Program Analysis, January–April 2011

Assignment 2, due 15 May 2011

Instructions for submitting solutions

Please submit your solutions electronically by email to facilitate evaluation (I am travelling and will not be able to pick up physical submissions).

Preferrably, send an electronic document in PDF (you can generate it in LaTeX or OpenOffice or Word or whatever, but send only PDF). If you can't do this, send scanned copies of handwritten pages.

All exercises are taken from *Principles of Program Analysis* by Flemming Nielson, Hanne Riis Nielson and Chris Hankin. Cross check if you think there are typos!

1. **Exercise 4.6**

   Show that all of

   - $\sqcup$
   - $\lambda(l_1, l_2).\top$
   - $\lambda(l_1, l_2).\begin{cases} l_1 & \text{if } l_2 \sqsubseteq l_1 \\ \top & \text{otherwise} \end{cases}$
   - $\lambda(l_1, l_2).\begin{cases} l_2 & \text{if } l_1 = \bot \\ l_1 & \text{if } l_2 \sqsubseteq l_1 \wedge l_1 \neq \bot \\ \top & \text{otherwise} \end{cases}$
   - $\lambda(l_1, l_2).\begin{cases} l_1 \sqcup l_2 & \text{if } l_1 \sqsubseteq l' \vee l_2 \sqsubseteq l_1 \\ \top & \text{otherwise} \end{cases}$

   are upper bound operators (where l' is some element of L). Determine which of them are also widening operators. Try to find sufficient conditions on l' such that the operator involving l' is a widening operator.

2. **Exercise 4.7**

   Show that if L satisfies the Ascending Chain Condition[1] then an operator on L is a widening operator if and only if it is an upper bound operator. Conclude that if L satisfies the Ascending Chain Condition then the least upper bound operator $\sqcup : L \times L \to L$ is a widening operator.

[1] **Ascending Chain Condition:** Each ascending chain $l_1 \sqsubseteq l_2 \sqsubseteq l_3 \sqsubseteq \cdots$ eventually stabilizes. That is, $\exists n : l_n = l_{n+1} = \cdots$.

3. **Exercise 4.18**

   Let $(\mathcal{P}(V_1), \alpha_1, \gamma_1, \mathcal{P}(D_1))$ and $(\mathcal{P}(V_2), \alpha_2, \gamma_2, \mathcal{P}(D_2))$ be Galois insertions. Define

$$\begin{array}{rcl} \alpha(VV) & = & \bigcup\{\alpha_1(\{v_1\}) \times \alpha_2(\{v_2\}) \mid (v_1, v_2) \in VV\} \\ \gamma(DD) & = & \{(v_1, v_2) \mid \alpha_1(\{v_1\}) \times \alpha_2(\{v_2\}) \subseteq DD\} \end{array}$$

   and determine whether or not $(\mathcal{P}(V_1 \times V_2), \alpha, \gamma, \mathcal{P}(D_1 \times D_2))$ is a Galois insertion.

4. **Exercise 4.19**

   Let $(L, \alpha_1, \gamma_1, M_1)$ and $(L, \alpha_2, \gamma_2, M_2)$ be Galois insertions. Define

$$\begin{array}{rcl} \alpha(l) & = & (\alpha_1(l), \alpha_2(l)) \\ \gamma(m_1, m_2) & = & \gamma(m_1) \sqcap \gamma(m_2) \end{array}$$

   and determine whether or not $(L, \alpha, \gamma, M_1 \times M_2)$ is a Galois insertion.