrac{\cfrac{x_1 = ctr_0 \quad x_1 = i_0 - 1}{ctr_0 = i_0 - 1} \quad ctr_1 = ctr_0 + 1}{ctr_1 = i_0} \quad y_1 = ctr_1}{y_1 = i_0} \quad y_1 \neq i_0}{\emptyset}$$

Proof of Unsatisfiability

The Present State…

Trace

```
1: x = ctr
2: ctr = ctr + 1
3: y = ctr
4: assume(x = i-1)
5: assume(y ≠ i)
```

… is all the information the executing program has ***here***

State…

1. … after executing trace *past (prefix)*
2. … knows *present values* of variables
3. … makes trace *future (suffix)* infeasible

At ***pc_4***, which predicate on ***present state*** shows infeasibility of ***future*** ?

What Predicate is needed ?

Trace	Trace Formula (TF)
1: `x = ctr`	$x_1 = ctr_0$
2: `ctr = ctr + 1`	$\wedge \quad ctr_1 = ctr_0 + 1$
3: `y = ctr`	$\wedge \quad y_1 = ctr_1$
4: `assume(x = i-1)`	$\wedge \quad x_1 = i_0 - 1$
5: `assume(y ≠ i)`	$\wedge \quad y_1 \neq i_0$

What Predicate is needed ?

Trace

```
1: x = ctr

2: ctr = ctr + 1

3: y = ctr

4: assume(x = i-1)

5: assume(y ≠ i)
```

Trace Formula (TF)

$x_1 = ctr_0$

$\wedge \quad ctr_1 = ctr_0 + 1$

$\wedge \quad y_1 = ctr_1$

$\wedge \quad x_1 = i_0 - 1$

$\wedge \quad y_1 \neq i_0$

Relevant Information

1. … after executing trace **prefix**

Predicate …

… implied by TF **prefix**

What Predicate is needed ?

Trace

```
1: x = ctr
2: ctr = ctr + 1
3: y = ctr
4: assume(x = i-1)
5: assume(y ≠ i)
```

Trace Formula (TF)

$x_1 = ctr_0$

$\wedge \quad ctr_1 = ctr_0 + 1$

$\wedge \quad y_1 = ctr_1$

$\wedge \quad x_1 = i_0 - 1$

$\wedge \quad y_1 \neq i_0$

Relevant Information

1. … after executing trace **prefix**
2. … has **present values** of variables

Predicate …

… implied by TF **prefix**

… on **common** variables

What Predicate is needed ?

Trace	Trace Formula (TF)
`1: x = ctr`	$x_1 = ctr_0$
`2: ctr = ctr + 1`	$\wedge \; ctr_1 = ctr_0 + 1$
`3: y = ctr`	$\wedge \; y_1 = ctr_1$
`4: assume(x = i-1)`	$\wedge \; x_1 = i_0 - 1$
`5: assume(y ≠ i)`	$\wedge \; y_1 \neq i_0$

Relevant Information

1. … after executing trace **prefix**
2. … has **present values** of variables
3. … makes trace **suffix** infeasible

Predicate …

… implied by TF **prefix**

… on **common** variables

… & TF **suffix** is **unsatisfiable**

Interpolant = Predicate !

Trace	Trace Formula
`1: x = ctr`	$x_1 = ctr_0$
`2: ctr = ctr + 1`	$\wedge\ ctr_1 = ctr_0 + 1$
`3: y = ctr`	$\wedge\ y_1 = ctr_1$
`4: assume(x = i-1)`	$\wedge\ x_1 = i_0 - 1$
`5: assume(y ≠ i)`	$\wedge\ y_1 \neq i_0$

Ψ^-

Ψ^+

Interpolate Φ

$y_1 = x_1 + 1$

Predicate at 4:
$y = x+1$

Craig Interpolant
[Craig 57]

Computable from Proof of Unsat

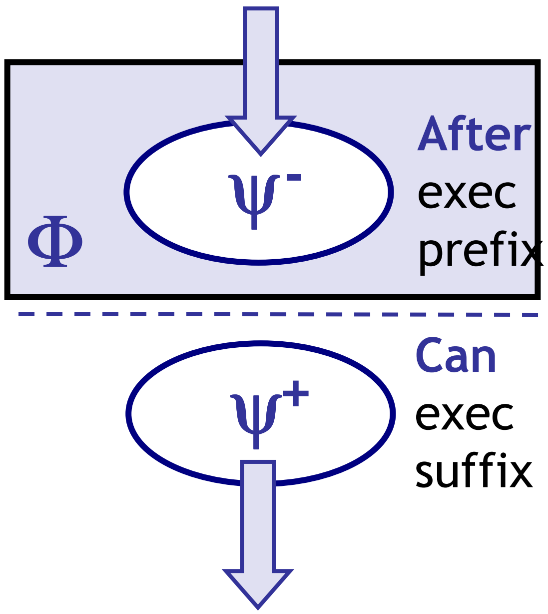
Predicate …

… implied by TF prefix

… on common variables

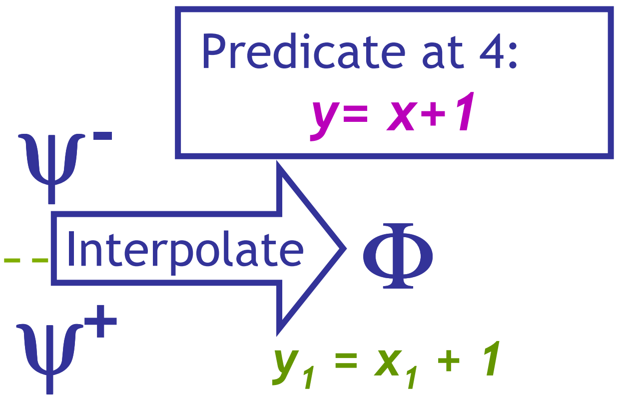
… & TF suffix is unsatisfiable

Another interpretation …



Trace Formula

$$x_1 = ctr_0$$
$$\wedge \quad ctr_1 = ctr_0 + 1$$
$$\wedge \quad y_1 = ctr_1$$
$$\wedge \quad x_1 = i_0 - 1$$
$$\wedge \quad y_1 \neq i_0$$



Unsat = Empty Intersection = Trace Infeasible

Interpolant Φ =

Overapproximation of states after prefix
that cannot execute suffix

Main Questions

Q. How to find good predicates ?
Where to track each predicate?

Q: How to compute interpolants?
(And do they always exist?)

Another Proof of Unsatisfiability

$$\cfrac{\cfrac{\cfrac{\cfrac{x_1 = ctr_0 \quad x_1 = i_0 - 1}{ctr_0 = i_0 - 1} \quad ctr_1 = ctr_0 + 1}{ctr_1 = i_0} \quad y_1 = ctr_1}{y_1 = i_0} \quad y_1 \neq i_0}{\emptyset}$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\overset{\boxed{\times(-1)}}{x_1 - ctr_0 = 0} \quad \overset{\boxed{\times 1}}{x_1 - i_0 + 1 = 0}}{ctr_0 - i_0 + 1 = 0} \quad \overset{\boxed{\times 1}}{ctr_1 - ctr_0 - 1 = 0}}{ctr_1 - i_0 = 0} \quad \overset{\boxed{\times 1}}{y_1 - ctr_1 = 0}}{y_1 - i_0 = 0} \quad \overset{\boxed{\times 1}}{y_1 - i_0 \neq 0}}{0 \neq 0}$$

Proof of Unsatisfiability   Rewritten Proof

Interpolant from Rewritten Proof ?

$x_1 = ctr_0$

$\wedge\ ctr_1 = ctr_0 + 1$

$\wedge\ y_1 = ctr_1$

- - - - - - - - - - - - - - - - -

$\wedge\ x_1 = i_0 - 1$

$\wedge\ y_1 \neq i_0$

Trace Formula

Interpolate

$$\dfrac{\dfrac{\dfrac{\dfrac{\overset{\boxed{\times (-1)}}{x_1 - ctr_0 = 0} \quad \overset{\boxed{\times 1}}{x_1 - i_0 + 1 = 0}}{ctr_0 - i_0 + 1 = 0} \quad \overset{\boxed{\times 1}}{ctr_1 - ctr_0 - 1 = 0}}{ctr_1 - i_0 = 0} \quad \overset{\boxed{\times 1}}{y_1 - ctr_1 = 0}}{y_1 - i_0 = 0} \quad \overset{\boxed{\times 1}}{y_1 - i_0 \neq 0}}{0 \neq 0}$$

Rewritten Proof

Interpolant from Rewritten Proof ?

$$x_1 = ctr_0$$
$$\wedge\ ctr_1 = ctr_0 + 1$$
$$\wedge\ y_1 = ctr_1$$
$$\wedge\ x_1 = i_0 - 1$$
$$\wedge\ y_1 \neq i_0$$

Trace Formula

Interpolate

$$x_1 - ctr_0 = 0 \quad \boxed{\times\ (-1)}$$
$$ctr_1 - ctr_0 - 1 = 0 \quad \boxed{\times\ 1}$$
$$y_1 - ctr_1 = 0 \quad \boxed{\times\ 1}$$
$$\overline{\boxed{y_1 - x_1 - 1 = 0}}$$
$$\boxed{y_1 = x_1 + 1}$$

Interpolant !

Building Predicate Maps

Trace	Trace Formula
`1: x = ctr`	$x_1 = ctr_0$
`2: ctr = ctr + 1`	$\wedge\ ctr_1 = ctr_0 + 1$
`3: y = ctr`	$\wedge\ y_1 = ctr_1$
`4: assume(x = i-1)`	$\wedge\ x_1 = i_0 - 1$
`5: assume(y ≠ i)`	$\wedge\ y_1 \neq i_0$

ψ^-

ψ^+

Interpolate $\Rightarrow$ $x_1 = ctr_0$

Predicate Map
2: $x = ctr$

- Cut + Interpolate at **each** point
- Pred. Map: pc_i □ Interpolant from cut i

Building Predicate Maps

Predicate Map
2: $x = ctr$
3: $x = ctr - 1$

Trace	Trace Formula
`1: x = ctr`	$x_1 = ctr_0$
`2: ctr = ctr + 1`	$\wedge\ ctr_1 = ctr_0 + 1$
`3: y = ctr`	$\wedge\ y_1 = ctr_1$
`4: assume(x = i-1)`	$\wedge\ x_1 = i_0 - 1$
`5: assume(y ≠ i)`	$\wedge\ y_1 \neq i_0$

Ψ^-

Interpolate

$x_1 = ctr_1 - 1$

Ψ^+

- Cut + Interpolate at **each** point
- Pred. Map: pc_i □ Interpolant from cut i

Building Predicate Maps

Predicate Map
2: $x = ctr$
3: $x = ctr - 1$
4: $y = x + 1$

Trace	Trace Formula
`1: x = ctr`	$x_1 = ctr_0$
`2: ctr = ctr + 1`	$\wedge\ ctr_1 = ctr_0 + 1$
`3: y = ctr`	$\wedge\ y_1 = ctr_1$
`4: assume(x = i-1)`	$\wedge\ x_1 = i_0 - 1$
`5: assume(y ≠ i)`	$\wedge\ y_1 \neq i_0$

ψ^-

Interpolate

ψ^+

$y_1 = x_1 + 1$

- Cut + Interpolate at **each** point
- Pred. Map: pc_i □ Interpolant from cut i

Building Predicate Maps

Predicate Map
2: $x = ctr$
3: $x = ctr - 1$
4: $y = x + 1$
5: $y = i$

Trace	Trace Formula
`1: x = ctr`	$x_1 = ctr_0$
`2: ctr = ctr + 1`	$\wedge\ ctr_1 = ctr_0 + 1$
`3: y = ctr`	$\wedge\ y_1 = ctr_1$
`4: assume(x = i-1)`	$\wedge\ x_1 = i_0 - 1$
`5: assume(y ≠ i)`	$\wedge\ y_1 \neq i_0$

ψ^-

Interpolate

ψ^+

$y_1 = i_0$

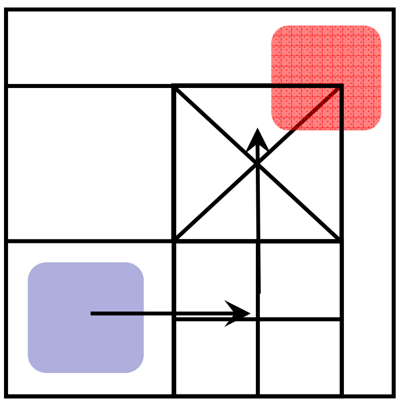
- Cut + Interpolate at **each** point
- Pred. Map: pc_i □ Interpolant from cut i

Local Predicate Use

Use predicates **needed** at **location**

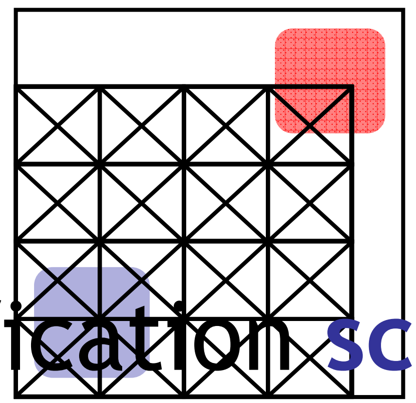
- #Preds. grows with program size
- #**Preds per location** small

Predicate Map
2: *x = ctr*
3: *x= ctr - 1*
4: *y= x + 1*
5: *y = i*



Local Predicate use

Ex: 2n states



Verification **scales …**

Global Predicate use

Ex: 2^n states

Question: When Do Interpolants Exist?

- Craig’s Theorem guarantees existence for first order logic
- But we are interpreting formulas over theories (arithmetic, theories of data structures)

The Good News

- Interpolants always exist for recursively enumerable theories
  - The proof is a simple application of compactness
- So: interpolants exist for Presburger arithmetic, sets with cardinality constraints, theory of lists, (quantifier-free) theory of arrays, multisets, …

The Bad News

- “The proof is a simple application of compactness”
  - May be algorithmically inefficient
  - Daunting engineering task to construct interpolating decision procedure for each individual theory

An Alternate Path: Reduction

- Want to compile formulas in a new theory to formulas in an old theory such that interpolation in the old theory imply interpolation in the new theory

- T reduces to R: can compile formulas in theory T to formulas in theory R
  - And use decision procedures for R to answer decision questions for T

- Technically: Given theories T and R, with $R \subseteq T$, a reduction is a computable map μ from T formulas to R formulas such that for any T-formula ϕ:
  - ϕ and $\mu(\phi)$ are T-equivalent
  - ϕ is T-satisfiable iff $\mu(\phi)$ is R-satisfiable

Example: Theory of Sets

Theory of sets reduces to theory of equality with uninterpreted functions

$x = y$	$\forall e.\ e \in x \Leftrightarrow e \in y$
$x = \emptyset$	$\forall e. e \notin x$
$x = U$	$\forall e. e \in x$
$x = \{e\}$	$e \in x \wedge \forall e'. e' \in x \Rightarrow e = e'$
$x = y \cup z$	$\forall e. e \in x \Leftrightarrow e \in y \vee e \in z$
$x = y \cap z$	$\forall e. e \in x \Leftrightarrow e \in y \wedge e \in z$

Example: Theory of Multisets

Theory of multisets reduces to the combination theory of equality with uninterpreted functions and linear arithmetic

$x = y$	$\forall$ e. count(x,e) = count(y,e)
$x=\emptyset$	$\forall$ e.count(x,e) = 0
x=[(e,n)]	count(x,e)=max(0,n)
	$\wedge\ \forall$ e'.e'$\neq$ e $\Rightarrow$ count(x,e')=0
$x=y\uplus z$	$\forall$ e.count(x,e)= count(y,e)+count(z,e)
$x=y\cup z$	$\forall$ e. count(x,e) = max(count(y,e), count(z,e))
$x=y\cap z$	$\forall$ e. count(x,e) = min(count(y,e), count(z,e))

Reduction and Interpolation

Ψ^- and Ψ^+ in Theory T

Reduction from T to R

Φ^- and Φ^+ in Theory R

Interpolate in R

Interpolant α in
Theory R as well as T

Eliminate quantifiers in T or R

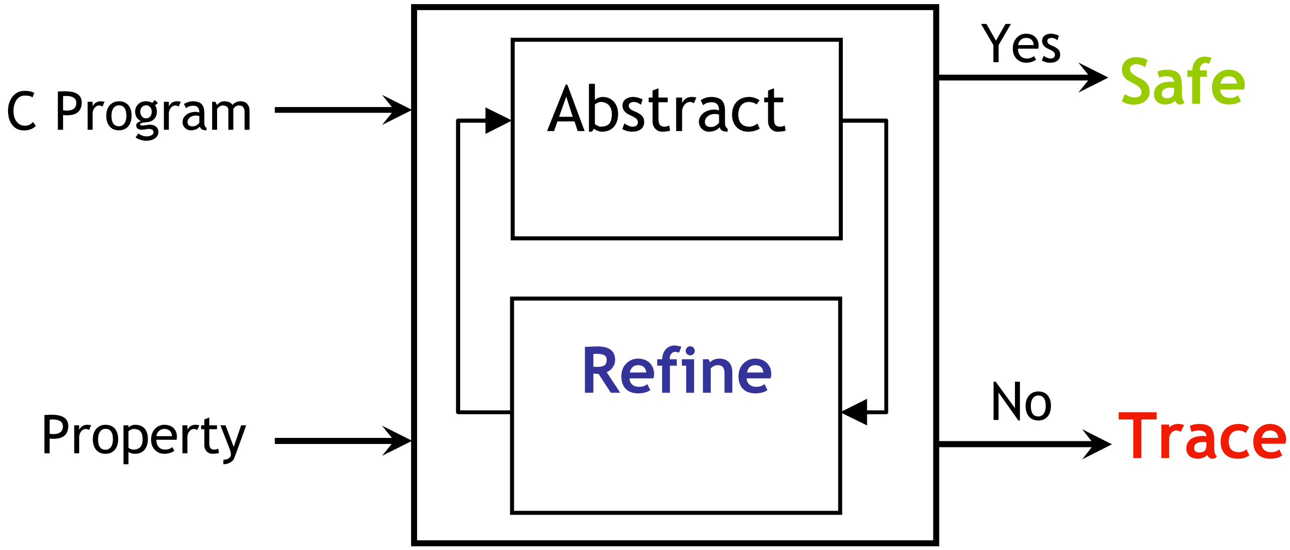
Quantifier-free
interpolant

[KapurM.Zarba06]

Reduction Theorem

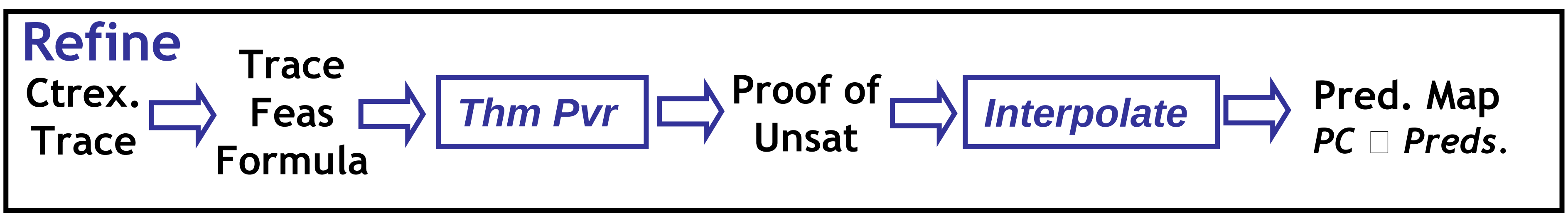
- Interpolants for the theory of arrays, sets, and multisets can be computed by reduction to the combination theory of linear arithmetic and equality with uninterpreted functions
  - We already have interpolating decision procedures for this latter theory

Lazy Abstraction



Problem: #Preds grows w/ Program Size

Solution: Localize pred. use, find where preds. needed



So far …

Lazy Abstraction

- Predicates:
  - Abstract infinite program states
- Counterexample-guided Refinement:
  - Find predicates tailored to prog, property

1. **Abstraction** : Expensive

   Reachability Tree

2. **Refinement** : Find predicates, use locations

   Proof of unsat of TF + **Interpolation**

So how well does all this work?

Quite well, if the program and property are control-dominated

Not so well when data is involved…

Localizing

Property3:

IRP Handler

Win NT DDK

Program	*Lines**	*Previous Time(mins)*	*Time (mins)*	*Predicates* Total	*Predicates* Average
kbfiltr	12k	1	3	72	*6.5*
floppy	17k	7	25	240	*7.7*
diskprf	14k	5	13	140	*10*
cdaudio	18k	20	23	256	*7.8*
parport	61k	DNF	74	753	*8.1*
parclss	138k	DNF	77	382	*7.2*

* Pre-processed

Refinement Failure: Unrolling Loops

```
x = 0; y = 50;
while ( x<100 ) {
  if ( x>=50 ) y = y+1;
  x = x+1;
}
assert( y==100 );
```

- counterexample:
  x=0; y=50; x>=100; y==100
  refinement: x==0
- counterexample:
  x=0; y=50; x<100; x=x+1; x>=100; y==100
  refinement: x==1
- counterexample:
  x=0; y=50; x<100; x=x+1; x<100; x=x+1;
  x>=100; y==100
  refinement: x==2
- ...

Refinement Failure: Unfolding Arrays

```
for (i=0; i<n; i++) {
  a[i]=i;
}
for (j=0; j<n; j++) {
  assert( a[j]==j );
}
```

- counterexample:
  i=0; i<n; a[i]=i; i++; i>=n;
  j=0; j<n; a[j]!=j
  refinement: a[0]==0
- counterexample:
  i=0; i<n; a[i]=i; i++; i<n; a[i]=i; i++; i>=n;
  j=0; j<n; a[j]==j; j++; j<n; a[j]!=j
  refinement: a[1]==1
- …

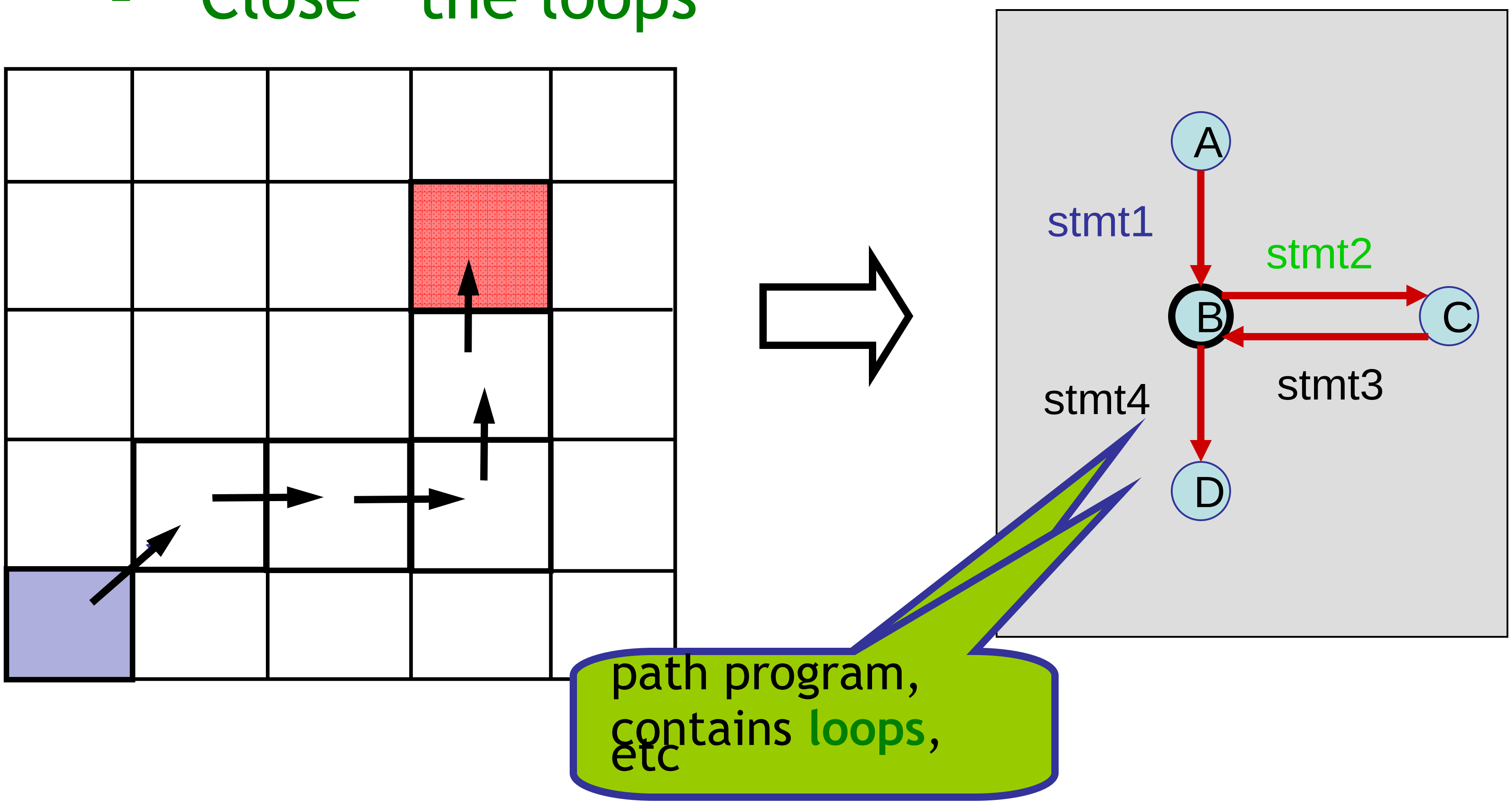
What went Wrong?

- Consider all unrolled counterexamples at once
  - Convergence of abstraction discovery
- Inspect families of counterexamples of unbounded length
  - Justification for unbounded universal quantification
- Looking at one counterexample path at a time is too weak [JhalaMcMillan05,JhalaMcMillan06]

[BeyerHenzingerM.Rybalchenko07]

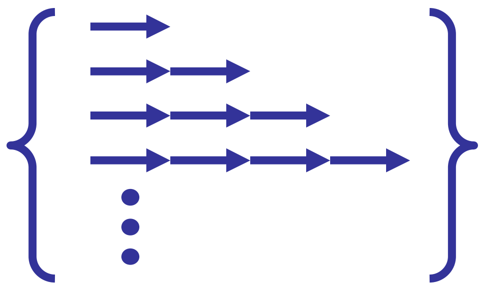
Path Programs

- Treat counterexamples as programs
  - “Close” the loops



Meaning of Path Programs

Path program ´ (Possibly unbounded) sets of counterexamples:



- Unbounded counterexamples

- Property-determined fragment of original program
  - Can be analyzed independently to find good abstractions

Path Invariants

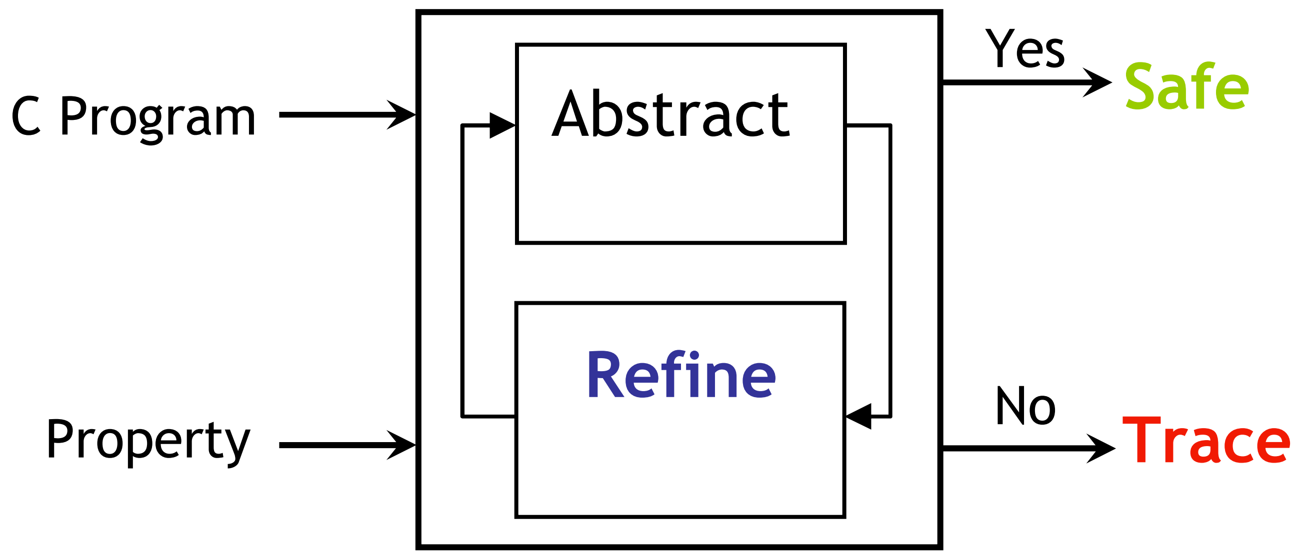
- Invariant for path programs ´ path invariant
- Abstraction refinement using path invariants
  - Elimination of **all** counterexamples **within** path program
  - Justification for **unbounded** quantification

Invariant Generation

- Given a path program, with a designated error location, find an invariant that demonstrates error is not reachable
  - Can scale: Reduced obligation to program fragment
  - Outer model checking loop integrates path invariants into program invariant

- Can use any technique
- We use constraint-based invariant generation [SankaranarayananSipmaManna04,BeyerHenzingerM.Rybalchenko07]

Lazy Abstraction



Problem: #Preds grows w/ Program Size

Solution: Localize pred. use, find where preds. needed

Refine

Ctrex. Trace ⇒ Trace Feas Formula ⇒ *Thm Pvr* ⇒ Proof of Unsat ⇒ *Interpolate* ⇒ Pred. Map *PC* □ *Preds.*

So far …

Lazy Abstraction

- Predicates:
  - Abstract infinite program states
- Counterexample-guided Refinement:
  - Find predicates tailored to prog, property

1. **Abstraction** : Expensive

   Reachability Tree

2. **Refinement** : Find predicates, use locations

   Proof of unsat of TF + **Interpolation**

Questions?

Lecture 3:
Technical Extensions and Termination

Rupak Majumdar

Recap …

Lazy Abstraction

- Predicates:
  - Abstract infinite program states
- Counterexample-guided Refinement:
  - Find predicates tailored to prog, property

1. **Abstraction** : Expensive
   Reachability Tree

2. **Refinement** : Find predicates, use locations
   Proof of unsat of TF + **Interpolation**

Technical Details

Q. How to analyze recursive procedures ?

An example

```
main(){
   □
1: if (flag){
2:    y = inc(x,flag);
3:    if (y<=x) ERROR;
   } else {
4:    y = inc(z,flag);
5:    if (y>=z) ERROR;
   }
   □
return;
}
```

```
inc(int a, int sign){
1: if (sign){
2:    rv = a+1;
   } else {
3:    rv = a-1;
   }
4: return rv;
}
```

Inline Calls in Reach Tree

```
main(){
    □
1: if (flag){
2:     y = inc(x,flag);
3:     if (y<=x) ERROR;
   } else {
4:     y = inc(z,flag);
5:     if (y>=z) ERROR;
   }
   □
return;
}
```

```
inc(int a, int sign){
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   }
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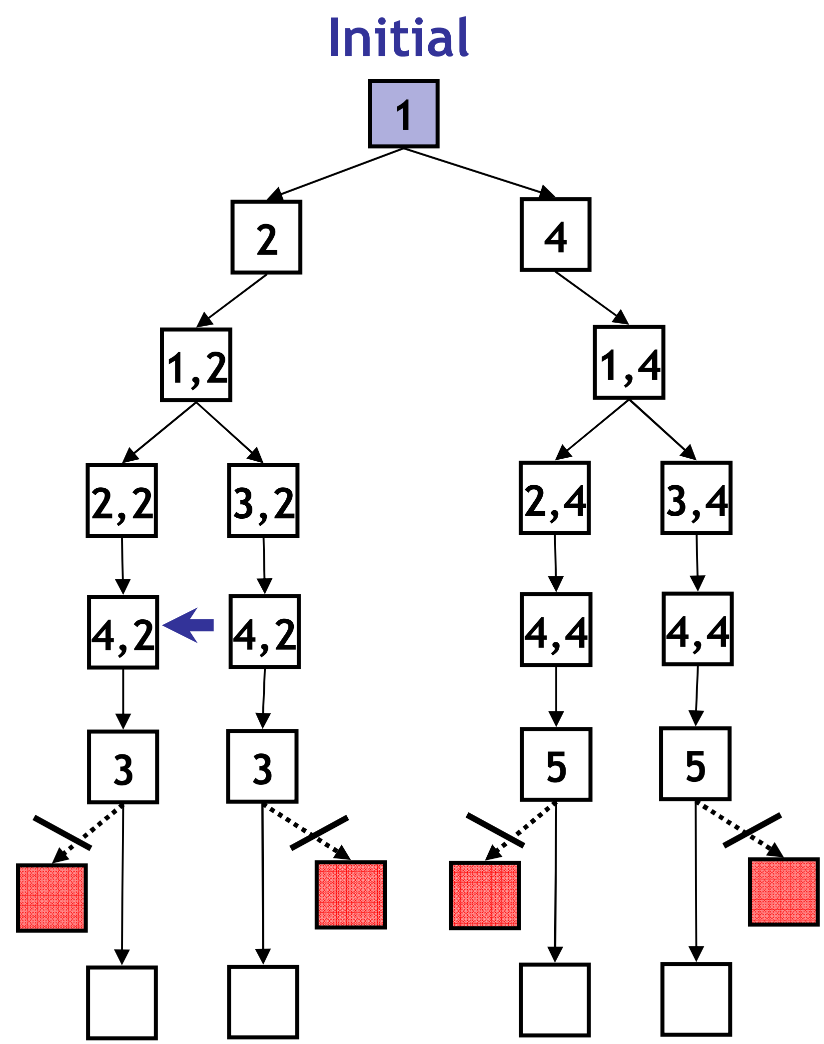
```



Inline Calls in Reach Tree

Problem

- Repeated analysis for "inc"
- Exploding call contexts

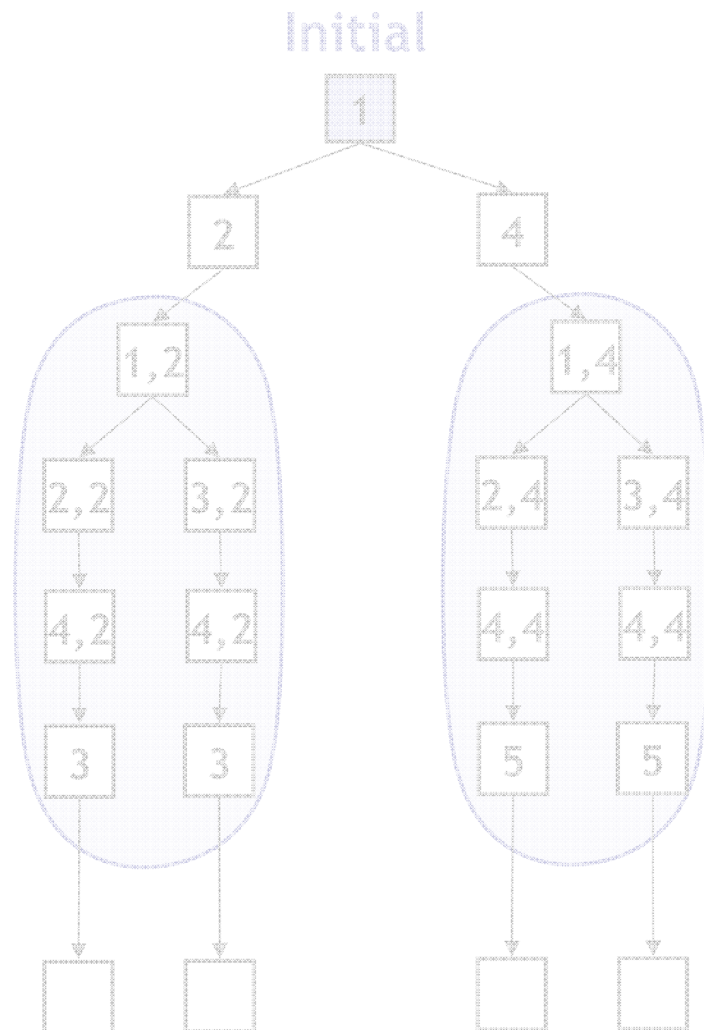
```
int x; //global
f1(){
```

```
f2(){
```

```
f3(){
```

```
f4(){
```

```
fn(){
1: x ++;
return;
}
```

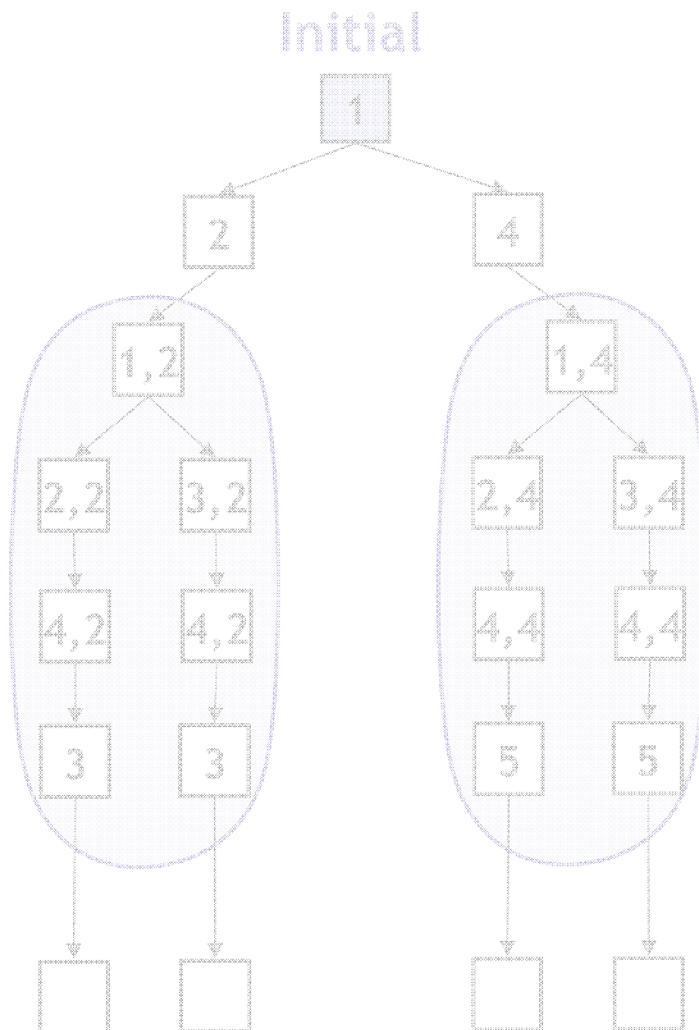


2^n nodes in Reach Tree

Inline Calls in Reach Tree

Problem

- Repeated analysis for “inc”
- Exploding call contexts
- Cyclic call graph (Recursion)
  - Infinite Tree!



Solution : Procedure Summaries

Summaries: Input/Output behavior

- Plug summaries in at each callsite

  … instead of inlining entire procedure

[Sharir-Pnueli 81, Reps-Horwitz-Sagiv 95]

- Summary = set of ***(F □ F')***
  - ***F*** : Precondition formula describing input state
  - ***F'*** : Postcondition formula describing output state

Solution : Procedure Summaries

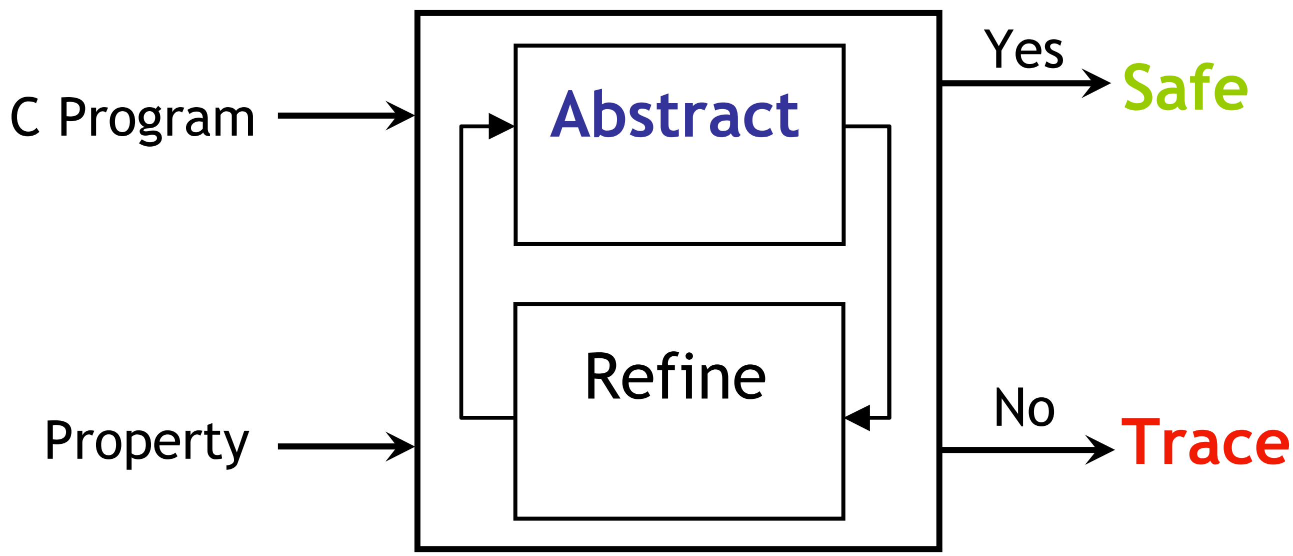
```
inc(int a, int sign){
1: if (sign){
2:   rv = a+1;
   } else {
3:   rv = a-1;
   }
4: return rv;
}
```

- *(¬ sign=0 □ rv > a)*
- *(sign = 0 □ rv < a)*

- Summary = set of *(F □ F')*
  - *F* : Precondition formula describing input state
  - *F'* : Postcondition formula describing output state

Q. How to compute, use summaries ?

Lazy Abstraction + Procedure Summaries



Q. How to compute, use summaries ?

Abstraction with Summaries

```
main(){
1: if (flag){
2:    y = inc(x,flag);
3:    if (y<=x) ERROR;
   } else {
4:    y = inc(z,flag);
5:    if (y>=z) ERROR;
   }

return;
}
```

```
inc(int a, int sign){
1: if (sign){
2:    rv = a+1;
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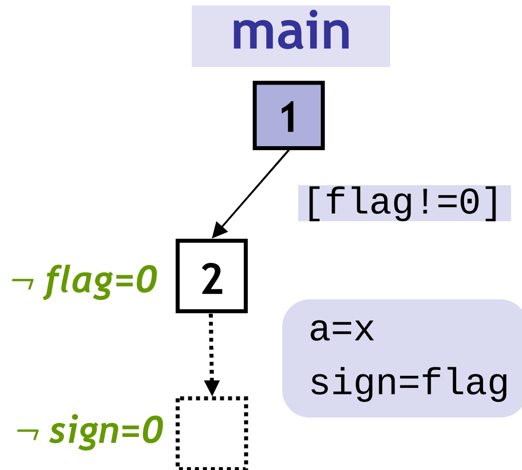
4: return rv;
}
```



Predicates: *flag=0 , y>x , y<z*

sign=0 , rv>a , rv<a

Abstraction with Summaries

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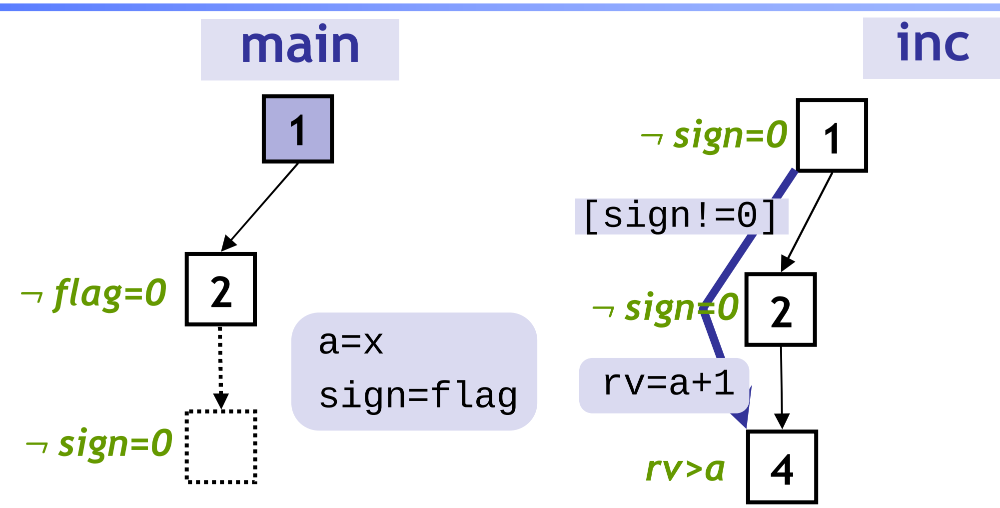

Predicates: *flag=0 , y>x , y<z*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*

Summary Successor

```
main(){
   □
1: if (flag){
2:    y = inc(x,flag);
3:    if (y<=x) ERROR;
   } else {
4:    y = inc(z,flag);
5:    if (y>=z) ERROR;
   }
   □
return;
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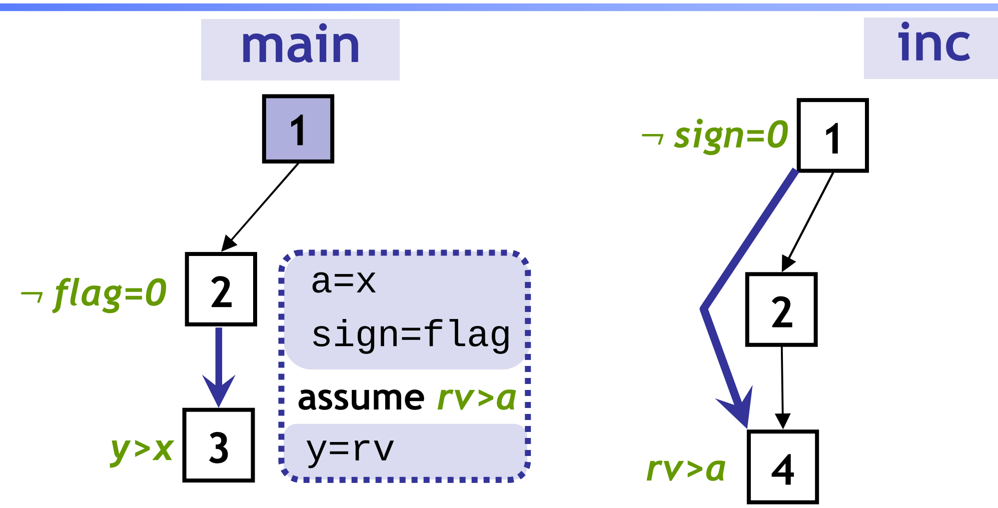

Predicates: *flag=0 , y>x , y<z*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*

Abstraction with Summaries

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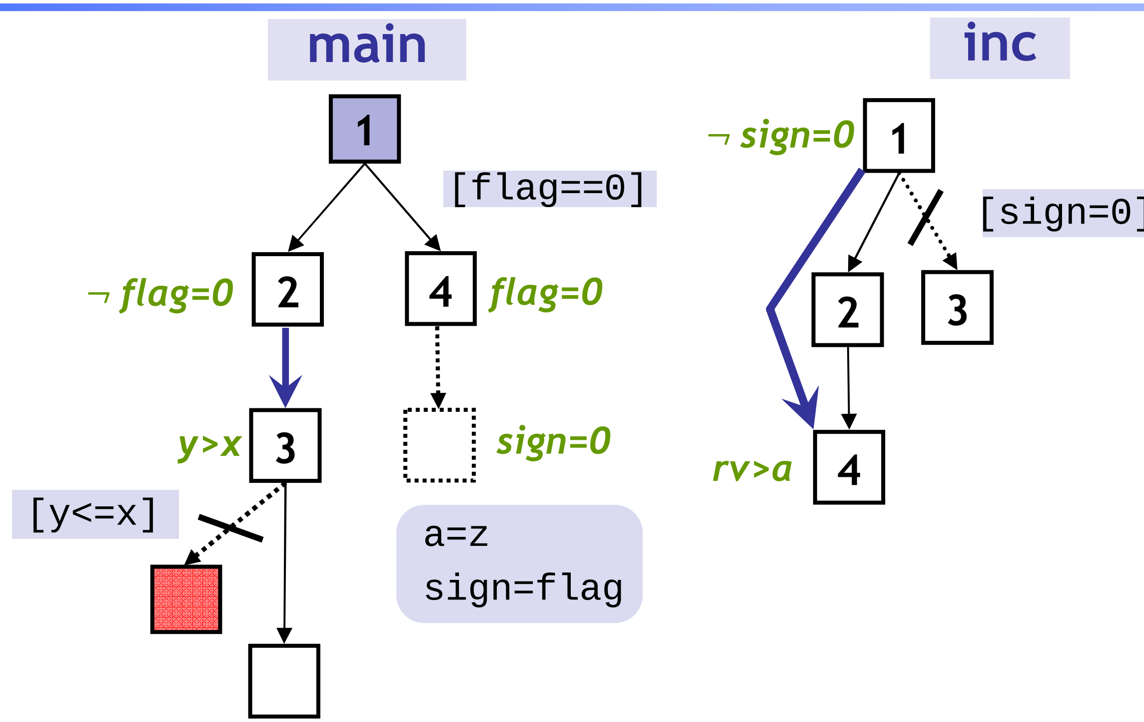

Predicates: *flag=0 , y>x , y<z*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*

Abstraction with Summaries

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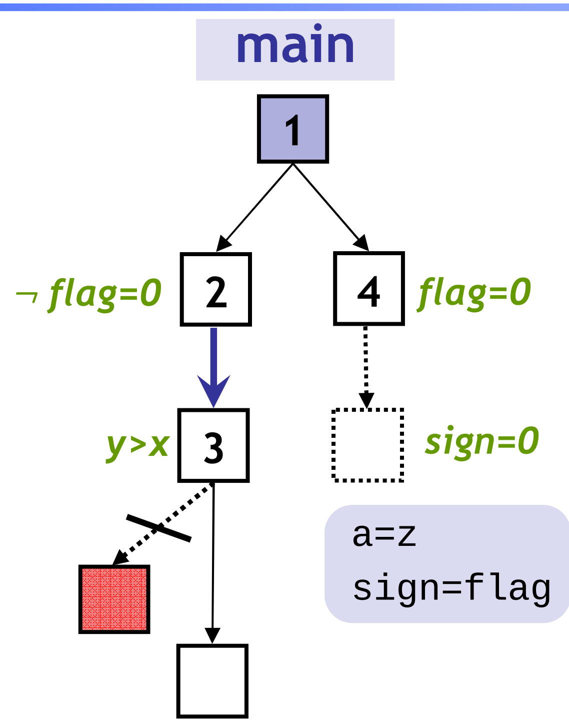

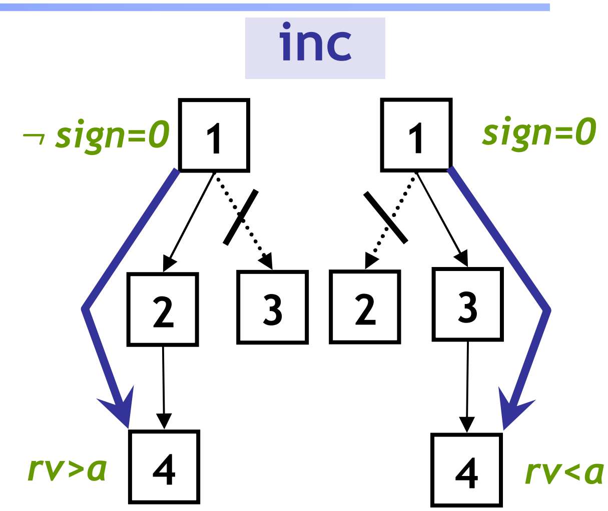

Predicates: *flag=0 , y>x , y<z*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*
(sign=0 □ rv<a)

Summary Successor

```
main(){
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1: if (flag){
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   }
   □
return;
}
```

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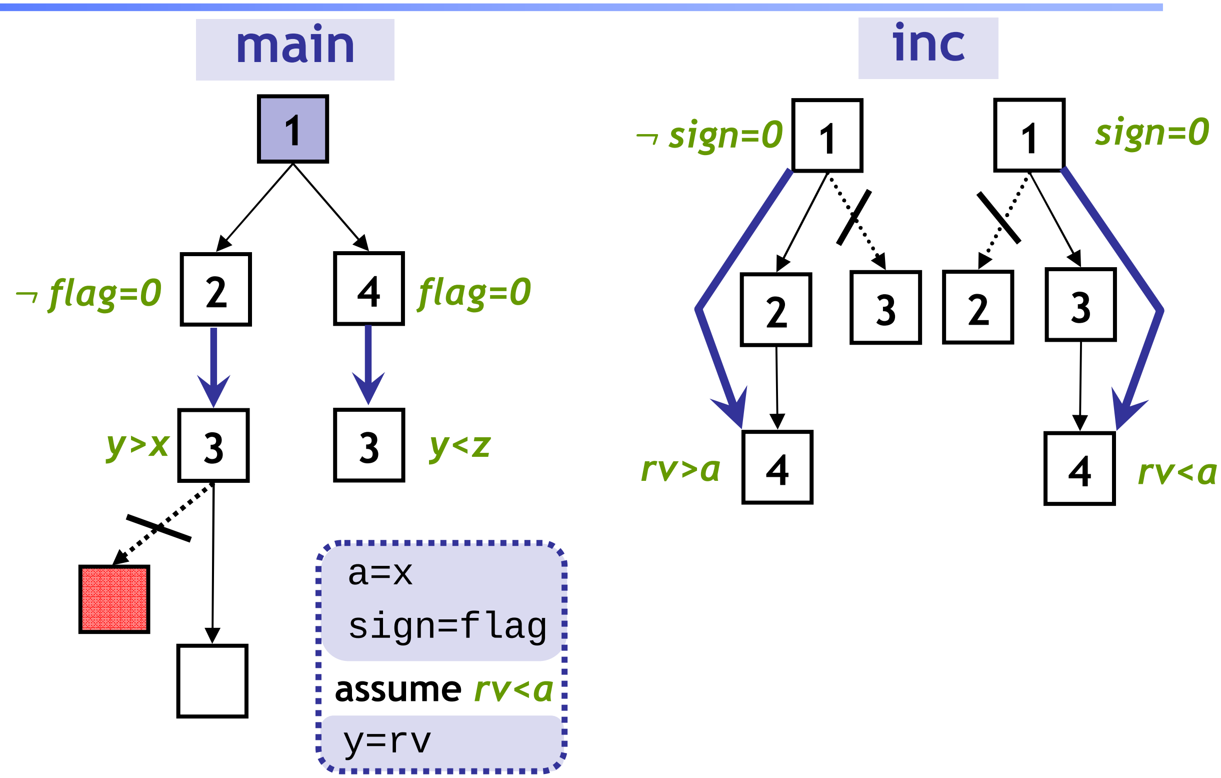

Predicates: *flag=0 , y>x , y<z*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0* □ *rv>a),*
(sign=0 □ *rv<a)*

Abstraction with Summaries

```
main(){
   □
1: if (flag){
2:    y = inc(x,flag);
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   }
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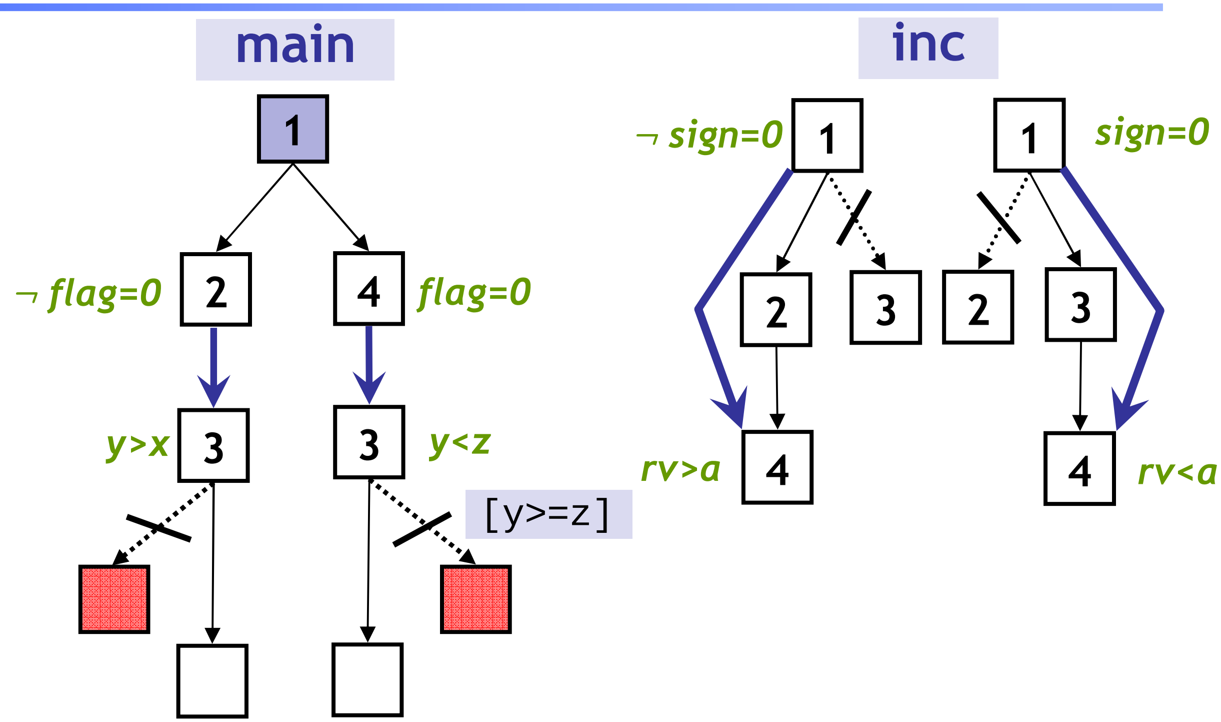


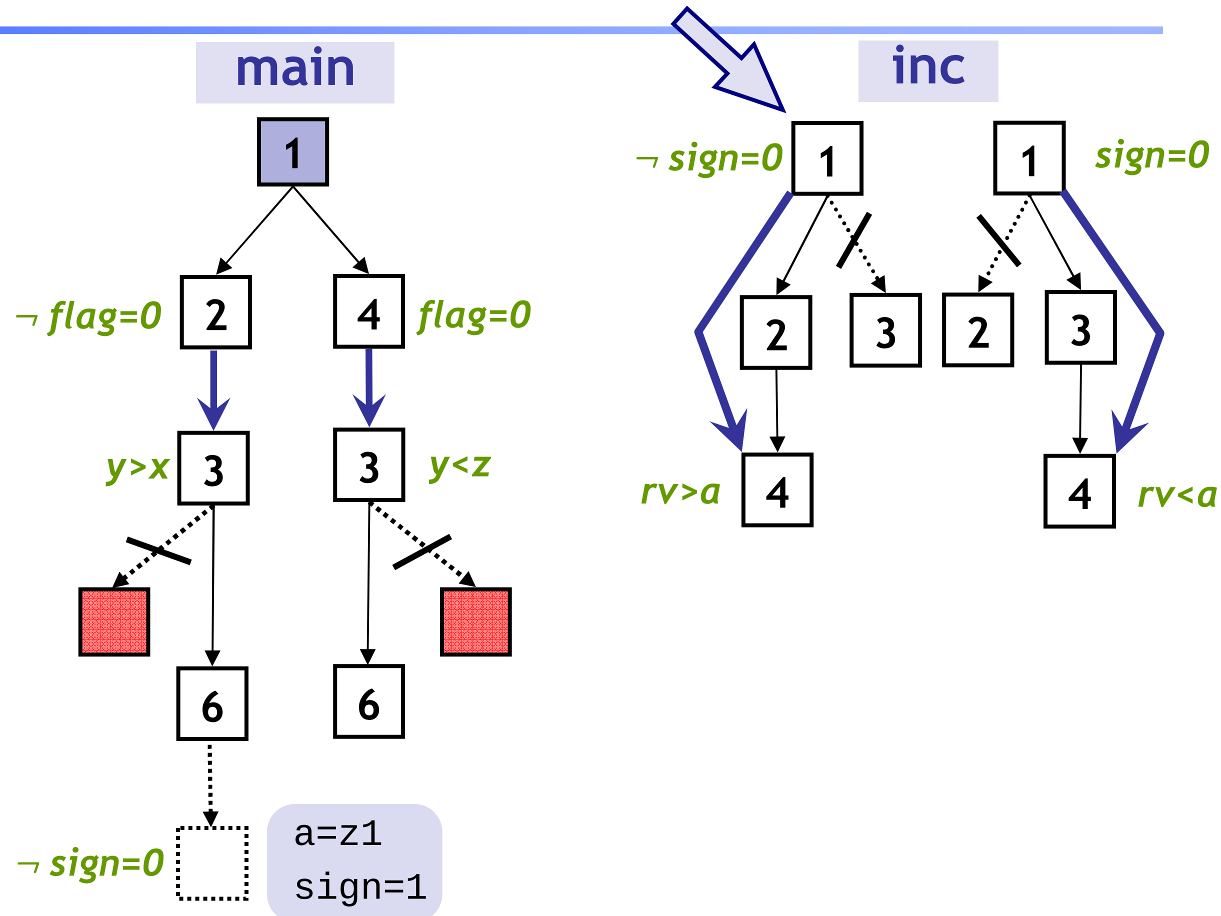
Predicates: *flag=0 , y>x , y<z*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*
(sign=0 □ rv<a)

Another Call …

```
main(){
   □
1: if (flag){
2:    y = inc(x,flag);
3:    if (y<=x) ERROR;
   } else {
4:    y = inc(z,flag);
5:    if (y>=z) ERROR;
   }
6: y1 = inc(z1,1);
7: if (y1<=z1) ERROR;
return;
}
```

```
inc(int a, int sign){
1: if (sign){
2:    rv = a+1;
   } else {
3:    rv = a-1;
   }
4: return rv;
}
```



Predicates: *flag=0 ,y>x,y<z, y1>z1*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*
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Another Call …

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main(){
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5:    if (y>=z) ERROR;
   }
6: y1 = inc(z1,1);
7: if (y1<=z1) ERROR;
return;
}
```

```
inc(int a, int sign){
1: if (sign){
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   } else {
3:    rv = a-1;
   }
4: return rv;
}
```
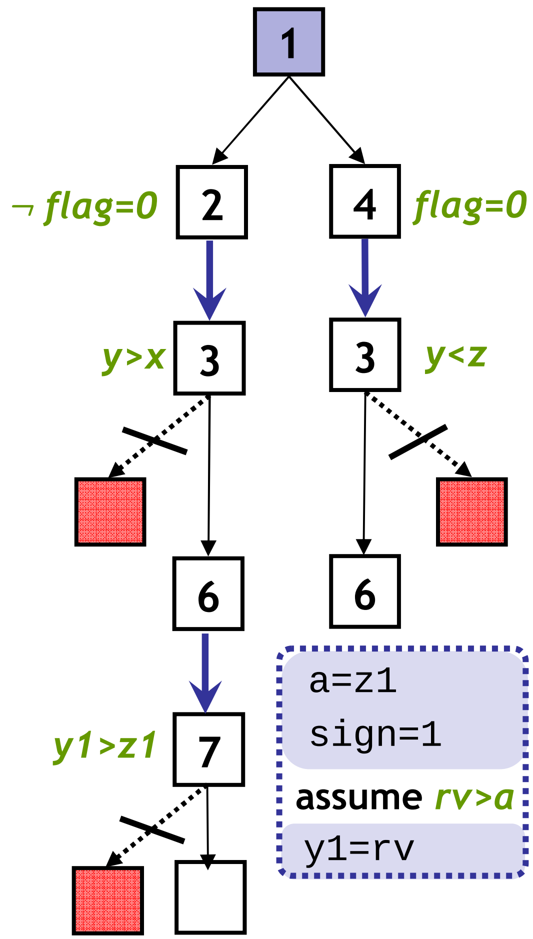
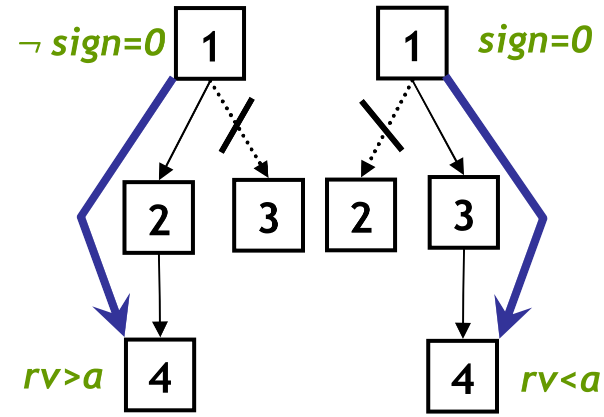

main



inc



SAFE

Predicates: *flag=0 ,y>x,y<z, y1>z1*
sign=0 , rv>a , rv<a

Summary: *(¬ sign=0 □ rv>a),*
(sign=0 □ rv<a)

Technical Details

Q. **How to perform interpolation in the presence of recursive calls?**

Traces with Procedure Calls

Trace | **Trace Formula**

pc_1: $x_1 = 3$

pc_2: assume $(x_1>0)$

pc_3: $x_3 = f_1(x_1)$

pc_4: $y_2 = y_1$

pc_5: $y_3 = f_2(y_2)$

pc_6: $z_2 = z_1+1$

pc_7: $z_3 = 2*z_2$

pc_8: return z_3

pc_9: return y_3

pc_{10}: $x_4 = x_3+1$

pc_{11}: $x_5 = f_3(x_4)$

pc_{12}: assume$(w_1<5)$

pc_{13}: return w_1

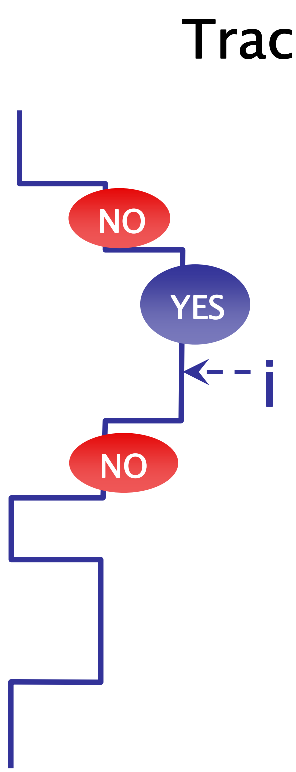
pc_{14}: assume $x_4>5$

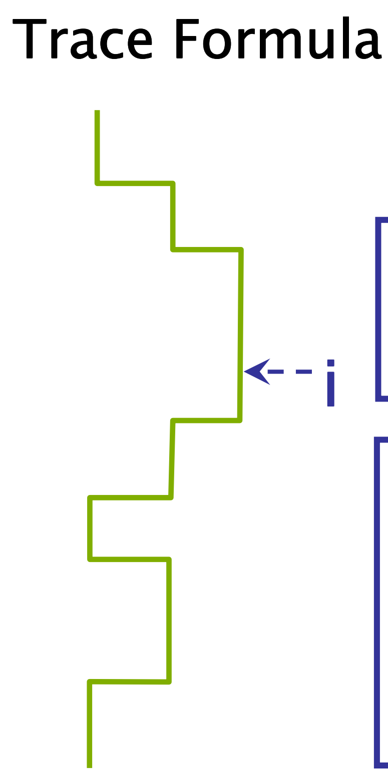
pc_{15}: assume$(x_1=x_3+2)$

i

i

Interprocedural Analysis





Find predicate needed at point i

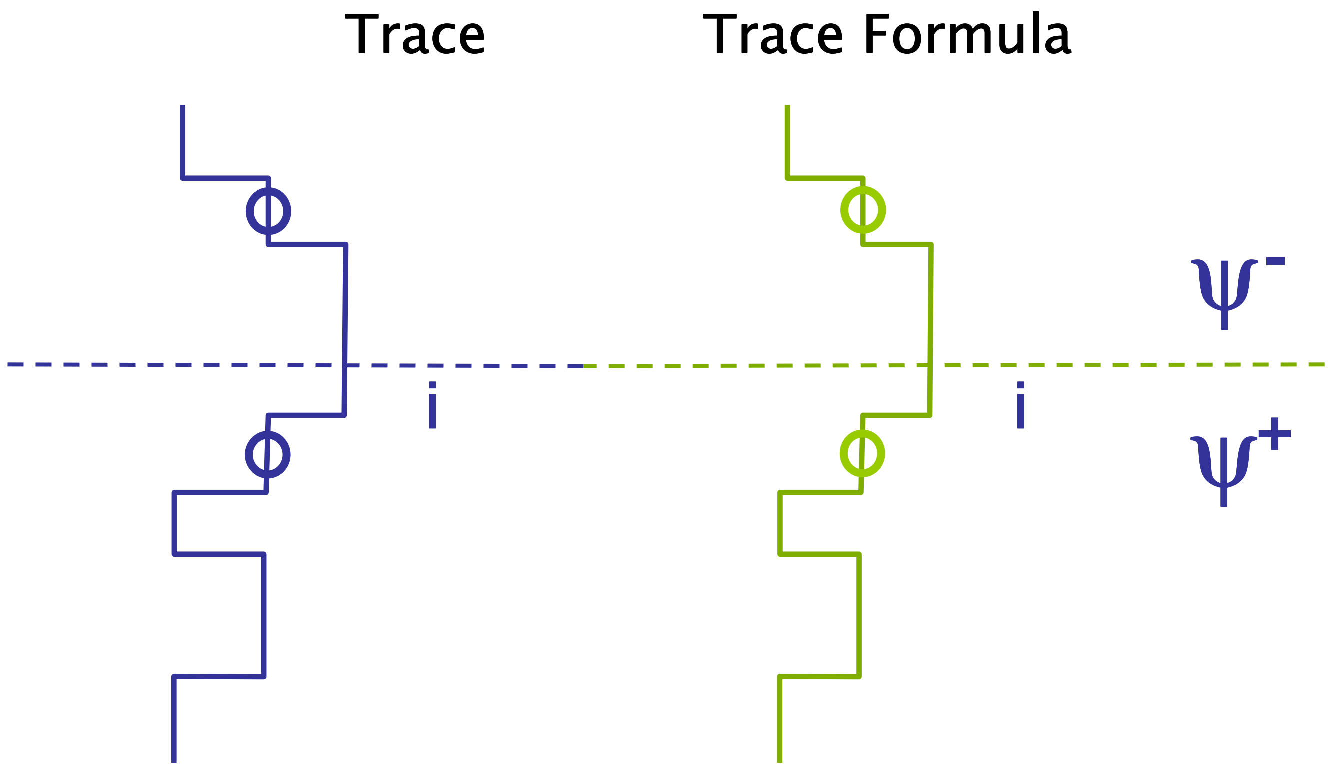
Require at each point i :

Scoped predicates

YES: Variables **visible** at i

NO: Caller's local variables

Problems with Cutting

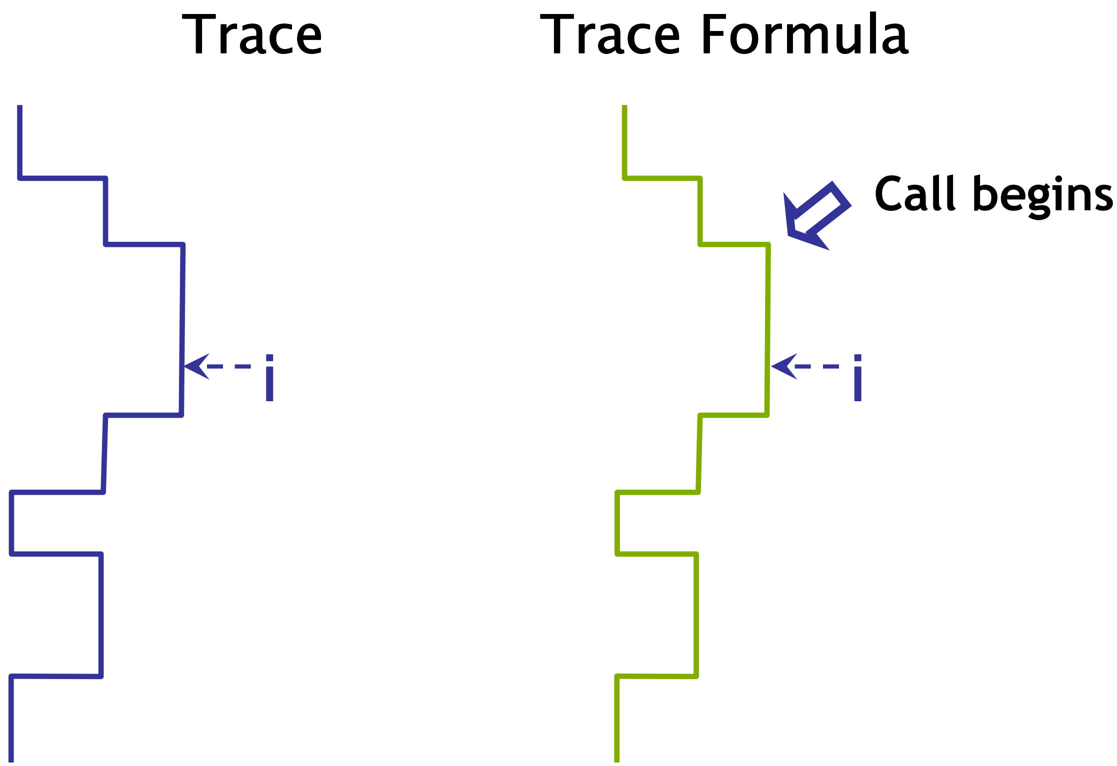


Caller variables common to ψ^- and ψ^+

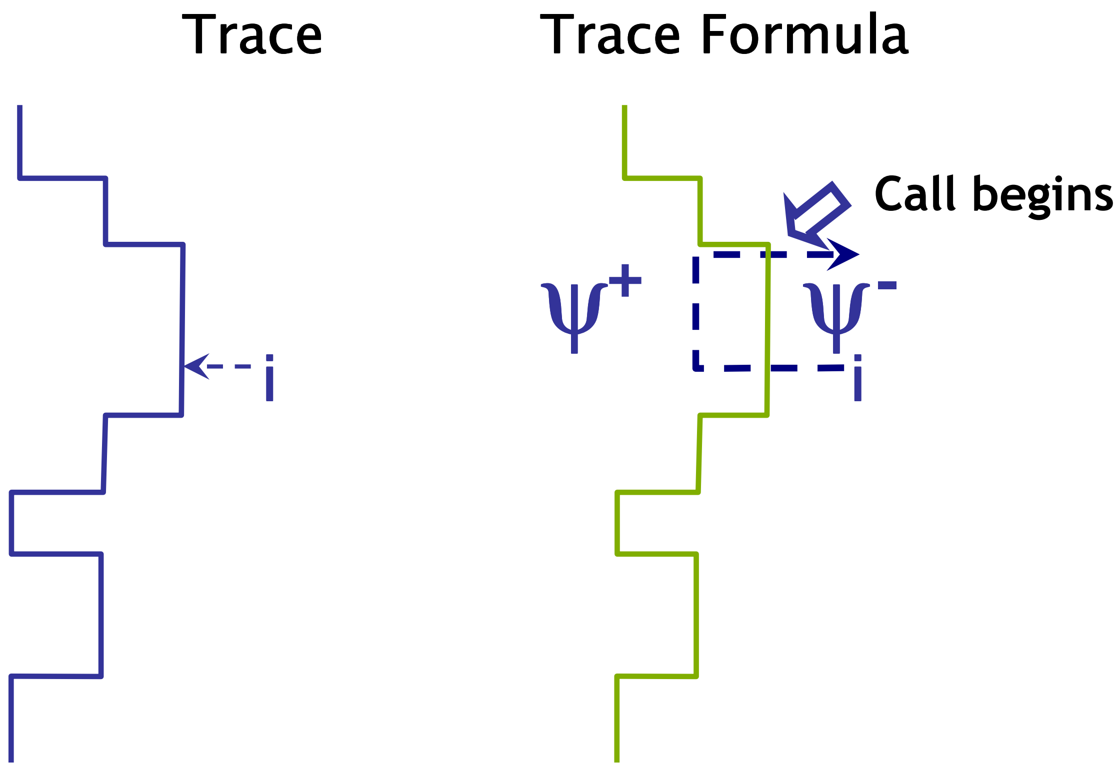
- Unsuitable interpolant: not well-scoped

Scoped Cuts

Trace Formula

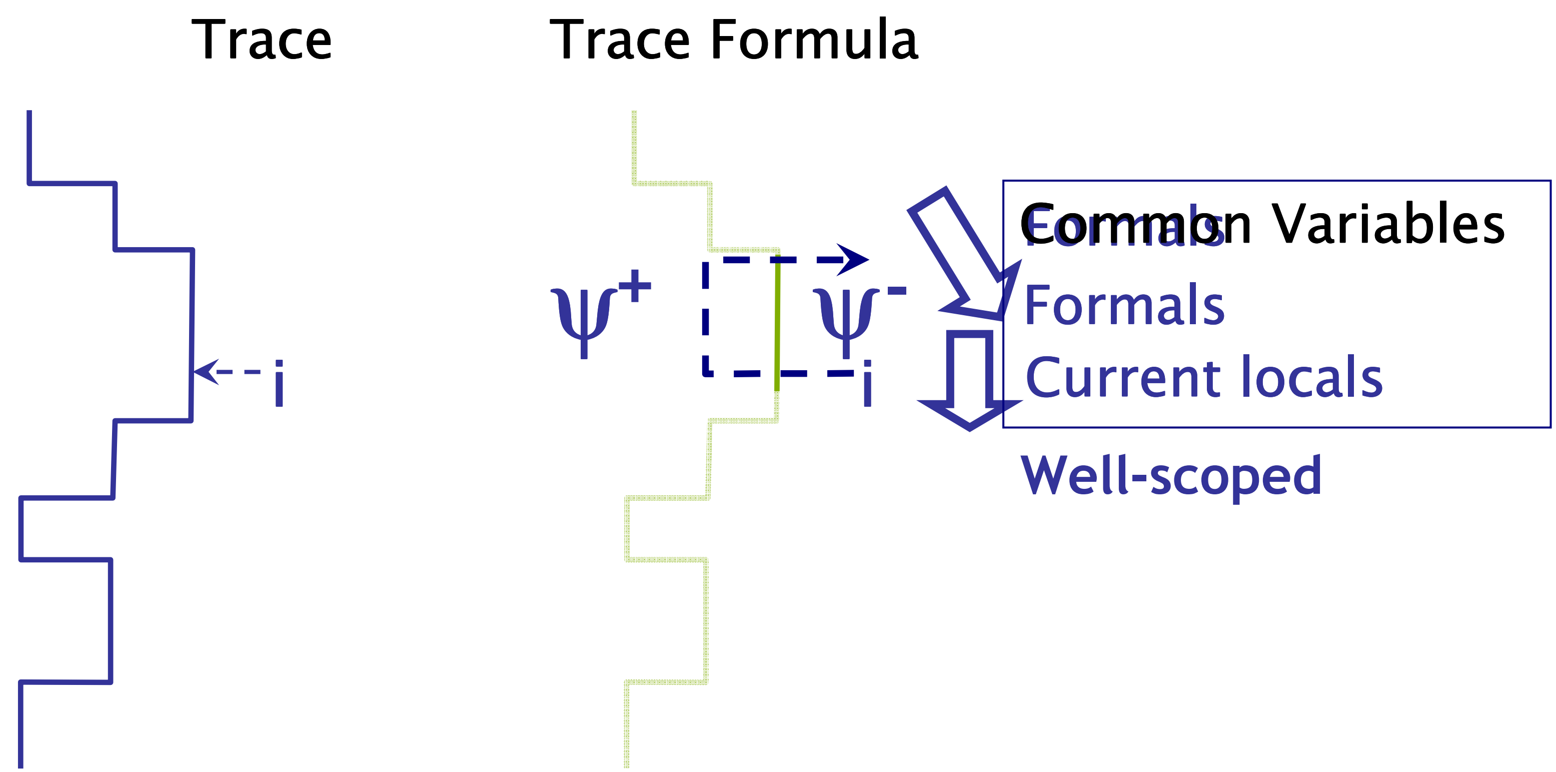


Scoped Cuts



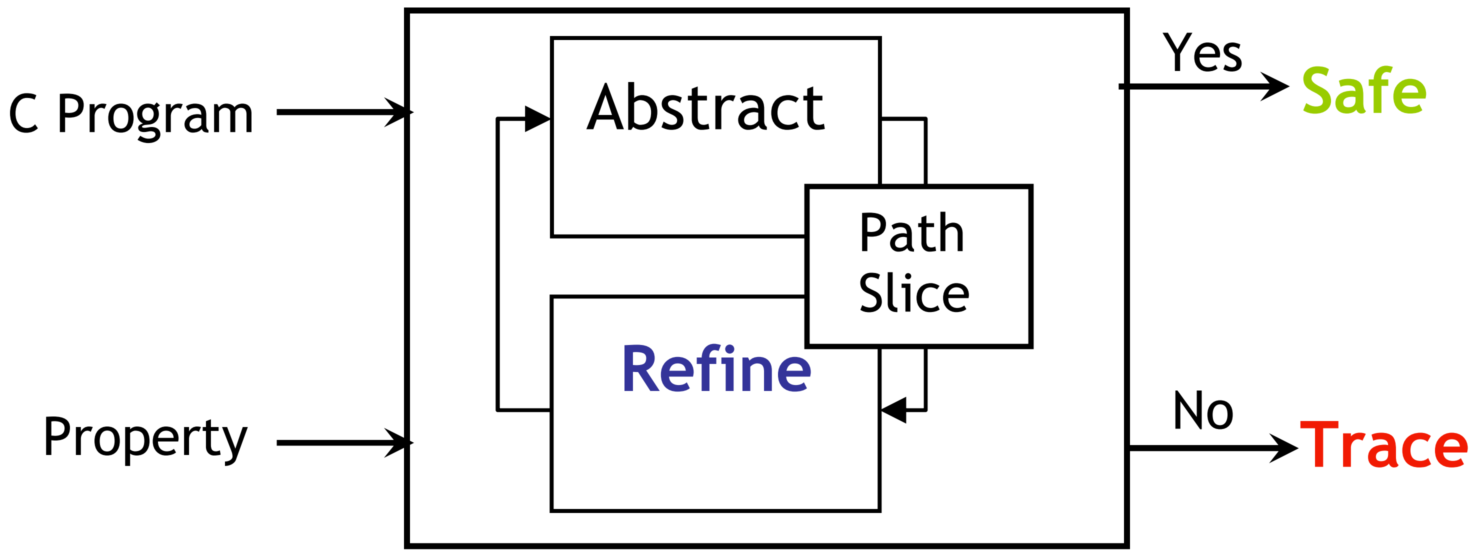
Predicate at pc_i = Interpolant from cut i

Common Variables



Predicate at pc_i = Interpolant from i–cut

Lazy Abstraction: Summary



Lazy Abstraction: Summary

- Predicates:
  - Abstract infinite program states
- Counterexample-guided Refinement:
  - Find predicates tailored to prog, property

1. **Abstraction** : Expensive
   Reachability Tree, Procedure summaries
2. **Refinement** : Find predicates, use locations
   Slice irrelevant details
   Proof of unsat of TF + Interpolation

Extension

Merging CEGAR and Abstract Domains

Comparing dataflow and CEGAR

	Dataflow	CEGAR with predicate abstraction
Precision	- Propagates facts over infeasible branches	+ Analysis considers only feasible branches
	- Dataflow facts lost at join points	+ Path sensitive analysis considers each path independently

Comparing dataflow and CEGAR

	Dataflow	**CEGAR with predicate abstraction**
Fact discovery	Eager Analysis **+** Quickly computes relevant facts **-** Not adaptive, may become overwhelmed with irrelevant facts	Lazy Analysis **-** Analysis expensive and predicates not always found **+** Discovers relevant facts through counterexamples
Domain-specific analyses	**+** Easily adapted, flexible representation of facts	**-** Predicates not convenient for domain-specific analyses

Can we combine the best of both approaches?

Predicated lattices

- For each program location, track a set of first-order predicates over program variables
- Rather than track dataflow facts directly, track a *predicated lattice*:
  - New dataflow facts:

    map: predicates → original dataflow facts
  - Only join facts associated with the same set of predicates
  - Default predicate: `true`
- Instance of *reduced cardinal powers of lattices* [Cousot² 79]

Example

```
1: out = fopen(...);

2: fprintf(out, ...);

3: fclose(out);
    .
    .
```

true out→O

OK

true out→C

Example

```
4: flag = {0,1};

5: if (flag)
6:    out = fopen(...);

7: else
      ;
    .
    .

8: if (flag)

9:    fprintf(out, ...);
```

true → out→C

true → out→O

true → out→C

Facts lost due to join

true → out→T

Add predicates:
flag!=0, flag==0

true → out→T

Error!

Repeat with new predicates

```
4: flag = {0,1};

5: if (flag)
6:   out = fopen(...);

7: else
       ;
    .
    .

8: if (flag)

9:   fprintf(out, ...);
```

Both facts kept at join

true → out→C

flag!=0 → out→O

flag==0 → out→C

flag!=0 → out→O

flag==0 → out→C

flag!=0 → out→O

OK

Symbolic execution lattice

- Tracks values of program variables and heap
  - Facts at each program point represented as map from *names* to *values*
  - *Symbolic execution* used to compute new facts after each statement
  - A “static” version of the DART algorithm

Example

```
int x = 0;
int y = 1;
int *z;
```

x→0, y→1, z→T

```
if (*) {
  z = &x;
```

x→0, y→1, z→x

```
}
else {
  z = &y;
```

x→0, y→1, z→y

```
}
...
```

x→0, y→1, z→{x,y}=>T

```
*z = 5;
...
```

x→T, y→T, z→{x,y}=>5

```
assert(*z == 5);
```

OK

How to extend this to liveness properties?

Specifically: Reasoning about termination

When does a Program Terminate?

- Iff its reachable transition relation is well-founded

- Reachable transition relation =
  $TR(x,x') \cap Reach(x) \times Reach(x')$ =
  Restriction of the transition relation to the set of reachable states

Well-Founded Relation

- A binary relation > is *well-founded* if there is no infinite descending sequence

- No s0, s1, s2,… such that
  s0 > s1 > s2 > …

Example: > on natural numbers

But not > on integers

Idea: Rank Functions

- Fix a set X, and > a wf relation on X

- Suppose I can map each reachable state s of the transition graph to a *rank* r(s)$\in$X s.t.
  $s \rightarrow s'$ implies $r(s) > r(s')$

Then the system must terminate

The converse is also true

Example

Input x, n

While(x <= n) x++;

Terminates, using (roughly) the rank function n-x

Does it, really?

Disjunctive Rank Functions

- In many cases, finding a single wf relation can be difficult

- Suppose I can find wf relations T1,…,Tk such that RTR $\subseteq$ T1 $\cup$ … $\cup$ Tk

- Does the program terminate?
  - Not in general (Why?)

Disjunctive Well-foundedness

If T1…Tk are wf relations and

$R^+ \subseteq T1 \cup \ldots \cup Tk$

Then: R is well-founded

Such R is called *disjunctively well-founded*

Disjunctive Well-foundedness

P terminates if $TR \cap Reach \times Reach$ is disjunctively well-founded

Useful: Can consider individual portions of the program independent of other parts

Incremental Termination

T = emptyset

While TR+ not included in T:

  invariant: T is a finite union of wf relations

  find abstract counterexample to wf

  if concretely feasible

    does not terminate

  otherwise find wf relation T'

  $T = T \cup T'$

Counterexample to Termination

- Lasso = Stem + Cycle
  - Represents infinite execution

  Stem Cycle Cycle …

Needs rank-finding technique to find a wf relation showing lasso cannot be executed arbitrarily (Heuristics exist)

Reduction to Safety

- How to check if $R^+ \subseteq T$ for the reachable transition relation?

- Can reduce check to safety
- Run program parallel with a monitor for T
  - runs in parallel with the program
  - inspects pairs of states wrt. T
  - goes to error if observes $(s, s') \notin T$
  - Use non-determinism to perform check

Reduction to Safety: Idea

```
selected := ⊥
phase := SELECT

while True {
    switch (phase) {
        SELECT: if ( nondet() ) {
                        selected := current
                        phase := CHECK
                    }
        CHECK: if ( (selected, current) ∉ T ) { ERROR: }
    }
}
```

Terminator

- Input: program written in C
- Language features supported
  - nested loops, gotos
  - aliasing
  - (mutually) recursive function calls
- Output:
  - termination proof: transition invariant
  - counterexample: lasso = stem + cycle
- Scalability: (on drivers from WinDDK)

Questions?

Lecture 4:
Some Techniques for Infinite State

Rupak Majumdar

Topics

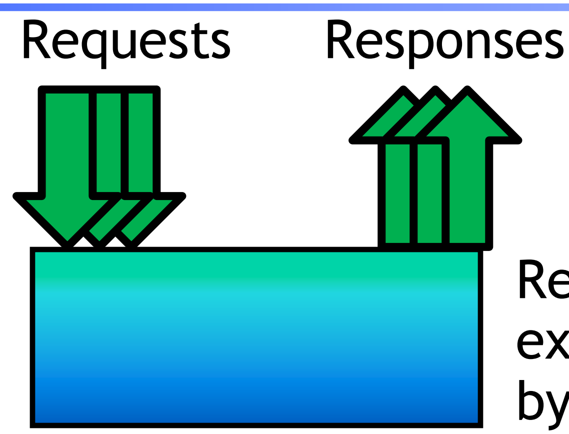
1. Safety and liveness verification for asynchronous programs

1. Case splitting and symmetry reduction for parameterized programs

Topic 1

Safety and liveness verification for asynchronous programs

Asynchronous Programs



Requests queued and executed asynchronously by cooperative scheduler

Useful Model:

- Distributed Systems
- Web Servers
- Embedded Systems

Languages and Libraries:

- LibAsync, LibEvent, …
- NesC and TinyOS
- Go
- AJAX

Asynchronous Programs

```
global bit b = 0;
```

```
main(){
 ...
 async h1();
 ...
}
```

```
h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}
```

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

- main ends in dispatch location
  - Calls asynchronously posted functions
- Async calls stored in task buffer
- Scheduler picks a pending task and runs it to completion

Asynchronous Program Execution

```
global bit b = 0;
```

PC

```
main(){
 ...
 async h1();
 ...
}
```

```
h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}
```

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

- Execution starts in main
- Task buffer empty

Pending Calls
h1

State: b = 0

Asynchronous Program Execution

```
global bit b = 0;

main(){
 ...
 async h1();
 ...
}

h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}

h2(){
 ...
 b = 1;
 ...
 return;
}
```

- Execution enters dispatch loop
- Picks pending call and executes it
- Returns to dispatch loop on return

Pending Calls
h1
h2

State: b = 0

Asynchronous Program Execution

```
global bit b = 0;
```

```
main(){
 ...
 async h1();
 ...
}
```

PC

```
h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}
```

PC

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

- Pick another pending call

Pending Calls
h1
h2
h1
h2

State: b = 0

Asynchronous Program Execution

```
global bit b = 0;
```

```
main(){
  ...
  async h1();
  ...
}
```

```
h1(){
  if(b == 0){
    async h1();

    async h2();

    return;
  }

}
```

```
h2(){
  ...
  b = 1;
  ...
  return;
}
```

PC

PC

- Pick some pending task

Pending Calls
h2
h1
h2

State: b = 1

Asynchronous Program Execution

```
global bit b = 0;
```

```
main(){
  ...
  async h1();
  ...
}
```

```
h1(){
  if(b == 0){
    async h1();

    async h2();

    return;
  }

}
```

```
h2(){
  ...
  b = 1;
  ...
  return;
}
```

PC

PC

- Pick some pending task

Pending Calls
h2
h1

State: b = 1

Asynchronous Program Execution

```
global bit b = 0;
```

```
main(){
  ...
  async h1();
  ...
}
```

```
h1(){
  if(b == 0){
    async h1();

    async h2();

    return;
  }

}
```

```
h2(){
  ...
  b = 1;
  ...
  return;
}
```

- And the program terminates

Pending Calls

h2

State: b = 1

Observations

```
global bit b = 0;

main(){
 ...
 async h1();
 ...
}

h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}

h2(){
 ...
 b = 1;
 ...
 return;
}
```

- Call stack for a handler can be unbounded
- Shared global state can change between posting and execution
- Task Buffer can be unbounded
  - Consider *n* runs of h1 before h2
  - Task buffer contains *n* h1, *1* h2

Properties: Safety

```
global bit b = 0;

main(){
 ...
 async h1();
 ...
}

h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}

h2(){
 ...
 b = 1;
 ...
 return;
}
```

Given a Boolean asynchronous program and a control location in a handler

Is there an execution which reaches the control location?

We do not care about the task buffer

- Handlers cannot take decisions based on the contents of the task buffer

Properties: Termination

```
global bit b = 0;
```

```
main(){
 ...
 async h1();
 ...
}
```

```
h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}
```

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

Given a Boolean asynchronous program,
Does it terminate?

- main *does not* terminate on all runs
  - What if h1 is chosen over h2 forever?

Fairness

```
global bit b = 0;
```

```
main(){
 ...
 async h1();
 ...
}
```

```
h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}
```

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

An infinite execution is *fair* if

- For every handler *h* that is pending
- The scheduler eventually picks and runs an instance of *h*

- So: the run choosing h1 over h2 always is not fair

- Will focus on *fair runs*
  - *Captures the intuition that scheduling is fair*

Asynchronous Program Analysis

Main Result:

Safety and Liveness decidable for asynchronous programs

[SenViswanathan06,JhalaM07,GantyMRybalchenko09,GantyM09]

This is hard because: Not finite state or context free:

- potentially unbounded stacks,
- potentially unbounded request queue

This is useful because:

- Asynchronous programs used in many correctness-critical settings
- The style breaks up control flow, making it difficult to reason about code

First Attempt: Does not Work

- Treat asynchronous calls as synchronous and use interprocedural reachability

- Why?
  - Global state can change between posting a task and executing it

First Attempt: Does not Work

```
global bit b = 0;
```

```
main(){
 ...
 b = 0;
 async p1(); async p2();
}
```

```
p1(){
  assert(b==0);
}
```

```
p2(){
 b = 1;
 return;
}
```

- In synchronous computation, the assert holds
- In asynchronous computation, p2 can execute before p1

Second Attempt

```
global bit b = 0;
```

```
main(){
 ...
 async h1();
 ...
}
```

```
h1(){
 if(b == 0){
   async h1();

   async h2();

   return;
 }

}
```

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

Reduce to sequential analysis

- *Add a counter for each handler (tracks number of pending instances)*
- *An async call increments counter*
- *Dispatch loop chooses a non-zero counter, decrements it and runs corresponding handler*

Second Attempt

```
global bit b = 0;
       ch1=0,ch2=0;
```

```
main(){
 ...
 ch1++;

 while(ch1>0||ch2>0){
   pick h s.t. ch > 0
   ch--;
   h();
 }
}
```

```
h1(){
 if(b == 0){
   ch1++;
   ch2++;
   return;
 }
}
```

```
h2(){
 ...
 b = 1;
 ...
 return;
}
```

Reduce to sequential analysis

- *Add a counter for each handler (tracks number of pending instances)*
- *An async call increments counter*
- *Dispatch loop chooses a non-zero counter, decrements it and runs corresponding handler*

- *Sound, but decidability not obvious*
  - *In general, analyses undecidable for sequential programs with counters*

Technique

1. Convert an AP to one without recursion

2. Convert recursion-free AP to Petri nets

Removing the Stack

- Need to summarize the effect of handlers on the task buffer
  - Observation: Just the *number* of async calls matter, *not the order* in which they are made

- Parikh's Lemma: For every *context free* language L, there is a *regular* language L' such that:
  - For every w in L, there is a permutation of w in L' and conversely

- So: can replace a recursive handler with a non-recursive one while maintaining the summary

Example

```
H() {
  if (*) {
    async a();
    H();
    async b()
  }
}
```

```
H() {
  while (*) {
    async a();
    async b();
  }
}
```

This gets rid of unbounded stacks in handlers
From now on, only consider *recursion-free AP*

But there is a second source of unboundedness (the task buffer)

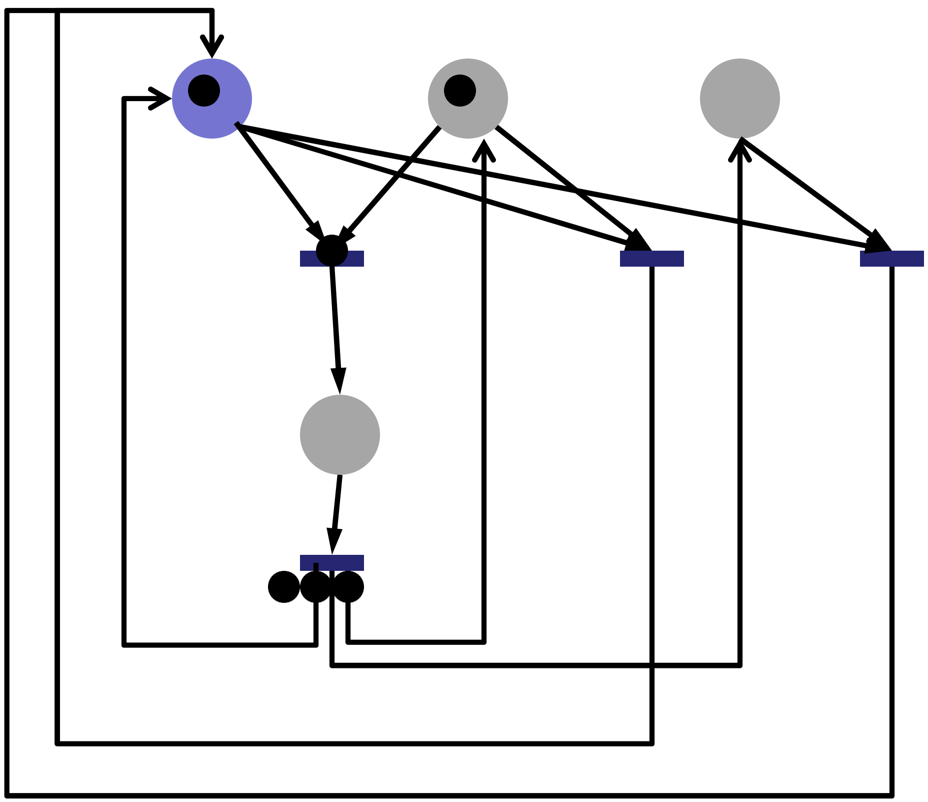
Petri Nets

Can convert an asynchronous program into a *Petri net (PN)* s.t. every execution of the async program is equivalent to an execution of the Petri net

In particular:

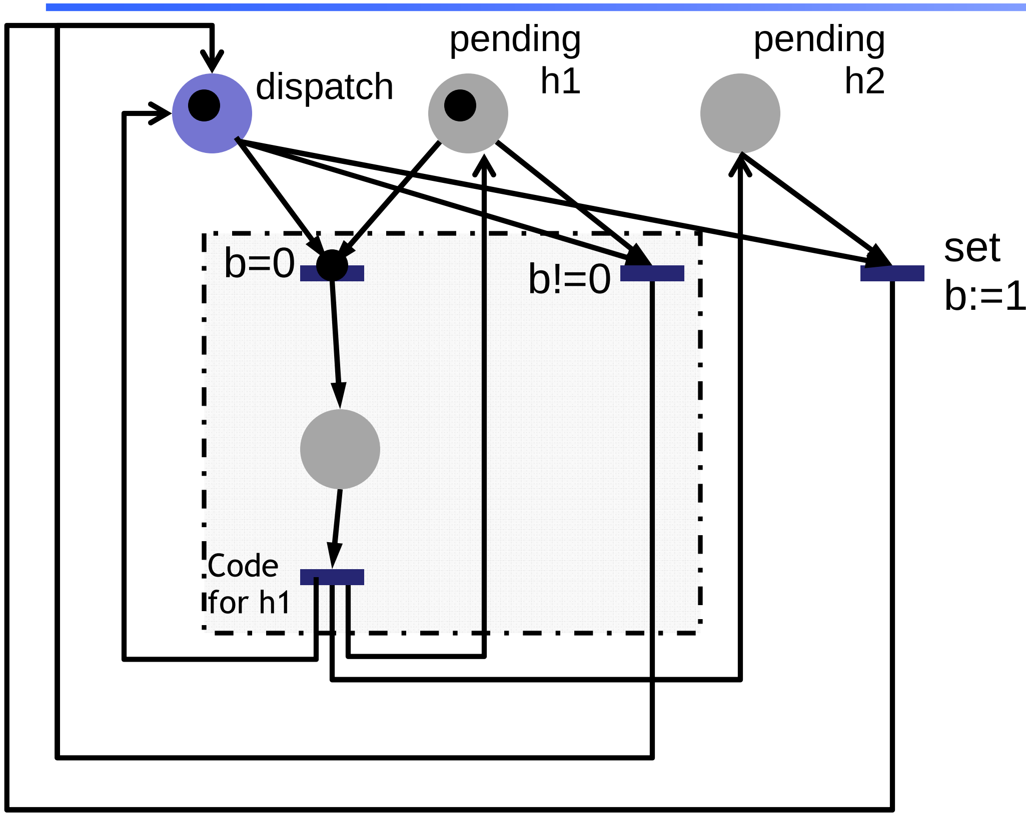
- The asynchronous program is safe iff the Petri net is coverable
- The asynchronous program fairly terminates iff the Petri net has no fair infinite runs

Petri Nets 101



- Set of places
- Set of transitions
- Places marked with tokens
- Transition function takes tokens from sources of transitions to destinations
- With an initial marking, defines an infinite state system
- But with good decidability properties

Petrification




```
global bit b = 0;

main(){
  ...
  async h1();
  ...
}

h1(){

  if(b == 0){
    async h1();

    async h2();

    return;
  }

}

h2(){
  ...
  b = 1;
  ...
  return;
}
```

Tokens:

- Control flow in each handler
- Pending handler calls

That's Good, Because…

Can apply algorithmic results on Petri Nets to reason about async programs

- Many strong decidability results

Coverability Problem for PN

- Given Petri net P
  - (Places, transitions, initial marking)
- Target marking m
  - Places of P -> tokens

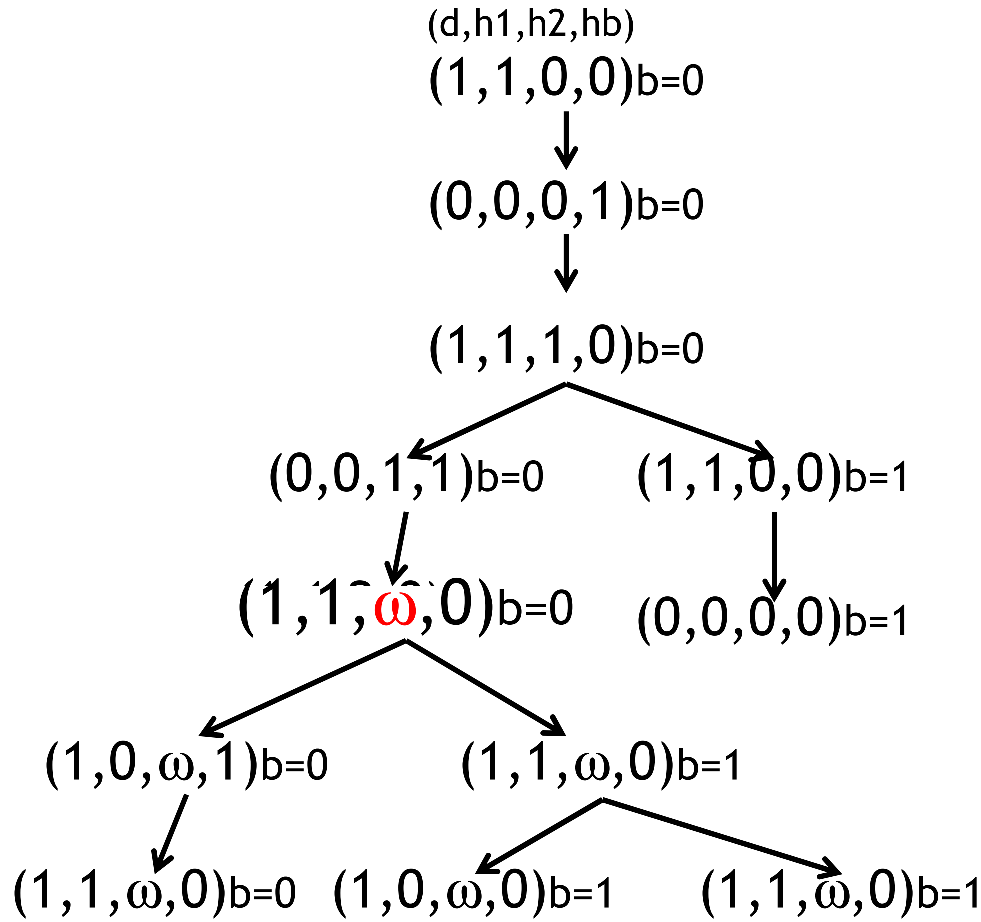
- Is there an execution of P to a marking m' with $m \leq m'$?
  - $m \leq m'$ if $m(p) \leq m'(p)$ for each place

Karp Miller Tree

- Build a tree with markings as nodes
- Transitions as edges

- Infinite tree
- But accelerate

- Terminates by Dickson's Lemma



Safety Verification

Corollary to Coverability Graph: Safety verification decidable

- Remove recursion using Parikh's lemma
- Build Petri net, construct coverability tree
- Check if there is a node in the coverability tree with a non-zero entry in the special location

- Similar proof in [SenViswanathan06] using multiset rewrite systems
- Independent, alternate algorithm in [JhalaM07]

Boundedness Verification

An AP is bounded if there is some N such that the task buffer always has fewer than N tasks

Reduce to checking boundedness of PN

- Algorithm based on coverability tree construction

Fair Termination

- Given:
  - An asynchronous program with Boolean variables
- Check:
  - There is no fair infinite run
- Two checks:

  (a)Each called handler terminates

  (b)There is no infinite fair execution of handlers

LTL model checking for pushdown systems [Walukiewicz,Steffen]

Hardness: PN Reachability

- Given Petri net P
  - (Places, transitions, initial marking)
- Target marking m
  - Places of P -> tokens
- Is there an execution of P to marking m?

- Known: Decidable, EXPSPACE-hard
- Best known algorithms: non-primitive recursive space

From FT to PN Reachability

- Given PN and place p, construct AP such that AP has fair infinite run iff PN reaches a marking where p has no tokens
  - Known to be equivalent to PN reachability
- Simulate a PN using an AP
  - One task for each place
  - One task to simulate transitions
- Post a task to guess when p has no tokens

Idea of Reduction

- Simulate a PN using an AP
  - One task for each place
  - One task to simulate transitions

- Post a task to guess when p has no tokens
  - If task for p is scheduled after guess, then guess was wrong, so shut down program
  - Otherwise execute a simple infinite loop

Fairness ensures the guess is eventually made and eventually falsified (if wrong)

Fair Termination is Decidable

- First part the same:
  - Remove stack using Parikh’s lemma
  - Construct Petri net
- Use a logic over Petri nets [Yen90] to encode the fairness condition
  - Complexity of the logic: polynomial reduction to PN reachability
  - Note: Original paper [Yen90] had a mistake

Main Result

Theorem: [GantyM.09]

Checking safety and fair termination is decidable for asynchronous programs

Safety EXPSPACE-complete for asynchronous automata

- Using a more careful construction

Observations

- For async programs, fair termination computable
  - Unlike threads [Ramalingam 00]
  - Known that threads and events are dual [NeedhamLauer]

  - So what gives?
  - We restrict global state to be finite!
  - A thread can encode unbounded information in its stack

Topic 2

Parameterized Verification of Software

A Simple Example

- Global int ctr = 0;

- Arbitrary number of threads

- Each thread executes:

```
ctr = ctr + 1;
assert (ctr  > 0)
ctr = ctr – 1;
```

Show assertion holds

Parameterized Verification

- Intuitively, the assertion holds
- For any fixed number N of threads, can prove the assertion by finite state model checking
  - Ctr bounded by N
- How can we prove the system for *any number of threads*?

Idea: Temporal Case Splitting

- To show the property for all threads,
  - fix an arbitrary thread, and
  - prove the property for that thread

- Abstraction: Consider thread tid, and environment "not tid" for all threads whose id is not tid

- In general, has Skolem variables and abstract system into Skolems and others

Conclusion

Verification by Theorem Proving

```
Example ( ) {
1: do{
        lock();
        old = new;
        q = q->next;
2:      if (q != NULL){
3:          q->data = new;
            unlock();
            new ++;
        }
4: } while(new != old);
5:   unlock ();
     return;
}
```

1. Loop Invariants
2. Logical formula
3. Check Validity

Invariant:

$$lock \wedge new = old$$

$$\vee$$

$$\neg lock \wedge new \neq old$$

Verification by Theorem Proving

```
Example ( ) {
1: do{
      lock();
      old = new;
      q = q->next;
2:    if (q != NULL){
3:       q->data = new;
         unlock();
         new ++;
      }
4: } while(new != old);
5:  unlock ();
    return;
}
```

1. Loop Invariants
2. Logical formula
3. Check Validity

- Loop Invariants
- Multithreaded Programs
+ Behaviors encoded in logic
+ Decision Procedures

Precise [ESC]

Verification by Program Analysis

```
Example ( ) {
1: do{
        lock();
        old = new;
        q = q->next;
2:      if (q != NULL){
3:          q->data = new;
            unlock();
            new ++;
        }
4: } while(new != old);
5:   unlock ();
     return;
}
```

1. Dataflow Facts
2. Constraint System
3. Solve constraints

- Imprecision due to fixed facts

+ Abstraction

+ Type/Flow Analyses

Scalable [CQUAL, ESP, MC]

Verification by **Model Checking**

```
Example ( ) {
1: do{
       lock();
       old = new;
       q = q->next;
2:     if (q != NULL){
3:         q->data = new;
           unlock();
           new ++;
       }
4: } while(new != old);
5:   unlock ();
     return;
}
```

1. (Finite State) Program
2. State Transition Graph
3. Reachability

- Pgm $\rightarrow$ Finite state model
- State explosion

\+ State Exploration

\+ Counterexamples

Precise [SPIN, SMV, Bandera,JPF]

Combining Strengths

Theorem Proving

- loop invariants

+ Behaviors encoded in logic

Refine

+ Theorem provers

Computing Successors,Refine

Program Analysis

- Imprecise

+ Abstraction

Shrink state space

Lazy Abstraction

Model Checking

- Finite-state model, state explosion

+ State Space Exploration

Path Sensitive Analysis

+ Counterexamples

Finding Relevant Facts

Toward More Reliable Systems

- Tools and theory are equally important
  - Tools reinforce process change, amplify programmer productivity, ensure quality
  - Theoretical progress drives better tools
  - Applications motivate good theory

- System development is too complex to be reduced to a single problem or solution
  - Model checking is not the only tool
  - Just tools will not make software perfect
  - How can developers and verifiers interact better?

Conclusions

- Software systems need to be more reliable
  - Enormous resources spent in stabilizing software
- Long way to go
  - Inadequate languages, inaccurate tools
  - Make system developers and system verifiers interact
- Tools, by themselves, are not enough
  - Tools and analysis is one view, but social aspects are also important

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